Equations: Combining Like Terms
Transcript

When solving an equation, I want to isolate the variable. I want to use my algebra skills to have x by itself on one side of the equal sign so that it is equal to the value on the other side of the equal sign. In other words, I want to have x with a coefficient of 1 equal to a constant. I am going to use two different sizes and colors of rectangles to model the equation $8 = 2x - 6x$, much like I can model operations with integers.

My goal is to get from this equation to this. *(Points to whiteboard)*

We are looking at the equation: $8 = 2x$, or $2x - 6x$, or $6x$. We know from our integer rules that $2 - 6$ is equivalent to $2x - 6x$. The rules that we use to combine integers hold true when we combine like terms with variables. We can rewrite our equation as $8 = 2x + -6x$.

As I model solving this equation, I will record the process representationally and algebraically here on the dry board.

We are going to represent the process using rectangles to represent variables and constants. Black rectangles that are one unit by one unit will represent 1.

White rectangles that are one unit by one unit will represent -1.

Black rectangles that are one unit by x units will represent x.

White rectangles that are one unit by x units will represent $-x$.

The line that I am drawing represents the equal sign. I will model the left side of the equation on the left side of this line and the right side of the equation on the right side of this line.

To represent the equation $8 = 2x - 6x$, I am placing 8 one unit rectangles here to represent the 8 on the left side of the equation.

On the right side of the line that represents the equal sign, I am placing two x rectangles to represent the $2x$, and six $-x$ rectangles to represent the $-6x$.

Remembering that the goal when solving this equation is to determine the value of the unknown, I need to have a single x rectangle all by itself, or isolated, on one side of the equal sign in order to determine its value. In this model, there are two x rectangles and six $-x$ rectangles on the right side of the equal sign.

Therefore, I need to combine the x rectangles, or combine like terms and simplify.

An x paired with a $-x$ has a value of 0, much like $1 + -1 = 0$.

So, I am going to remove these pairs that have a value of 0 to simplify the right side of the equation model.

I am going to pause and record what I have modeled.

In the model, I combined, or paired, two x rectangles with two $-x$ rectangles.

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This results in $8 = 0+\text{-}4x$ or $8 = \text{-}4x$.

When I look at the model, I see that I have four -x's on the right side of the model. I need to determine the value that is equal to $1x$.

At this point, I need to determine the value of each -x. I need to determine the value of each unknown.

I can do this by separating or dividing each side of my model into four equal groups.

Each -x is equivalent to two units.

I divided each side of the model into four equal groups.

$\frac{8}{4} = 2$, and $\frac{\text{-}4x}{4}$

$\frac{\text{-}4x}{4} = \text{-}1x$.

Because we know that 2 is equal to each -x, we can remove all but one of the groups.

Notice that in the model and the algebraic representation, the x, or the unknown value, is negative.

However, we want to identify the value of x rather than the value of -x. I can use reasoning to determine x. In other words, reasoning tells us -x means the opposite of x. So, if 2 is equal to the opposite of x, then $\text{-}2 = x$.

How can I show this with a model? I need to model the opposite of each side of the equation: $2 = \text{-}x$.

I need to think, "What is the opposite of -x? x!"

"What is the opposite of 2? -2!"

How do I record this? Multiplying a value by $\text{-}1$ will result in the opposite of that value. So, I am going to multiply each side of my equation by $\text{-}1$.

This result is the equation $\text{-}2 = x$.

This looks like many steps to solve an equation. As students progress in their understanding of how to solve equations with integer or rational number values, they will become more proficient with symbolic steps, going from $8 = 2x - 6x$ to $8 = \text{-}4x$.

Some students may then divide both sides by 4 and then multiply both sides by $\text{-}1$ as we modeled.

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Some may see that dividing both sides by 4 and then multiplying by -1 gives the same result as dividing both sides by -4, resulting in $-2 = x$.

This equation is considered a one-step equation because it requires one operation applied to both sides of the equation to identify the value of x. Though we modeled dividing by 4 and taking the opposite by multiplying by -1, these two actions are really one when simplified: dividing by -4.

Some may consider the combining of $2x$ and $-6x$ as a step. Because this happened only on one side of the equation, it is not considered a step.

Let's look at some other equations that are considered one-step equations.

$3x = \frac{1}{2}$, $7x - 5x + 7$, or $-9 + 3 = \frac{1}{3}x$.

Each of these equations will require one operation applied to both sides of the equation to determine the value of x. It does not matter if the x is on the left side of the equation or on the right side of the equation.