Equations: Rational Number Coefficients
Transcript

When solving an equation, my goal is to determine the value of the unknown. Therefore, I want to isolate the variable to determine the value of x. I am going to use two different rectangles to model and solve the equation. A square tile will be used to model one unit or 1.

A rectangle that has the same height as the square and an unknown length will be used to model x, the unknown value.

I can use these models to solve the equation $\frac{2}{3}x=4$.

This equation tells us that even though we don't know the value for x, we do know that

$\frac{2}{3}x$, our unknown, is 4.

Remember, the goal is to use the models to determine the value of the unknown, x.

Let's move over to the left side of the board to work with our equation model. This line represents the equal sign. I will model the left side of the equation on the left side of this line and the right side of the equation on the right side.

To represent the equation $\frac{2}{3}x=4$, I am placing four positive unit tiles here to represent the 4 on the right side of the equation.

On the left side of the line that represents the equal sign, I need to model $\frac{2}{3}x$.

If this bar is $1x$, then this bar represents $\frac{1}{3}x$, and this bar represents $\frac{2}{3}x$.

I am going to place the model of $\frac{2}{3}x$ on the left side of the line.

To determine the value of x, I am going to determine how many unit tiles are used to make a complete whole, or an entire x.

I need to think about how much of the model of x is missing.

I am going to trade out my model of $\frac{2}{3}x$ for this model.

It still models $\frac{2}{3}x$. It just represents $\frac{2}{3}$ as two $\frac{1}{3}$s.

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I am going to rotate my model of $\frac{2}{3}x$, so that it is placed vertically on the left side of my equation model.

My model tells me that $\frac{2}{3}$ of the unknown value equals four units. By sharing the four units equally between the two $\frac{1}{3}$s, I can determine the value of $\frac{1}{3}$. In order to do this, I will place $\frac{1}{2}$ of the unit tiles in line with the first $\frac{1}{3}$ and the other half of the unit tiles in line with the second $\frac{1}{3}$. So, each $\frac{1}{3}$ of the unknown is equal to two unit tiles or 2.

In modeling the equation, we are determining the value of the whole unknown, or in this case, $1x$. This means that if this $\frac{1}{3}$ has a value of 2, and this $\frac{1}{3}$ has a value of 2, then this $\frac{1}{3}$ will also have a value of 2.

Remembering that the goal is to determine the value of the unknown, I have determined that $1x$ equals six units.

I am going to pause and record algebraically what I have modeled. I determined that $\frac{2}{3}x$ equals four units. In the model, I placed half of the units, or two units, with each of the $\frac{1}{3}$s. So, I can represent that algebraically by multiplying both sides of the equation by $\frac{1}{2}$.

To determine the value of the unknown, x, I needed to determine the value of three groups of $\frac{1}{3}x$, and three groups of 2. Algebraically, this is represented by multiplying both sides of the equation by 3.

I can rewrite the product of $\frac{1}{2}$ and 3 as $\frac{3}{2}$. $3x\frac{1}{2} = \frac{3}{2}$.

Next, I am going to simplify the expressions on each side of the equation.

I notice that $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals of each other. When I multiply two reciprocals, the product is 1. The product of 1 and x is x.

Algebraically, if I simplify my factors before I multiply, I see that the 4 simplifies to 2, which is then multiplied by 3. This sequence of operations matches the actions of the model.

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This may look like many steps to solve an equation. However, as students progress in their understanding of how to solve equations with rational number coefficients, they will become more proficient with the use of the reciprocal of the coefficient and its relationship to the model.

Also, as students become even more proficient in solving equations with rational number coefficients, they will experience less of a need to model the equation as they begin to think about the numbers themselves.

This equation is considered a one-step equation because it requires one operation applied to both sides of the equation to determine the value of x. Though we modeled dividing by 2 and multiplying by 3, these two actions are really one when simplified; that is multiplying by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

Another equation that is considered to be a one-step equation with rational number coefficients is $-3x = 27$. I can solve this equation by multiplying each side by $-\frac{1}{3}$, which is the reciprocal of -3, or by dividing each side by -3, which represents the inverse operation of multiplying by -3.

A third equation is $12 = \frac{4}{3}x$.

Each of these equations will require one operation applied to both sides of the equation to determine the value of x. It does not matter if the unknown, x, is on the left side of the equation or on the right side of the equation.

