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Algebraic Reasoning – Mathematics

TEKS	Supporting Information
(a) General requirements. Students shall be awarded one credit for successful completion of this course. Prerequisite: Algebra I.	The TEKS include descriptions of prerequisite coursework. Algebra I is a required prerequisite.
(b) Introduction. (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.	A well-balanced mathematics curriculum includes the Texas College and Career Readiness Standards (CCRS). A focus on mathematical fluency and solid understanding allows for rich exploration of the key ideas of Algebraic Reasoning.
(b) Introduction. (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.	This paragraph occurs second in the TEKS, preceding the content descriptions. This highlights the emphasis of student use of the mathematical process standards to acquire and demonstrate mathematical understanding. This introductory paragraph includes generalization and abstraction in the text from AR(1)(B). This introductory paragraph includes computer programs in the text from AR(1)(C). This introductory paragraph states, "Students will use mathematical relationships to generate solutions and make connections and predictions," instead of the text from AR(1)(E).
(b) Introduction. (3) In Algebraic Reasoning, students will build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I, continue with the development of mathematical reasoning related to algebraic understandings and processes, and deepen a foundation for studies in subsequent mathematics courses. Students will broaden their knowledge of functions and relationships, including linear, quadratic, square root, rational, cubic, cube root, exponential, absolute value, and logarithmic functions. Students will study these functions through analysis and application that includes explorations of patterns and structure, number and algebraic methods, and modeling from data using tools that build to workforce and college readiness such as probes, measurement tools, and software tools, including spreadsheets.	Specifics about Algebraic Reasoning mathematics content is summarized in this paragraph. This summary follows the paragraph about the mathematical process standards. This supports the notion that the TEKS should be learned in a way that integrates the mathematical process standards in an effort to develop fluency. The paragraph also connects the key concepts found in Algebraic Reasoning to prior content and the CCRS.
(b) Introduction. (4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.	The State Board of Education approved the retention of some "such as" statements within the TEKS where needed for clarification of content. The phrases "including" and "such as" should not be considered as limiting factors for the student expectations (SEs) in which they reside.

Additional Resources are available online including

[Vertical Alignment Charts](#)

[Texas Mathematics Resource Page](#)

[Texas College and Career Readiness Standards](#)

Algebraic Reasoning – Mathematics

TEKS: Mathematical Process Standards.

Supporting Information

AR(1)(A) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to apply mathematics to problems arising in everyday life, society, and the workplace.

This SE emphasizes application. The opportunities for application have been consolidated into three areas: everyday life, society, and the workplace.

This SE, when paired with a content SE, allows for increased relevance through connections within and outside mathematics. *Example:* When paired with AR(3)(D), the student may be asked to determine the function in order to calculate the cost and tax of an item and then combine them using arithmetic operation to create a new function that models this situation.

AR(1)(B) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

This process standard applies the same problem-solving model and is included in the TEKS for kindergarten through grade 12.

This is the traditional problem-solving process used in mathematics and science. Students may be expected to use this process in a grade-appropriate manner when solving problems that can be considered difficult relative to mathematical maturity.

AR(1)(C) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

The phrase “as appropriate” indicates that students are assessing which tools and techniques to apply rather than trying only one or all of those listed. *Example:* When paired with AR(7)(E), students may be expected to determine if a linear function models a situation by choosing from a variety of techniques and tools.

AR(1)(D) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Students may be expected to address three areas: mathematical ideas, reasoning, and implications of these ideas and reasoning.

Communication can be through the use of symbols, diagrams, graphs, or language. The phrase “as appropriate” implies that students may be expected to assess which communication tool to apply rather than trying only one or all of those listed.

The use of multiple representations includes translating and making connections among the representations. *Example:* When paired with AR(3)(C), students may be expected to determine if two functions are inverses using symbols, tables, and graphs.

AR(1)(E) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to create and use representations to organize, record, and communicate mathematical ideas.

The expectation is that students use representations for three purposes: to organize, record, and communicate mathematical ideas.

Representations include verbal, graphical, tabular, and algebraic representations. As students create and use representations, the students will evaluate the effectiveness of the representations to ensure that those representations are communicating mathematical ideas with clarity. *Example:* When paired with AR(2)(D), students may be expected to organize various data to approximate the rate of change in a given context.

AR(1)(F) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to analyze mathematical relationships to connect and communicate mathematical ideas.

Students may be expected to analyze relationships and form connections with mathematical ideas.

Students may form conjectures about mathematical representations based on patterns or sets of examples and non-examples. Forming connections with mathematical ideas extends past conjecturing to include verification through a deductive process. *Example:* When paired with AR(5)(E), students may be expected to develop a matrix based upon a situation before providing a solution.

AR(1)(G) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

The expectation is that students speak and write with precise mathematical language to explain and justify the work. This includes justifying a solution. *Example:* When paired with AR(6)(C), the student may be expected to justify a solution to a real-world application of exponential, logarithmic, square root, or cubic functions.

Algebraic Reasoning – Mathematics

TEKS: Patterns and structure.

Supporting Information

AR(2)(A) **Patterns and structure.** The student applies mathematical processes to connect finite differences or common ratios to attributes of functions.

The student is expected to determine the patterns that identify the relationship between a function and its common ratio or related finite differences as appropriate, including linear, quadratic, cubic, and exponential functions.

AR(2)(B) **Patterns and structure.** The student applies mathematical processes to connect finite differences or common ratios to attributes of functions.

The student is expected to classify a function as linear, quadratic, cubic, and exponential when a function is represented tabularly using finite differences or common ratios as appropriate.

AR(2)(C) **Patterns and structure.** The student applies mathematical processes to connect finite differences or common ratios to attributes of functions.

The student is expected to determine the function that models a given table of related values using finite differences and its restricted domain and range.

AR(2)(D) **Patterns and structure.** The student applies mathematical processes to connect finite differences or common ratios to attributes of functions.

The student is expected to determine a function that models real-world data and mathematical contexts using finite differences such as the age of a tree and its circumference, figurative numbers, average velocity, and average acceleration.

This SE builds on the introduction of slope in grade 8 [8(4)(A), (B), and (C)].

AR(2)(A), (B), (C), and (D) introduce the student to the concept of average rate of change for non-linear functions and provide the ground work for the concept of a derivative in Calculus courses.

Students may be expected to determine the common ratio, first finite difference, second finite difference, and third finite difference to appropriately classify a table of points as fitting one of the four listed function families.

When paired with AR(1)(G), students may be expected to justify their choice.

When paired with AR(1)(D), students may be expected to demonstrate first differences graphically, in much the same manner as for slope in grade 8 [8(4)(A)].

When paired with AR(1)(A), students are expected to apply the concept of average rate of change to a context that can be modeled by both linear and non-linear functions.

Algebraic Reasoning – Mathematics

TEKS: Patterns and structure.

Supporting Information

AR(3)(A) **Patterns and structure.** The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e ; functions and their inverses; and key attributes of these functions.

The student is expected to compare and contrast the key attributes, including domain, range, maxima, minima, and intercepts, of a set of functions such as a set comprised of a linear, a quadratic, and an exponential function or a set comprised of an absolute value, a quadratic, and a square root function tabularly, graphically, and symbolically.

AR(3)(B) **Patterns and structure.** The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e ; functions and their inverses; and key attributes of these functions.

The student is expected to compare and contrast the key attributes of a function and its inverse when it exists, including domain, range, maxima, minima, and intercepts, tabularly, graphically, and symbolically.

AR(3)(C) **Patterns and structure.** The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e ; functions and their inverses; and key attributes of these functions.

The student is expected to verify that two functions are inverses of each other tabularly and graphically such as situations involving compound interest and interest rate, velocity and braking distance, and Fahrenheit-Celsius conversions.

AR(3)(D) **Patterns and structure.** The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e ; functions and their inverses; and key attributes of these functions.

The student is expected to represent a resulting function tabularly, graphically, and symbolically when functions are combined or separated using arithmetic operations such as combining a 20% discount and a 6% sales tax on a sale to determine $h(x)$, the total sale, $f(x) = 0.8x$, $g(x) = 0.06(0.8x)$, and $h(x) = f(x) + g(x)$.

AR(3)(E) **Patterns and structure.** The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e ; functions and their inverses; and key attributes of these functions.

The student is expected to model a situation using function notation when the output of one function is the input of a second function such as determining a function $h(x) = g(f(x)) = 1.06(0.8x)$ for the final purchase price, $h(x)$ of an item with price x dollars representing a 20% discount, $f(x) = 0.8x$ followed by a 6% sales tax, $g(x) = 1.06x$.

In this SE, students may be expected to compare the attributes from functions of different families whereas in Algebra II, students are simply examining the attributes of a given function [2A(2)(A)].

When paired with AR(1)(B) and (G), students may be expected to contrast attributes of several functions and their inverses to identify common relationships such as a function and its inverse being reflections across the line $y = x$.

Students are not expected to use composition as in 2A(2)(D). Students may be expected to recognize the juxtaposition of x and y values of functions and their inverse represented either in a graph or a table.

Students are expected to create new functions by using arithmetic operations.

This SE is more general than those in Algebra I and Algebra II, where students are expected to add, subtract, multiply, and divide polynomials.

This SE can be considered the inverse of AR(3)(F), constructing new functions as opposed to deconstructing new functions.

Students may or may not be asked to construct the new function symbolically first.

This SE provides the groundwork for composition found in Precalculus [P(2)(A), (B), and (C)].

This SE focuses on the individual domain values as they proceed through each layer of a composed function to reach the corresponding range values.

When paired with AR(1)(D), students may represent this composition in a series of three or more tables.

Algebraic Reasoning – Mathematics

TEKS: Patterns and structure.

AR(3)(F) **Patterns and structure.** The student applies mathematical processes to understand the connections among representations of functions and combinations of functions, including the constant function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 10 or e ; functions and their inverses; and key attributes of these functions.

The student is expected to compare and contrast a function and possible functions that can be used to build it tabularly, graphically, and symbolically such as a quadratic function that results from multiplying two linear functions.

Supporting Information

This SE can be considered the inverse of AR(3)(D), deconstructing new functions as opposed to constructing new functions. For example, if $f(x) = 2x - 3$ and $g(x) = 3x - 2$, students may be asked to compare the x -intercepts of these two functions with those of $h(x) = (2x - 3)(3x - 2)$.

Algebraic Reasoning – Mathematics

TEKS: Number and algebraic methods.

Supporting Information

AR(4)(A) **Number and algebraic methods.** The student applies mathematical processes to simplify and perform operations on functions represented in a variety of ways, including real-world situations.

The student is expected to connect tabular representations to symbolic representations when adding, subtracting, and multiplying polynomial functions arising from mathematical and real-world situations such as applications involving surface area and volume.

AR(4)(B) **Number and algebraic methods.** The student applies mathematical processes to simplify and perform operations on functions represented in a variety of ways, including real-world situations.

The student is expected to compare and contrast the results when adding two linear functions and multiplying two linear functions that are represented tabularly, graphically, and symbolically.

AR(4)(C) **Number and algebraic methods.** The student applies mathematical processes to simplify and perform operations on functions represented in a variety of ways, including real-world situations.

The student is expected to determine the quotient of a polynomial function of degree three and of degree four when divided by a polynomial function of degree one and of degree two when represented tabularly and symbolically.

AR(4)(D) **Number and algebraic methods.** The student applies mathematical processes to simplify and perform operations on functions represented in a variety of ways, including real-world situations.

The student is expected to determine the linear factors of a polynomial function of degree two and of degree three when represented symbolically and tabularly and graphically where appropriate.

This SE focuses on the relationship between the symbolic representation and both the effects of the arithmetic combination of two or more polynomials as well as the effect on an assortment of points with the same x -values evaluated using all of the functions involved. The student may be expected to examine the relationship of the points in the table below.

x	$l = x$	$w = 2x$	$h = 3x$	$V = 6x^3$
1	1	2	3	6
2	2	4	6	48
3	3	6	9	162
4	4	8	12	384
5	5	10	15	750

This SE focuses on different representations of combinations of linear functions as opposed to the Algebra I skills, which focus on the arithmetic operations [A(10)(A) and (B)].

This SE extends the Algebra I skill to include higher degree polynomial dividends but also adds the examination of the relationship of the corresponding range values for the dividend, divisor, and quotient in multiple representations [A(10)(C)].

This SE extends the Algebra I skills but also adds the examination of the relationship of the corresponding range values for the factors and product in multiple representations [A(10)(E) and (F)].

Algebraic Reasoning – Mathematics

TEKS: Number and algebraic methods.

Supporting Information

AR(5)(A) **Number and algebraic methods.** The student applies mathematical processes to represent, simplify, and perform operations on matrices and to solve systems of equations using matrices.

This is the first SE where students are expected to combine matrices arithmetically.

The student is expected to add and subtract matrices.

AR(5)(B) **Number and algebraic methods.** The student applies mathematical processes to represent, simplify, and perform operations on matrices and to solve systems of equations using matrices.

This is the first SE where students are expected to multiply matrices.

The student is expected to multiply matrices.

AR(5)(C) **Number and algebraic methods.** The student applies mathematical processes to represent, simplify, and perform operations on matrices and to solve systems of equations using matrices.

When paired with AR(1)(G) and AR(5)(C), students may be expected to explain the difference in matrix multiplication and scalar multiplication.

The student is expected to multiply matrices by a scalar.

AR(5)(D) **Number and algebraic methods.** The student applies mathematical processes to represent, simplify, and perform operations on matrices and to solve systems of equations using matrices.

When paired with AR(1)(A), students may be expected to demonstrate how multiplying by a scalar is used in a real-world application such as computer animation.

When paired with AR(1)(G) and AR(5)(B), students may be expected to explain the difference in matrix multiplication and scalar multiplication.

The student is expected to represent and solve systems of two linear equations arising from mathematical and real-world situations using matrices.

This SE extends A(5)(C) to include the technique of solving systems of equations with matrices.

AR(5)(E) **Number and algebraic methods.** The student applies mathematical processes to represent, simplify, and perform operations on matrices and to solve systems of equations using matrices.

This SE extends AR(5)(D) to three equations with three unknowns.

The student is expected to represent and solve systems of three linear equations arising from mathematical and real-world situations using matrices and technology.

Algebraic Reasoning – Mathematics

TEKS: Number and algebraic methods.

AR(6)(A) **Number and algebraic methods.** The student applies mathematical processes to estimate and determine solutions to equations resulting from functions and real-world applications with fluency.

The student is expected to estimate a reasonable input value that results in a given output value for a given function, including quadratic, rational, and exponential functions.

AR(6)(B) **Number and algebraic methods.** The student applies mathematical processes to estimate and determine solutions to equations resulting from functions and real-world applications with fluency.

The student is expected to solve equations arising from questions asked about functions that model real-world applications, including linear and quadratic functions, tabularly, graphically, and symbolically.

AR(6)(C) **Number and algebraic methods.** The student applies mathematical processes to estimate and determine solutions to equations resulting from functions and real-world applications with fluency.

The student is expected to approximate solutions to equations arising from questions asked about exponential, logarithmic, square root, and cubic functions that model real-world applications tabularly and graphically.

Supporting Information

This SE is a precursor to solving quadratic, rational, and exponential functions and can be thought of as finding the intersection of one of the listed curves and a horizontal line.

Solutions may be irrational.

When paired with AR(1)(C), students may be expected to choose the tool(s) necessary to find a solution such as the bisection method.

This SE along with AR(6)(A) and (C) allow students the opportunity to be introduced to simplistic numerical analysis techniques such as, but not limited to, the bisection method.

Solutions may be irrational.

When paired with AR(1)(C), students may be expected to choose the tool(s) necessary to find a solution such as the bisection method.

Algebraic Reasoning – Mathematics

TEKS: Modeling from data.

Supporting Information

AR(7)(A) **Modeling from data.** The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions.

This SE extends the Algebra I skills to include interval notation $[A(2)(A), A(6)(A), A(9)(A)]$.

The student is expected to represent domain and range of a function using interval notation, inequalities, and set (builder) notation.

There is no limit to the family of functions that can be represented for this SE.

AR(7)(B) **Modeling from data.** The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions.

This SE extends AR(7)(A) to include the difference between the mathematical and reasonable domain and range as dictated by a given real-world situation. For example, a linear function that models the number of coats produced may have a mathematical domain of all real numbers, but a reasonable domain of nonnegative integers.

The student is expected to compare and contrast between the mathematical and reasonable domain and range of functions modeling real-world situations, including linear, quadratic, exponential, and rational functions.

AR(7)(C) **Modeling from data.** The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions.

This SE is related to AR(2)(A), (B), (C), and (D). In this case, the focus is on data and application of the average rate of change.

The student is expected to determine the accuracy of a prediction from a function that models a set of data compared to the actual data using comparisons between average rates of change and finite differences such as gathering data from an emptying tank and comparing the average rate of change of the volume or the second differences in the volume to key attributes of the given model.

AR(7)(D) **Modeling from data.** The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions.

This SE extends AR(2)(C) and AR(7)(C) to include real-world situations.

The student is expected to determine an appropriate function model, including linear, quadratic, and exponential functions, for a set of data arising from real-world situations using finite differences and average rates of change.

AR(7)(E) **Modeling from data.** The student applies mathematical processes to analyze and model data based on real-world situations with corresponding functions.

This SE may include non-linear data.

The student is expected to determine if a given linear function is a reasonable model for a set of data arising from a real-world situation.