Mathematics TEKS
SUPPORTING INFORMATION

Mathematical Models with Applications

Texas Education Agency

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TEKS	Supporting Information
(a) General requirements. Students can be awarded one-half to one credit for successful completion of this course. Prerequisite: Algebra I.	The TEKS include descriptions of prerequisite coursework. Algebra I is a required prerequisite.
(b) Introduction. (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.	A well-balanced mathematics curriculum includes the Texas College and Career Readiness Standards. A focus on mathematical fluency and solid understanding allows for rich exploration of the key ideas of Mathematical Models with Applications.
(b) Introduction. (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.	This paragraph occurs second in the TEKS, preceding the content descriptions. This highlights the emphasis of student use of the mathematical process standards to acquire and demonstrate mathematical understanding. The concept of generalization and abstraction in the text from M(1)(B) included in the introductory paragraphs from elementary TEKS may be considered subsumed in this language. Computer programs may be included under technology in the text from M(1)(C). This introductory paragraph states, "Students will use mathematical relationships to generate solutions and make connections and predictions," instead of the text from M(1)(E).
(b) Introduction. (3) Mathematical Models with Applications is designed to build on the knowledge and skills for mathematics in Kindergarten-Grade 8 and Algebra I. This mathematics course provides a path for students to succeed in Algebra II and prepares them for various post-secondary choices. Students learn to apply mathematics through experiences in personal finance, science, engineering, fine arts, and social sciences. Students use algebraic, graphical, and geometric reasoning to recognize patterns and structure, model information, solve problems, and communicate solutions. Students will select from tools such as physical objects; manipulatives; technology, including graphing calculators, data collection devices, and computers; and paper and pencil and from methods such as algebraic techniques, geometric reasoning, patterns, and mental math to solve problems.	Specifics about Mathematical Models with Applications mathematics content is summarized in this paragraph. This summary follows the paragraph about the mathematical process standards. This supports the notion that the TEKS should be learned in a way that integrates the mathematical process standards in an effort to develop fluency. The paragraph also connects the key concepts found in Mathematical Models with Applications to prior content and the Texas College and Career Readiness Standards.

(b) Introduction. (4) In Mathematical Models with Applications, students will use a mathematical modeling cycle to analyze problems, understand problems better, and improve decisions. A basic mathematical modeling cycle is summarized in this paragraph. The student will: (A) represent: (i) identify the variables in the problem and select those that represent essential features; and (ii) formulate a model by creating and selecting from representations such as geometric, graphical, tabular, algebraic, or statistical that describe the relationships between the variables; (B) compute: analyze and perform operations on the relationships between the variables to draw conclusions; (C) interpret: interpret the results of the mathematics in terms of the original problem; (D) revise: confirm the conclusions by comparing the conclusions with the problem and revising as necessary; and (E) report: report on the conclusions and the reasoning behind the conclusions.	This paragraph provides general statements about Mathematical Models with Applications and the use of these TEKS.
(b) Introduction. (5) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.	The State Board of Education approved the retention of some "such as" statements within the TEKS for clarification of content. The phrases "including" and "such as" should not be considered as limiting factors for the student expectations (SEs) in which they reside.

Additional Resources are available online including

- Vertical Alignment Charts
- Texas Mathematics Resource Page
- Texas College and Career Readiness Standards

TEKS: Mathematical Process Standards.	Supporting Information
M(1)(A) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to apply mathematics to problems arising in everyday life, society, and the workplace.**	This SE emphasizes application. The opportunities for application have been consolidated into three areas: everyday life, society, and the workplace. This SE, when paired with a content SE, allows for increased relevance through connections within and outside mathematics. *Example*: When paired with M(6)(B), the student may be asked to determine how the volume is affected after changing the height of a cylindrical tank by 6 feet as opposed to changing both the radius and height by 3 feet each.
M(1)(B) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.**	This process standard applies the same problem-solving model and is included in the TEKS for kindergarten through grade 12. This is the traditional problem-solving process used in mathematics and science. Students may be expected to use this process in a grade appropriate manner when solving problems that can be considered difficult relative to mathematical maturity.
M(1)(C) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.**	The phrase "as appropriate" indicates that students are assessing which tools and techniques to apply rather than trying only one or all of those listed. *Example:* When paired with M(5)(C), the student is expected to choose the appropriate tool(s), which may include the use of technology-based regression tools to model motion using quadratic functions.
M(1)(D) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**	Students may be expected to address three areas: mathematical ideas, reasoning, and implications of these ideas and reasoning. Communication can be using symbols, diagrams, graphs, or language. The phrase "as appropriate" implies that students may be expected to assess which communication tool to apply rather than trying only one or all of those listed. The use of multiple representations includes translating and making connections among the representations. *Example*: When paired with M(2)(B), students may be expected to communicate their solution processes using multiple representations.
M(1)(E) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to create and use representations to organize, record, and communicate mathematical ideas.**	The expectation is that students use representations for three purposes: to organize, record, and communicate mathematical ideas. Representations include verbal, graphical, tabular, and algebraic representations. As students create and use representations, the students will evaluate the effectiveness of the representations to ensure that those representations are communicating mathematical ideas with clarity. *Example*: When paired with M(3)(D), students may be expected to organize amortization tables for various vehicles in an effort to compare value.
M(1)(F) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to analyze mathematical relationships to connect and communicate mathematical ideas.**	Students may be expected to analyze relationships and form connections with mathematical ideas. Students may form conjectures about mathematical representations based on patterns or sets of examples and non-examples. Forming connections with mathematical ideas extends past conjecturing to include verification through a deductive process. *Example*: When paired with M(8)(A), students may be expected to communicate the Fundamental Counting Principle and its use to determine how many couples are possible with 36 distinct tennis players.
M(1)(G) **Mathematical process standards.** The student uses mathematical processes to acquire and demonstrate mathematical understanding. **The student is expected to display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.**	The expectation is that students speak and write with precise mathematical language to explain and justify the work. This includes justifying a solution. *Example*: When paired with M(6)(C), the student may be expected to explain how to use the Pythagorean Theorem to determine horizontal distance across an obstacle such as a mountain or lake.

TEKS: Mathematical modeling in personal finance.0.17	Supporting Information
M(2)(A) **Mathematical modeling in personal finance**. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. **The student is expected to use rates and linear functions to solve problems involving personal finance and budgeting, including compensations and deductions.**	Though direct variation is not explicitly stated in this SE, it is subsumed within linear functions. When paired with M(1)(D), rates and functions may be represented graphically or numerically.
M(2)(B) **Mathematical modeling in personal finance**. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. **The student is expected to solve problems involving personal taxes.**	When paired with M(1)(D), students may be expected to communicate the process at which a solution was reached using multiple representations.
M(2)(C) **Mathematical modeling in personal finance**. The student uses mathematical processes with graphical and numerical techniques to study patterns and analyze data related to personal finance. **The student is expected to analyze data to make decisions about banking, including options for online banking, checking accounts, overdraft protection, processing fees, and debit card/ATM fees.**	Specificity has been added to include online banking options, checking accounts, overdraft protection, processing fees, and debit card/ATM fees. When paired with M(1)(B), students may be expected to use the problem-solving process to analyze the data from multiple sources in order to make an informed decision.

TEKS: Mathematical modeling in personal finance.	Supporting Information
M(3)(A) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. **The student is expected to use formulas to generate tables to display series of payments for loan amortizations resulting from financed purchases.**	This SE includes loan amortizations for any financed purchase. Students are expected to use formulas to generate amortization tables to represent a series of payments.
M(3)(B) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. **The student is expected to analyze personal credit options in retail purchasing and compare relative advantages and disadvantages of each option.**	Specificity focuses on personal credit options. Personal credit options may include deferred payments, credit cards, and personal loans. These options may include differences in costs, interest rates, term lengths, and payment timelines.
M(3)(C) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. **The student is expected to use technology to create amortization models to investigate home financing and compare buying a home to renting a home.**	Specificity includes the use of technology to create amortization tables. When paired with M(1)(D), students may use a variety of representations to make the prescribed comparison.
M(3)(D) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, graphs, and amortization modeling to solve problems involving credit. **The student is expected to use technology to create amortization models to investigate automobile financing and compare buying a vehicle to leasing a vehicle.**	Specificity includes the use of technology to create amortization tables. When paired with M(1)(E), students may be expected to organize amortization tables for various vehicles in an effort to compare value.

TEKS: Mathematical modeling in personal finance.	Supporting Information
M(4)(A) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. **The student is expected to analyze and compare coverage options and rates in insurance.**	Types of insurance include but are not limited to personal, life, health, car, homeowner's, and rental insurance.
M(4)(B) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. **The student is expected to investigate and compare investment options, including stocks, bonds, annuities, certificates of deposit, and retirement plans.**	Specificity includes certificates of deposit. When paired with M(1)(D), students may be expected to use multiple representations to make this comparison.
M(4)(C) **Mathematical modeling in personal finance.** The student uses mathematical processes with algebraic formulas, numerical techniques, and graphs to solve problems related to financial planning. **The student is expected to analyze types of savings options involving simple and compound interest and compare relative advantages of these options.**	Types of savings options may include savings accounts, money market accounts, and certificates of deposit. These options may be analyzed with varying interest rates, safety of returns, flexibility, and liquidity of assets.

TEKS: Mathematical modeling in science and engineering.	Supporting Information
M(5)(A) **Mathematical modeling in science and engineering**. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. **The student is expected to use proportionality and inverse variation to describe physical laws such as Hook's Law, Newton's Second Law of Motion, and Boyle's Law**.	Direct variation has been restated as proportionality. This SE specifically addresses Newton’s Second Law of Motion.
M(5)(B) **Mathematical modeling in science and engineering**. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. **The student is expected to use exponential models available through technology to model growth and decay in areas, including radioactive decay.**	The use of geometric models has been specified as exponential models to include all real inputs instead of only integer inputs. Specificity includes the area of radioactive decay as a context for modeling exponential functions. Other areas may address related topics in science and engineering.
M(5)(C) **Mathematical modeling in science and engineering**. The student applies mathematical processes with algebraic techniques to study patterns and analyze data as it applies to science. **The student is expected to use quadratic functions to model motion.**	When paired with M(1)(C), students may be expected to use technology-based regression tools to model motion using quadratic functions. The focus is on sets of data related to science and engineering that may be modeled with quadratic functions.

TEKS: Mathematical modeling in science and engineering.	Supporting Information
M(6)(A) **Mathematical modeling in science and engineering.** The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. **The student is expected to use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in architecture.**	This SE focuses on mathematical patterns and structure in architecture as it relates to science and engineering. Specificity has been added with similarity. Geometric transformations include dilations, which generate similar figures.
M(6)(B) **Mathematical modeling in science and engineering.** The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. **The student is expected to use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields.**	The focus is on the application of scale factors in fields of study related to science and engineering. When paired with M(1)(A), the student may be asked to determine how the volume is effected after changing the height of a cylindrical tank by 6 feet as opposed to changing both the radius and height by 3 feet each.
M(6)(C) **Mathematical modeling in science and engineering.** The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. **The student is expected to use the Pythagorean Theorem and special right-triangle relationships to calculate distances.**	The focus is on the application of the Pythagorean Theorem to calculate distance in fields of study related to science and engineering. When paired with M(1)(G), the student may be expected to explain how to use the Pythagorean Theorem to determine horizontal distance across an obstacle such as a mountain or lake.
M(6)(D) **Mathematical modeling in science and engineering.** The student applies mathematical processes with algebra and geometry to study patterns and analyze data as it applies to architecture and engineering. **The student is expected to use trigonometric ratios to calculate distances and angle measures as applied to fields.**	Fields of study may include, but are not limited to, physics or mechanical engineering.

TEKS: Mathematical modeling in fine arts.	Supporting Information
M(7)(A) **Mathematical modeling in fine arts.** The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. **The student is expected to apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles.**	This SE specifies modeling periodic behavior in art and music. When paired with M(1)(G), students may be expected to identify and explain the use of the golden ratio in classical art and architecture.
M(7)(B) **Mathematical modeling in fine arts.** The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. **The student is expected to use similarity, geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and photography.**	Specificity has been added with similarity. Geometric transformations include dilations, which generate similar figures. Proportionality as an area of study for mathematical patterns and structure in fine arts is included.
M(7)(C) **Mathematical modeling in fine arts.** The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. **The student is expected to use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.**	When paired with M(1)(A), students may be expected to compare the waveform differences of the notes of an arpeggio.
M(7)(D) **Mathematical modeling in fine arts.** The student uses mathematical processes with algebra and geometry to study patterns and analyze data as it applies to fine arts. **The student is expected to use scale factors with two-dimensional and three-dimensional objects to demonstrate proportional and non-proportional changes in surface area and volume as applied to fields.**	The focus is on the application of scale factors in fields related to the fine arts. When paired with M(1)(G), students may be expected to explain why people use scale models as a proof of concept.

TEKS: Mathematical modeling in social sciences.	Supporting Information
M(8)(A) **Mathematical modeling in social sciences.** The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. **The student is expected to determine the number of ways an event may occur using combinations, permutations, and the Fundamental Counting Principle.**	The focus is on events related to the social sciences. When paired with M(1)(A), students may be expected to determine how many couples are possible with 36 people working together in a group.
M(8)(B) **Mathematical modeling in social sciences.** The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. **The student is expected to compare theoretical to empirical probability.**	The focus is on events related to the social sciences. When paired with M(1)(B), a student may be expected to determine if an outcome is reasonable.
M(8)(C) **Mathematical modeling in social sciences.** The student applies mathematical processes to determine the number of elements in a finite sample space and compute the probability of an event. **The student is expected to use experiments to determine the reasonableness of a theoretical model such as binomial or geometric.**	Clarification has been made to use binomial and geometric models. Students should be provided the theoretical model in order to design an appropriate experiment. The focus is on conducting an experiment related to the social sciences.

TEKS: Mathematical modeling in social sciences.	Supporting Information
M(9)(A) **Mathematical modeling in social sciences.** The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. **The student is expected to interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, dot plots, stem-and-leaf plots, and box and whisker plots, to draw conclusions from the data and determine the strengths and weaknesses of conclusions.**	Line plots are identified as dot plots for vertical alignment with the kindergarten–grade 8 TEKS. However, in this case, a line graph is a piece-wise linear function that connects bivariate data from left to right in order to indicate the general direction of data. The focus is on sets of data related to the social sciences. This SE includes determining the strengths or weaknesses of the conclusions drawn from data.
M(9)(B) **Mathematical modeling in social sciences.** The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. **The student is expected to analyze numerical data using measures of central tendency (mean, median, and mode) and variability (range, interquartile range or IQR, and standard deviation) in order to make inferences with normal distributions.**	Specificity includes the identification of the measures of central tendency (mean, median, and mode) and the measures of variability (range, interquartile range or IQR, and standard deviation). In 6(12)(C), central tendency is referred to as center, and variability is referred to as spread.
M(9)(C) **Mathematical modeling in social sciences.** The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. **The student is expected to distinguish the purposes and differences among types of research, including surveys, experiments, and observational studies.**	When paired with M(1)(G), a student may be expected to explain why a given type of research is more appropriate for a given situation.
M(9)(D) **Mathematical modeling in social sciences.** The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. **The student is expected to use data from a sample to estimate population mean or population proportion.**	Students are expected to estimate the population mean or population proportion. Students may be expected to use a sample mean or sample proportion to validate the estimation.
M(9)(E) **Mathematical modeling in social sciences.** The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. **The student is expected to analyze marketing claims based on graphs and statistics from electronic and print media and justify the validity of stated or implied conclusions.**	This SE includes statistics in addition to graphs when analyzing claims. Specificity includes electronic and print media, which may include journals and newspapers. This SE includes justifying implied conclusions in addition to stated arguments or conclusions. The focus is on sets of data related to the social sciences.
M(9)(F) **Mathematical modeling in social sciences.** The student applies mathematical processes and mathematical models to analyze data as it applies to social sciences. **The student is expected to use regression methods available through technology to model linear and exponential functions, interpret correlations, and make predictions.**	The focus is on sets of data and prediction related to the social sciences. Specificity includes the use of regression to model linear and exponential functions. Students are expected to interpret correlations for linear and exponential models. When paired with M(1)(A), (C), and (D), students may be expected to analyze a problem situation, select a model, and interpret the information from that model. The focus is on sets of data related to the social sciences that may be modeled with linear and exponential functions. Additional applications of this material can be found in M(5)(B). When paired with M(1)(B) or (G), the appropriateness of the model impacts the reasonableness of the solution. Students may be expected to justify or make an argument for the chosen model. Correlations may or may not be used to justify predictions.

TEKS: Mathematical modeling in social sciences.	Supporting Information
M(10)(A) **Mathematical modeling in social sciences.** The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. **The student is expected to formulate a meaningful question, determine the data needed to answer the question, gather the appropriate data, analyze the data, and draw reasonable conclusions.**	The knowledge and skills statement simplifies the context for this student expectation with the phrase, "design a study." The focus is on conducting a study related to the social sciences. The question may be answered with qualitative data, quantitative data, or both.
M(10)(B) **Mathematical modeling in social sciences.** The student applies mathematical processes to design a study and use graphical, numerical, and analytical techniques to communicate the results of the study. **The student is expected to communicate methods used, analyses conducted, and conclusions drawn for a data-analysis project through the use of one or more of the following: a written report, a visual display, an oral report, or a multi-media presentation.**	The knowledge and skills statement simplifies the context for this student expectation with the phrase "design a study." Specificity in the knowledge and skills statement includes graphical, numerical, and analytical techniques to communicate the results of the study. The focus is on conducting a study related to the social sciences. The question may be answered with qualitative data, quantitative data, or both. Specificity includes affirmation that students may be expected to combine presentation formats when communicating the results of the study.