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## Your Definitions

Computational Fluency

Mathematical Proficiency

Automaticity

Conceptual Understanding

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 12

	Algebraic Manipulation	Automaticity	Computational Fluency	Mathematical Proficiency
<b>Addition and Subtraction</b>	<p>3(3)(F) Represent equivalent fractions</p> <p>4(3)(B) Decompose fractions with models</p> <p>4(3)(E) Represent and solve addition and subtraction of fractions with equal denominators using models</p> <p>4(3)(F) Evaluate for reasonableness of sums and differences using benchmark fractions</p> <p>5(3)(H) Represent and solve for addition/subtraction of fractions using objects</p> <p>6(3)(C) Represent integer operations</p>		<p>4(4)(A) Add/subtract whole numbers and decimals using the standard algorithm</p> <p>5(3)(K) Add/subtract positive rational numbers fluently</p> <p>6(3)(D) Add/subtract integers fluently</p> <p>7(3)(A) Add/subtract rational numbers fluently</p>	<p>7(3)(B) Solve problems using addition and subtraction of rational numbers</p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 12

	Algebraic Manipulation	Automaticity	Computational Fluency	Mathematical Proficiency
<b>Multiplication and Division</b>	<p>5(3)(D) Represent multiplication of decimals</p> <p>5(3)(F) Represent quotients of decimals</p> <p>5(3)(I) Represent and solve problems involving multiplication of a whole number and a fraction</p> <p>5(3)(J) Represent and solve problems involving division of a unit fraction by a whole number</p> <p>6(3)(A) Recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values</p> <p>6(3)(B) Determine if a quantity is increased or decreased when multiplied by a fraction</p> <p>6(3)(C) Represent integer operations</p>		<p>5(3)(E) Solve for products of decimals</p> <p>5(3)(G) Solve for quotients of decimals</p> <p>6(3)(D) Multiply/divide integers fluently</p> <p>6(3)(E) Multiply/divide positive rational numbers fluently</p> <p>7(3)(A) Multiply/divide rational numbers fluently</p>	<p>7(3)(B) Solve problems using multiplication and division of rational numbers</p>

**Vertical Learning Progression Recording Sheet**

**Grades K - 12**

	<b>Algebraic Manipulation</b>	<b>Automaticity</b>	<b>Computational Fluency</b>	<b>Mathematical Proficiency</b>
<b>Addition and Subtraction</b>				

**Vertical Learning Progression Recording Sheet**

**Grades K - 12**

	<b>Algebraic Manipulation</b>	<b>Automaticity</b>	<b>Computational Fluency</b>	<b>Mathematical Proficiency</b>
<b>Multiplication and Division</b>				

## Developing Mathematical Proficiency

Pairing a content standard with a process standard to solve problems allows students to become mathematically proficient with the content for each grade level.



How does pairing a process standard with a content standard allow students to become mathematically proficient?

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Why is it important that the student expectations in the mathematical proficiency column be coupled with the process standards?

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Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Francesca's Fractions

Below is Francesca's work from her class today.

$$\begin{aligned} & \frac{2}{3} + \frac{3}{4} \\ & \frac{8}{12} + \frac{9}{12} \\ & \left( \frac{8}{12} + \frac{4}{12} \right) + \frac{5}{12} \\ & \frac{12}{12} + \frac{5}{12} \\ & 1\frac{5}{12} \end{aligned}$$

What was her strategy? Complete the four problems below using her strategy.

1 $\frac{2}{3} + \frac{5}{9}$	3 $\frac{5}{8} + \frac{2}{3}$
2 $\frac{1}{2} + \frac{4}{5}$	4 $\frac{7}{10} + \frac{3}{4}$

What patterns did you notice?

*Grade 5 Fluency Activity – 5(3)(K) The student is expected to add and subtract positive rational numbers fluently.*

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Inigo's Integers

Below is Inigo's work from his class today.

$$\begin{aligned} & -5 + 8 \\ & (-5 + 5) + 3 \\ & 0 + 3 \\ & 3 \end{aligned}$$

What was his strategy? Complete the four problems below using his strategy.

1 $-6 + 2$	3 $-3 + 15$
2 $7 + (-12)$	4 $8 + (-5)$

What patterns did you notice?

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Ra'Neisha's Rationals

Below is Ra'Neisha's work from her class today.

$$\begin{aligned} & -1.2 + 3.4 \\ & (-1.2 + 1.2) + 2.2 \\ & 0 + 2.2 \\ & 2.2 \end{aligned}$$

What was her strategy? Complete the four problems below using her strategy.

1 $-6.4 + 2.3$	3 $-3.7 + 15.4$
2 $7.9 + (-12.4)$	4 $3.4 + (-1.2)$

What patterns did you notice?

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Millie's Multiplication

Millie used the strategy shown to multiply  $(3x - 1)(4x^2 - 5x - 3)$ .

	$4x^2$	$-5x$	$-3$
$3x$			
$-1$			

	$4x^2$	$-5x$	$-3$
$3x$	$12x^3$	$-15x^2$	$-9x$
$-1$	$-4x^2$	$5x$	$3$

	$4x^2$	$-5x$	$-3$
$3x$	$12x^3$	$-15x^2$	$-9x$
$-1$	$-4x^2$	$5x$	$3$

$-19x^2$     $-4x$

$12x^3 - 19x^2 - 4x + 3$

What was her strategy? Complete the four problems below using her strategy.

<b>1</b> $(5a - 1)(-2a^2 + 4a - 3)$	<b>3</b> $(x - 8)(7x + 4)$
<b>2</b> $(3y + 7)(2y + 7)$	<b>4</b> $-3b(b^2 - 4b + 6)$

What patterns did you notice?

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Fred's Factoring

Fred used the strategy shown to factor  $27x^2 + 42x - 5$ .

$27x^2$	
	$-5$

	$3x$	$-5$
$9x$	$27x^2$	$-45x$
$1$	$3x$	$-5$

$-42x$

wrong  
sign

	$3x$	$5$
$9x$	$27x^2$	$45x$
$-1$	$-3x$	$-5$

$42x$

$$(9x - 1)(3x + 5)$$

What was his strategy? Complete the four problems below using his strategy.

<b>1</b>	$6x^2 - 19x + 15$	<b>3</b>	$2x^2 - 11x + 5$
<b>2</b>	$3x^2 - 11x - 4$	<b>4</b>	$-10x^2 + 11x + 6$

What patterns did you notice?

*High School Fluency Activity – A(10)(E) The student is expected to factor, if possible, trinomials with real factors in the form  $ax^2 + bx + c$ , including perfect square trinomials of degree two.*

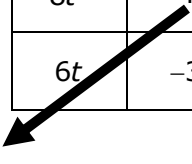
Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Darian's Division

Below is Darian's work from his class today for the problem  $\frac{8t^2 + 2t - 3}{2t - 1}$ .

$2t$	$-1$
$8t^2$	
	$-3$

	$2t$	$-1$
$4t$	$8t^2$	$-4t$
$3$	$6t$	$-3$

$2t$  

$$(2t - 1)(4t + 3)$$

$$\frac{8t^2 + 2t - 3}{2t - 1} = \frac{\cancel{(2t - 1)}(4t + 3)}{\cancel{2t - 1}} = 4t + 3$$

What was his strategy? Complete the four problems below using his strategy.

<b>1</b> $(3x^2 - 12x - 15) \div (x - 5)$	<b>3</b> $\frac{6y^2 + 11y - 10}{3y - 2}$
<b>2</b> $\frac{x^2 - x - 20}{x + 4}$	<b>4</b> $(2w^2 + 5w - 12) \div (w + 4)$

What patterns did you notice?

*High School Fluency Activity – A(10)(C) The student is expected to factor, if possible, trinomials with real factors in the form  $ax^2 + bx + c$ , including perfect square trinomials of degree two.*



## Drill or Practice?

**Drill** refers to repetitive, non-problem-based exercises designed to improve skills or procedures already acquired.

**Practice** refers to different problem-based tasks or experiences, spread over numerous class periods, each addressing the same basic ideas.

Van De Walle, J. (2004). *Elementary and middle school mathematics*. Boston, MA: Pearson.

## Case Study Recording Sheet

Examine the case study documents provided for each student. What evidence do you see for each of the categories?

Student A			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

Student B			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

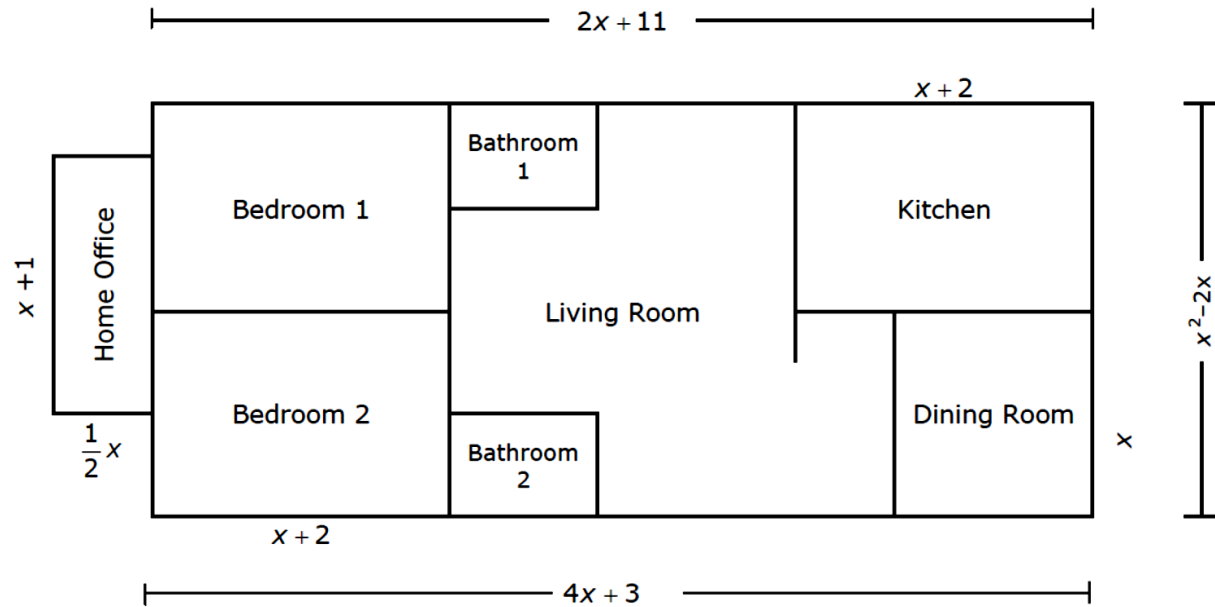


## Case Study Student A

### Work Sample

James has just purchased a house and wants to lay carpet in the living room, both bedrooms, and his home office. The house has a square dining room and 2 bathrooms that are each  $6 \text{ m}^2$ .

Write an algebraic expression that represents the area that James wants to carpet, then determine the value of  $x$ . Justify your response.



Carpeted Area = living room + bedroom 1+2 + office

LR  $4x+3-(x+2)$  DINING  $(x^2-3x)(x+2)$   
 $x^3+2x^2-3x^2-6x$   
 $x^3-x^2-6x$  Kitchen Bath.

$$A = (3x+1)(x^2-2x) - x^2 - (x^3-x^2-6x) - 12$$

$$A = \boxed{3x^3} - \cancel{6x^2} + \cancel{1x^2} - 2x - \cancel{x^2} - \boxed{x^3} + \cancel{1x^2} + 6x - 12$$

$$A = 2x^3 - 5x^2 + 4x - 12$$

Bed Rooms

$$A = (x+2)(x^2-2x)$$

$$A = x^3 - 2x^2 + 2x^2 - 4x$$

$$A = x^3 - 4x$$

Office

$$A = \frac{1}{2}x(x+1)$$

$$A = \frac{1}{2}x^2 + \frac{1}{2}x$$

Total Area =  $\boxed{2x^3} - \cancel{5x^2} + \cancel{4x} - 12 + \boxed{x^3} - \cancel{4x} + \cancel{\frac{1}{2}x^2} + \cancel{\frac{1}{2}x}$

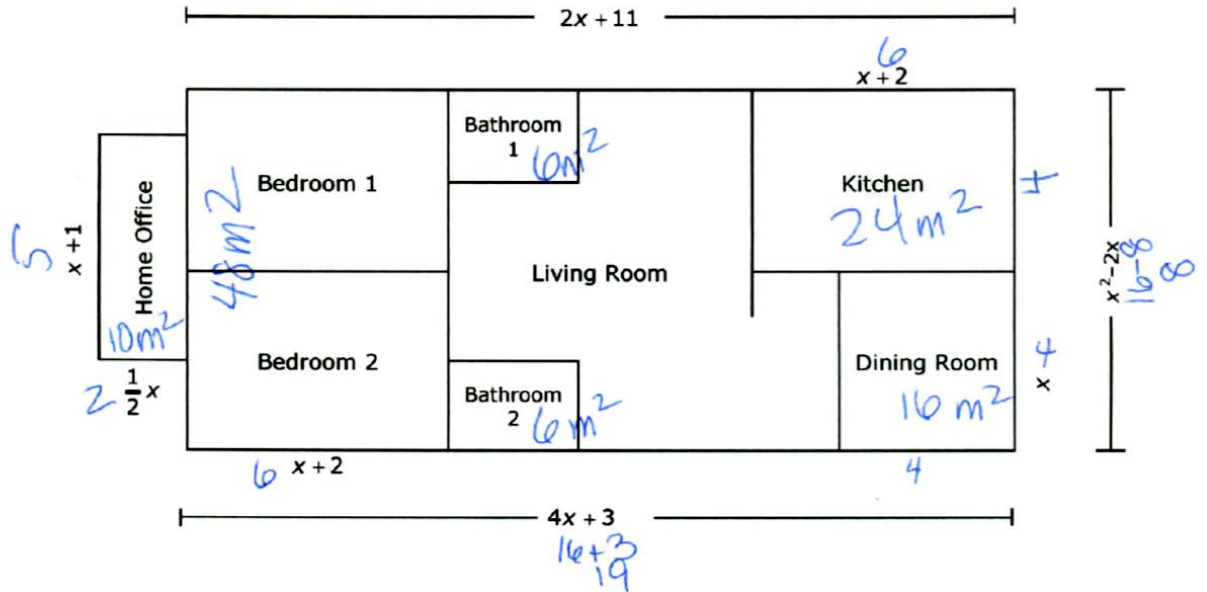
T.A. =  $3x^3 - 4\frac{1}{2}x^2 + \frac{1}{2}x - 12$  \*

## Case Study Student B

### Work Sample

James has just purchased a house and wants to lay carpet in the living room, both bedrooms, and his home office. The house has a square dining room and 2 bathrooms that are each  $6 \text{ m}^2$ .

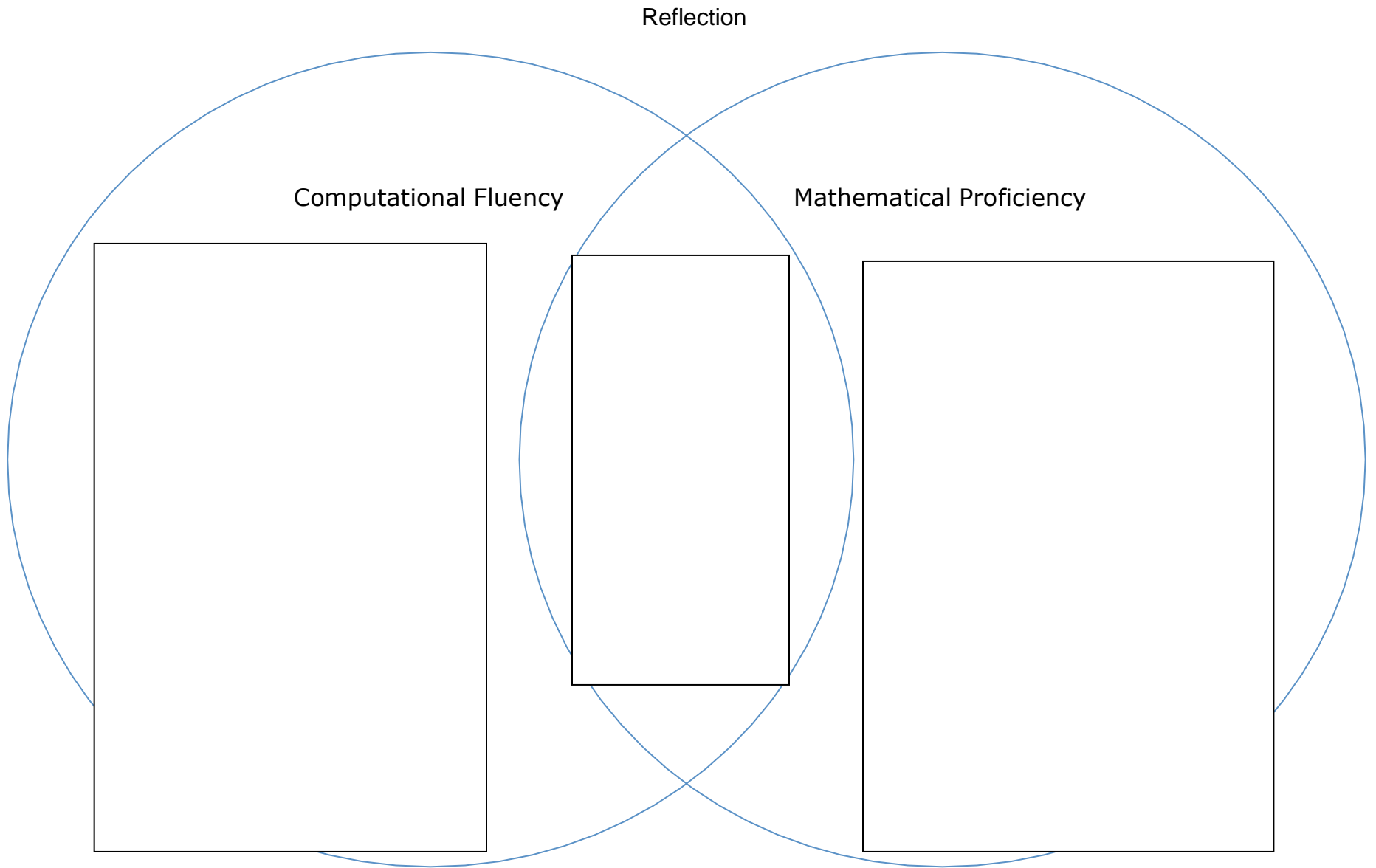
Write an algebraic expression that represents the area that James wants to carpet, then determine the value of  $x$ . Justify your response.



$$\begin{array}{r}
 2x + 11 = 4x + 3 \\
 -4x \quad -4x \\
 \hline
 -2x + 11 = 3 \\
 -11 \quad -11 \\
 \hline
 -2x = -8 \\
 \frac{-2}{-2} \quad \frac{-8}{-2} \\
 \hline
 x = 4
 \end{array}$$

$$\begin{array}{l}
 \text{House + Office} = \text{Total Area} \\
 19.8 + 10 = \text{Total Area} \\
 152 + 10 = \text{T.A.} \\
 \underline{162 \text{ m}^2}
 \end{array}$$

$$\begin{array}{l}
 \text{Total} - \text{Kitchen} - \text{Dining/Room} - \text{BR} \\
 162 - 24 - 16 - 12 \\
 * \boxed{110 \text{ m}^2 \text{ get carpet}} *
 \end{array}$$



## Reference Page

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