Grade 8- Slope Triangles

8(4)(A) **Proportionality.** The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $(y_2 - y_1) / (x_2 - x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line.

8(8)(D) **Expressions, equations, and relationships.** The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Materials:

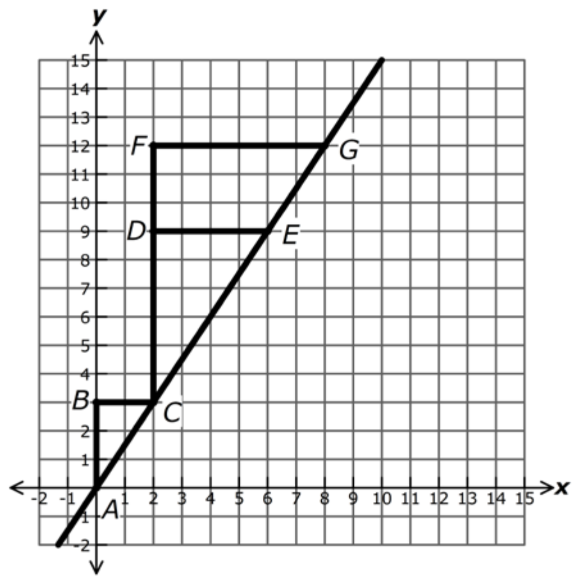
- **Slope Triangles**

Prior knowledge: It is expected that students will have explored special angle pairs formed when parallel lines are cut by a transversal prior to completing this activity.

Slope Triangles

Two triangles are **similar** if corresponding angles are ____________ and corresponding side lengths are ___________.

Right triangles *ABC*, *CDE*, and *CFG* are all represented on the following coordinate plane.



1. If $\overline{AB} \parallel \overline{CD}$, what is true about $\angle BAC$ and $\angle DCE$? Justify your answer.

2. If $\overline{BC} \parallel \overline{DE}$ and $\overline{DE} \parallel \overline{FG}$, what is true about $\angle BCA$, $\angle DEC$, and $\angle FGE$? Justify your answer.

3. Complete the table.

	Triangle *ABC*	Triangle *CDE*	Triangle *CFG*
$\frac{\text{length of vertical leg}}{\text{length of horizontal leg}}$			

4. Use the angle relationships and the ratios of the lengths of the legs to verify that triangles *ABC*, *CDE*, and *CFG* are all similar to each other.

5. If a new right triangle with a hypotenuse on the same line is added to the graph, do you think it would be similar to triangles *ABC*, *CDE*, and *CFG*? Why or why not?

6. Complete the table.

Hypotenuse	Endpoints	Use the coordinates of the endpoints to determine the lengths of the legs.	
$\overline{AC}$	*A* (____, ____)	Vertical leg:	Horizontal leg:
	C (____, ____)		

$\overline{CE}$	*C* (____, ____)	Vertical leg:	Horizontal leg:
	E (____, ____)		

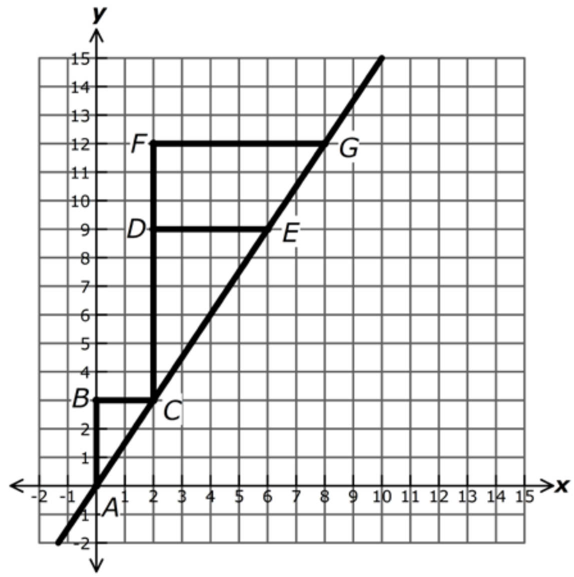
$\overline{CG}$	*C* (____, ____)	Vertical leg:	Horizontal leg:
	G (____, ____)		

7. A new right triangle with a hypotenuse that lies on the same line is added to this graph. The coordinates of the endpoints of the hypotenuse are (x_1, y_1) and (x_2, y_2). How could you determine the ratio of the length of the vertical leg to the length of the horizontal leg of this triangle?

Answer Key:

Two triangles are **similar** if corresponding angles are **congruent** and corresponding side lengths are **proportional**.

Right triangles *ABC*, *CDE*, and *CFG* are all represented on the following coordinate plane.



1. If $AB \parallel CD$, what is true about $\angle BAC$ and $\angle DCE$? Justify your answer.
Since $AB \parallel CD$, $\angle BAC$ and $\angle DCE$ are corresponding angles formed when parallel lines are cut by a transversal. Therefore, $\angle BAC \cong \angle DCE$.

2. If $BC \parallel DE$ and $DE \parallel FG$, what is true about $\angle BCA$, $\angle DEC$, and $\angle FGE$? Justify your answer.
Since, $BC \parallel DE$ and $DE \parallel FG$, $\angle BCA$, $\angle DEC$, and $\angle FGE$ are all corresponding angles formed when parallel lines are cut by a transversal. Therefore, $\angle BCA \cong \angle DEC \cong \angle FGE$.

3. Complete the table.

	Triangle *ABC*	Triangle *CDE*	Triangle *CFG*
$\dfrac{\text{length of vertical leg}}{\text{length of horizontal leg}}$	$\dfrac{AB}{BC} = \dfrac{3}{2}$	$\dfrac{CD}{DE} = \dfrac{6}{4} = \dfrac{3}{2}$	$\dfrac{CF}{FG} = \dfrac{9}{6} = \dfrac{3}{2}$

4. Use the angle relationships and the ratios of the lengths of the legs to verify that triangles *ABC*, *CDE*, and *CFG* are all similar to each other.
Corresponding side lengths are proportional because the ratios of the lengths of the vertical legs to the lengths of the horizontal legs are congruent.
Corresponding angles are congruent because $AB \parallel CD$, so $\angle BAC$ and $\angle DCE$ form corresponding angles when parallel lines are cut by a transversal, and are congruent. Also, $BC \parallel DE$ and $DE \parallel FG$, $\angle BCA$, $\angle DEC$, and $\angle FGE$ are also corresponding when parallel lines are cut by a transversal, and are also congruent.

5. If a new right triangle with a hypotenuse on the same line is added to the graph, do you think it would be similar to triangles *ABC*, *CDE*, and *CFG*? Why or why not?
Possible response includes the following: Yes, because the ratio of the length of the vertical

leg to the length of the horizontal leg would be the same as these three triangles. The horizontal leg would be parallel to these horizontal legs and the vertical leg would be parallel to these vertical legs. I could form corresponding angles when parallel lines are cut by a transversal to show the angles are congruent.

6. Complete the table.

Hypotenuse	Endpoints	Use the coordinates of the endpoints to determine the lengths of the legs.	
$\overline{AC}$	*A* (**<u>0</u>**, **<u>0</u>**) *C* (**<u>2</u>**, **<u>3</u>**)	Vertical leg: **3 – 0 = 3**	Horizontal leg: **2 – 0 = 2**

$\overline{CE}$	*C* (**<u>2</u>**, **<u>3</u>**) *E* (**<u>6</u>**, **<u>9</u>**)	Vertical leg: **9 – 3 = 6**	Horizontal leg: **6 – 2 = 4**

$\overline{CG}$	*C* (**<u>2</u>**, **<u>3</u>**) *G* (**<u>8</u>**, **<u>12</u>**)	Vertical leg: **12 – 3 = 9**	Horizontal leg: **8 – 2 = 6**

7. A new right triangle with a hypotenuse that lies on the same line is added to this graph. The coordinates of the endpoints of the hypotenuse are (x_1, y_1) and (x_2, y_2). How could you determine the ratio of the length of the vertical leg to the length of the horizontal leg of this triangle?

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Note that this ratio represents a rate of the vertical change to horizontal change between two points.