Learning Progression RN:
Understanding Positive Rational Numbers

UNDERSTANDING FRACTIONS

MAGNITUDE

LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
1	**Number Line Structure**	**RN.A. 1.1**	**i.** The student creates number lines with evenly spaced intervals. **ii.** The student understands that a number has a specific location on the number line based on what is "next" in the list of numbers (ordinal). **iii.** The student understands that numbers "to the right" are larger.	**i. (E)** Fails to mark or interpret intervals correctly. **ii. (E**) Incorrectly counts the "numbers" as opposed to "intervals" when associating numbers with location on a number line. **(E)** Incorrectly counts with the "next" number in the list regardless of equal intervals/distance. **iii. (M)** Does not grasp the idea that the coordinates (numbers) on a number line represent the distance the coordinate is away from 0.
1	**Magnitude as Distance**	**RN.A. 1.2**	**ii.** The student understands that a number has a specific location on the number line based on what is "next" in the list of numbers (ordinal), and that numbers represent a distance or quantity from 0 (cardinal). The student understands the similarities between a number line and ruler. **iii.** The student understands that numbers "to the right "are larger, related to the idea of magnitude. The student understands that fractions can be represented on a number line, and are also larger "to the right." **iv.** The student understands the magnitude of "common" fractions (e.g., 1/2, 1/4), and uses "common" fractions to estimate magnitude or distance.	**ii. (M)** Unable to count with any other distance except for 1 (e.g. student does not connect to "counting" with "fifths"). **(M)** Understands the end point as the distance, regardless of the beginning point (e.g., the distance between 3 and 6 is 6). (NOTE: this interpretation is only valid when the left-most endpoint is at 0). **iv. (M)** Cannot generalize the magnitude of a specific fraction, *a*/*b*. **(E)** Incorrectly estimates the distance between two rational numbers **(E)** Incorrectly thinks *n*/*n* is the same as *n*.
1	**Part-to-Whole Relationships**	**RN.A. 1.3**	**iv.a.** The student understands the magnitude or distance between two numbers, *a* and *b*, is related to counting the number of equal intervals between *a* and *b*. **iv.b.** The student understands that fractions, 1/*b*, are located by dividing 1 into *b* equal intervals (e.g., 1/4 as dividing 1 into 4 equal intervals). The student compares the value of a fraction to 1 when the numerator is greater than the denominator and when the numerator is less than the denominator (e.g., 3/3=1, so 5/3>1 and 3/5<1). **v.** The student understands the location of a mixed number on a number line (e.g., 2 ¼ is the same distance from 2 as ¼ is from 0).	**iv.a. (M)** Views fractions only as part:whole relationships and not as numbers in their own right (i.e., they view 1/4 in relation to 1, but not as its own number, 1/4). **(E)** Incorrectly "counts" intervals between 2/5 and 6/5 as "4." **iv.b. (M)** Does not grasp that fractions are a quantity (cardinal), measured as a distance from 0. **(M)** Does not recognize that when dividing a whole into equal parts, each part has the same magnitude and the sum of the parts sum to equal the whole (e.g., 1/4 + 1/4 + 1/4 +1/4 = 4/4) **(M)** Views a number written as a fraction over itself as division rather than composition of parts (i.e., 5/5=1 because 5 divided by 5 is 1, and not because 5 "fifths" is 1). **v. (E)** Cannot correctly convert a mixed number to an improper fraction.
1	**Unit Fractions**	**RN.A. 1.4**	**iv.a.** The student understands that the magnitude or distance between two numbers is related to counting the number of equal intervals, including fractional intervals. (i.e., correctly "counts" intervals between 2/5 and 6/5 as "4/5") **iv.b.** The student partitions the number line between 0 and 1 into *b* equal intervals, and recognizes that each interval is the same fractional unit size, 1/*b*. They can locate the number 7/4 as the distance of seven 1/4 intervals from 0. **v.** The student understands a mixed number as a number between two whole numbers (i.e., 2 ¼ is between 2 and 3). The student can also represent a mixed number as a sum of fractions and can locate the mixed number by counting the intervals (i.e., 8/8 + 8/8+ 1/8 = 17/8 is the same as 17 intervals of "eighths").	

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	LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
EQUIPARTITIONING	2	**Partitioning Wholes**	**RN.A. 2.1**	**i.** The student partitions **rectangles** and **circles** into equal regions using paper strips and pictorial representations. **ii.** The student makes the connection that a whole is composed of 2 halves, 3 thirds, 4 fourths, etc.	**i. (E)** Represents fractions with unequal parts. For example: Saying that 1/3 of the strip is shaded. **(M)** Does not recognize 1/*b* of a circle and 1/*b* of a rectangle as shapes partitioned into the same number of parts because they are focused on the size of the region. **ii. (M)** Does not recognize that the number of partitions is represented by the denominator of the fraction.
	2	**Partitioning Different-Sized Shapes**	**RN.A. 2.2**	**i.** The student partitions **shapes** into equal regions (with equal areas) using paper strips and pictorial representations. The student recognizes that shapes of different sizes can be partitioned equally and still represent unit fractions. The student understands that a unit fraction is not contingent upon the size of the shape, but the number of parts (i.e., a circle with a radius of 2 can be divided into two equal parts in the same way as a circle with a radius of 10). **ii.** The student makes the connection that a whole is composed of 2 halves, 3 thirds, 4 fourths, and that the number of parts or partitions is the denominator of the unit fraction. The student can label each part of the partitioned whole as a fraction.	**i. (M)** Does not recognize that for fraction models involving area, two parts may look different but hold the same relationship to the whole (i.e., A square can be folded in half to form a triangle or a rectangle. They are both halves of the same whole and will have the same area). **ii. (M)** Does not make the connection that as the number of partitions(represented by the denominator) increases, the unit fraction gets smaller.
	2	**Division as Partitioning**	**RN.A. 2.3**	**ii.a.** The student understands that partitioning a whole into equal parts is the same as dividing into equal groups. **ii.b.** The student models and describes division using concrete objects in equivalent sets (i.e., a student can model that 20 can be divided into 4 equal groups of 5, 5 equal groups of 4, 2 equal groups of 10, 10 equal groups of 2, or 20 equal groups of 1). The student partitions discrete number of objects into equal sets with no remainder. (i.e., 1/4 of 8 circles is 2 circles.)	**ii.a. (E)** Switches the divisor and dividend which results in an incorrect quotient. **(E)** Uses repeated subtraction to divide into equal groups but misinterprets the meaning of 0 (i.e., 36 divided into 6 groups: 36-6=30, 30-6=24, 24-6=18, 18-6=12, 12-6=6, 6-6=0; interprets quotient as 0 because it is the last number in their process instead of 6 which represents the number of times 6 was subtracted to get equal groups). **ii.b. (M)** Does not recognize that a whole, composed of a set (i.e., a set of circles can represent the whole), can be partitioned into any number of parts equally (i.e., 1/4 of 7 circles is 1¾ circles).
	2	**Combining Partitioned Parts**	**RN.A. 2.4**	**ii.a.** The student understands that the denominator represents the number of parts into which the whole was partitioned, making the connection between division and partitioning. **ii.b.** The student understands that a set can be composed of several wholes that can be partitioned into equal parts (i.e., if a set is composed of 2 circles, when this set is partitioned into halves, each half can be represented by a whole circle). **iii.** The student recognizes that partitioned parts can be combined and an amount is 1/*b* if *b* copies of it are equal to 1. The student shows, using pictures (or diagrams) that a fraction is made up of many pieces of a unit fraction (i.e., the fraction 3/4 is composed of 3 pieces that are each 1/4 by reasoning about the role of the denominator).	

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DECOMPOSITION

LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
3	**Composition of Models**	**RN.A. 3.1**	**i.** The student composes a fraction using a visual model (i.e., combine 1/8, 1/8, 1/8 to make 3/8) by counting the number of parts. The student makes the connection that if all parts, *b*, of the whole are counted, the whole can be represented as a fraction *b*/*b* (i.e., 8/8).	**i. (E)** Ignores the denominator of the fraction when composing fractions and only gives attention to the number of parts counted (i.e., student's response is 3 instead of 3/8) **(M)** Does not make the connection that composition of fractions can be represented in various ways (i.e., 2/8 and 1/8 is 3/8); only sees it as composition of the unit fraction (1/8, 1/8, 1/8). **(M)** Able to compose a fraction, but unable to decompose a fraction.
3	**Representations of Compositions**	**RN.A. 3.2**	**i.** The student composes and decomposes a fraction with and without a visual model. The student also represents a fraction greater than 1 in a model(i.e., 16/8). **ii.** The student reasons, with composition and decomposition, by understanding addition of fractions with the same whole as combining different representations of the unit fraction (i.e., 1/8 + 1/8 +1/8 = 4/8 - 1/8 = 2/8 + 1/8).	**i. (E)** Views the numerator and denominator as whole numbers and sums both the numerators and denominators when adding (i.e., 1/8 + 1/8 + 1/8 = 3/24). **(E)** Inverts the fraction to put the larger number as the denominator. **(M)** Views the quantities 2 and 8/4 as different, based on differences between "counting" versus "division". **ii. (E)** Unable to represent addition of fractions with the same whole given a fraction *a*/*b* (limited to only composition with unit fractions).
3	**Composition as Multiplication**	**RN.A. 3.3**	**i.** The student is able to reason with composition and decomposition to represent whole numbers as fractions (i.e., 2=2/1=8/4). **ii.** The student reasons, with composition and decomposition, by understanding addition of fractions with the same whole as combining different representations of any fraction *a*/*b*. **iii.** The student understands multiplication of whole numbers as repeated addition. The student extends their understanding of multiplication with whole numbers to work with fractions by modeling addition of unit fractions with the same whole as multiplication (e.g., ¼ + ¼ + ¼ = 3 x ¼).	**i.(M)** Unable to convert a whole number into a fraction (i.e., 7 = 7/1) **iii.(M)** Relates multiplication as addition, but only to unit fractions; Does not relate 3x4/5 as 4/5+4/5+4/5, which becomes 12/5.
3	**Decomposition as Repeated Addition**	**RN.A. 3.4**	**iii.** The student models decomposition of fractions and whole numbers as a repeated addition equation. The student works flexibly with fractions *a*/*b* when *a*>1 (e.g., 4/5 + 4/5 + 4/5 = 3*4/5 =12/5).	

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REPRESENTATIONS OF POSITIVE RATIONAL NUMBERS

EQUIVALENT FRACTIONS

LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
4	**Representing Equivalence**	**RN.B. 4.1**	**i.** Given a model, the student recognizes that it represents an equivalent fraction. The student understands that equivalent fractions will always be located at the same point on the number line. **ii.** The student understands that the number and size of the parts differ even though the two fractions themselves are equivalent. (i.e., 3/4 has 3 "bigger" parts, and 6/8 has 6 "smaller" parts.) **iii**. The student identifies common factors of the numerator and denominator to simplify the fraction.	**i. (E)** Recognizes equivalence when given a model, but cannot generate equivalent fractions. When asked if two fractions are equivalent, the student makes mistakes based on estimating partitions (i.e., concludes that 3/5 and 6/10 are not equivalent because in their drawing the points did not exactly match up). **ii. (M)** Does not recognize when "denominators" are easily related as multiples of each other. (i.e., that denominators of 6 and 12 are easily related; but 3 and 5 are not as easily related.) **(M)** Does not connect the multiplicative process for equivalent fractions with division. **iii. (M)** Does not view n/n as equivalent to 1. **(E)** Does not reduce to the simplest form when simplifying fractions. **(E)** Believes that big numbers do not have common factors, and cannot reduce.
4	**Generating Equivalent Models**	**RN.B. 4.2**	**i.** The student generates simple equivalent fractions using a visual model. **ii.** The student finds common denominators needed to write equivalent fractions (e.g., 3/4 as 18/24). The student knows to perform the same operation to the numerator and denominator to generate equivalent fractions. **iii.** The student identifies the greatest common factor to simplify the fraction to its standard form.	**i. (E)** Confuses relative equivalence and absolute equivalence. The fractional representation may be equivalent but the value is not equivalent (i.e., 1/4 of a meter is not the same distance as 3/12 of a kilometer). **ii. (M**) Thinks adding to the numerator and denominator will generate equivalent fractions. **(M)** Thinks multiplying the numerator and denominator by the same number is x times the fraction [$(ax/bx \neq x(a/b)$] **(M)** Cannot relate the process of finding equivalent fractions to the model of the whole (i.e., increasing the denominator by a factor of x (resulting in smaller parts) results in a numerator that is also increased by a factor of x (a larger # of parts) in order to be equivalent to the original fraction).
4	**Equivalence with Magnitude**	**RN.B. 4.3**	**i.** Given a model, the student understands that different fractions can represent the same magnitude.	**i. (E)** Not able to generate equivalent fractions without being given a model.
4	**Lawfulness of Equivalence**	**RN.B. 4.4**	**ii.** The student understands the mathematical reasoning behind generating equivalent fractions ($n/n * a/b = a/b$), including that a number divided by itself is 1 ($n/n = 1$), and the identity property of multiplication ($n * 1 = n$). The student understands dividing the denominator into "n" equal parts results in numerator that is exactly "n" times as big.	

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	LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
DECIMALS	5	**Decimals as Numbers**	**RN.B. 5.1**	**i.** The student approximates length as a decimal and the location of a decimal on a number line to the tenths place.	**i. (M)** Does not understand the meaning of place value (e.g., .08 ≠ .8). **(E)** Incorrect interpretation of place value language. (e.g., student does not distinguish between hundreds and hundredths interpreting 62 hundredths (0.62) as 6200.
DECIMALS	5	**Decimal Notation**	**RN.B. 5.2**	**i.** The student approximates length as a decimal and the location of a decimal on a number line to the hundredths place. **ii.** The student recognizes equivalence between representations of fractions and decimals (for fractions with denominators 10 or 100), as related to money.	**i. (M)** Does not understand equivalence within place value (e.g. that .8 and .80 are the same). **(E)** Mislabeling on number line because of misunderstanding in place value language. The student is not able to locate a decimal value on a number line due to limited knowledge of place value terms. **ii. (M)** Decimal understanding is limited to money situations (e.g., \$3.42). Does not understand the value of digits past the hundredths place. **(E)** Makes mistakes when comparing decimals.
DECIMALS	5	**Decimal Comparison**	**RN.B. 5.3**	**i.** The student measures length as a decimal and the location of a decimal on a number line accurately for any decimal number. **ii.** The student recognizes equivalence between fractional representation and decimal notation (for fractions with denominators 10^k, e.g., 342/1,000) **iii.** The student compares decimals that have the same place value correctly (e.g., 2.34 < 2.45)	**ii. (M)** Does not understand that the fractional portion of a decimal number represents a part of the whole. (e.g., 3.142 is represents 3 whole units and 142 thousandths of a whole) **iii. (M)** Misinterprets decimals with more digits as larger. The student is distracted by the length of a number instead of focusing on the place value (e.g., 0.54 > 0.7). **(M)** Misinterprets decimals with fewer digits as greater values (e.g., thinks 0.5 is larger than 0.77.
DECIMALS	5	**Decimal Representation**	**RN.B. 5.4**	**ii.a.** The student uses place value to read and write decimals (i.e., 5,781.1 = 5*1,000 + 7* 100 + 8*10 + 1 + 1*1/10). The student understands that a digit in the tens place is 10 times the digit in the ones place and that the digit in the ones place is 1/10th of the digit in the tens place. **ii.b.** The student understands that the decimal portion of a number represents a part of another unit (i.e., 3.142 represents 3 whole units and 142 thousandths of another unit). **iii.** The student compares decimals based on the value of the digit in each place. The student understands and applies the strategy for using place value in comparisons (i.e., work left to right in comparing magnitude; lawfulness of adding zeros to place value).	

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COMPARING TWO FRACTIONS	6	**Denominator Reasoning with Models**	**RN.B. 6.1**	**i.** The student compares fractions using models based on the size of denominators (i.e., 1/3 of a pizza is larger than 1/4 of the same pizza).	***(M)*** *Uses whole number reasoning to compare fractions (i.e., 2 is smaller than 4 so 1/2 is smaller than 1/4).* **i. (E)** Without a visual model, the student cannot compare fractions. **(E)** When comparing fractions with unlike denominators, the student only compares the denominator (i.e., 1/4 > 1/3 since 4>3).
COMPARING TWO FRACTIONS	6	**Numerator Reasoning**	**RN.B. 6.2**	**i.** The student compares any two fractions with the same denominators by reasoning about the number of parts with or without a model.	**i. (E)** When comparing fractions with unlike denominators, the student only compares numerators (i.e. 7/10 > 5/3 because 7 > 5). **(E)** Unable to convert fractions to fractions with a common denominator.
COMPARING TWO FRACTIONS	6	**Reasoning with Unlike Denominators**	**RN.B. 6.3**	**i.** The student compares any two fractions with unlike denominators by finding a common denominator. With or without a model, the student reasons about the quantity and size of each part.	
CONVERSION BETWEEN REPRESENTATIONS (connection to division)	7	**Fraction and Decimal Equivalence**	**RN.B. 7.1**	**i.** The student recalls basic fraction/decimal equivalencies, such as 1/2=0.5, 1/4=0.25, 3/4=0.75, 1/3=0.333... **ii.** Given an improper fraction, the student determines the whole based on an understanding of division (i.e. 13/5 has a whole of 2).	***(M)*** *Two equivalent fractions must have different decimal values because they don't "look the same" (e.g. the decimal representation of 32/5 must be different than 64/10).* **i. (M)** Incorrectly assumes the fraction bar, /, represents the decimal point. (e.g., equates 1/3 with 1.3 or 3/5 with 3.5). **(M)** Cannot write decimal equivalencies when fractions are greater than 1 (e.g., they know 1/4=0.25, but cannot connect 3 1/4=3.25). **ii. (M)** When using long division to convert a fraction to decimal form, student is only able to identify the number of "whole" groups; cannot discuss a remainder as a fraction of the whole. **(M)** Thinks division must result in a quotient greater than 1 (i.e., dividing 3 things among 4 people cannot be done). **(E)** Always places the bigger number inside when performing long division. **(M)** Does not connect fraction representation to the division of two numbers (i.e., *a* ÷*b* to the fractional representation a/b.)
CONVERSION BETWEEN REPRESENTATIONS (connection to division)	7	**Fractions as Equivalent to Division**	**RN.B. 7.2**	**i.** The student will use basic unit fractions to generate other equivalences. (i.e., 1/4=0.25, then 3/4=3(1/4)=3(0.25)=0.75). **ii.** The student is able to connect division to fractional representation (e.g., 1/4 is "one-fourth" which is also "one divided by four"). The student is able to convert an improper fraction to a mixed number and/or mixed number to an improper fraction.	**i. (M)** For non-standard fractions cannot convert to a decimal. [i.e., cannot convert 9/8; however, could convert 5/4 by 5(0.25)]. **ii. (E)** Improperly places the decimal point in long division. **(E)** Incorrectly adds the denominator, whole number, and numerator when converting a mixed number to an improper fraction.
CONVERSION BETWEEN REPRESENTATIONS (connection to division)	7	**Using Division to Determine Decimal Equivalents for Fractions**	**RN.B. 7.3**	**i.** The student converts fractions to decimals by using long division. The student places the decimal point correctly (i.e., 3/4 is equivalent to 3.00/4). **ii.** The student is able to justify why division is equivalent to counting unit fractions (13÷5 is the same as 13 fifths).	

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APPLICATIONS OF POSITIVE RATIONAL NUMBERS

	LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
PERCENTS AND RATIOS	8	**Ratios as Percents**	**RN.C. 8.1**	**i.** Given a part and a percentage, the student finds the whole using the same thinking used with rate problems (i.e., If I know 4% is 18, then I can multiply 4% by 25 to get to 100% and 18 by 25 to get to the amount of the whole). **ii.** The student uses proportional reasoning to find a given % of a number (e.g., 32% of 64 is...).	**i. (E)** For situations where the unit rate does not have a whole number factor of 100, the student cannot identify the whole (e.g. does not recognize they must multiply by 100/40 or 5/2 to get 100%). **(E)** Has difficulty setting up proportions (equivalent ratios) to find the percent of the whole (e.g., 32 is what % of 68; 32 is 68% of what number). **(M)** Does not fully understand the connection between different represenations of ratios: fractions, decimals and percents. Recognizes and can rewrite fractions with denominators of 10 and 100 as percents but not others (i.e., 1/10 = 10% but 1/5 ≠ 5%). **(M)** Does not make the connection that percents represent rational numbers. (M) Thinks percents cannot be greater than 100 or less than 1 (i.e., writing 1.45 as .145% or .02% as .02). **(E)** When asked to find a "percent" the student is unable to recognize when a problem requires an exact answer or when an estimation is acceptable.
	8	**Converting Between Percents and Ratios**	**RN.C. 8.2**	**i.** The student understands that a percent is a ratio (*a*/*b*) where the whole is 100, or "per 100". (e.g., 75% is equivalent to 75/100 or "75 out of 100" 2). The student understands the equivalence of different representations of a rational number: percents, fractions, decimals (e.g., 20% = 20/100 = 1/5 = 0.20).	

APPLICATIONS OF POSITIVE RATIONAL NUMBERS

PROPORTIONAL REASONING

LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
9	**Identifying Ratios**	**RN.C. 9.1**	**i.** The student writes verbal statements that describe the relationship between *a* and *b* [e.g., In a classroom of 15 students that include 5 boys and 10 girls, 5:15 (part-whole) represents the ratio between the number of boys and the number of students; 5:10 (part-part) represents the ratio between the number of boys and the number of girls. When simplified 5:10 is equivalent to 1:2 or for every one boy in the class, there are 2 girls].	**i. (M)** Does not understand situational contexts and cannot determine if the problem describes a part-to-whole or part-to-part relationship. **(E)** Does not attend to units when reading and writing ratios. **(M)** Misunderstands a ratio as a part-to-whole when recorded in fraction notation instead of the possibility of part:part. Interprets ratios only as fractions (part-whole) instead of the comparison of two quantities, part-whole AND part-part.
9	**Representing Ratios and Rates**	**RN.C. 9.2**	**ii.** The student uses ratio tables to solve problems without use of a unit rate, by applying multiplicative properties or solving proportional equations. **iii.** The student uses multiplicative reasoning to solve proportions given various models: a strip (tape) diagram, double number line, graph, or equation. **iv.** The student writes a proportional equation to represent two equivalent ratios.	**ii. (M)** Thinks proportions can only be setup one way before solving (i.e., a proportion 4/6 = 6/x can also be written as 6/4 = x/6 and will yield the same answer). **(E)** Ignores key information when setting up a proportion to solve. When representing two equivalent ratios, the student does not attend to labels. **iii. (M)** Has difficulty applying multiplicative reasoning to proportion problems [i.e., given the example, If I earn $15 in 4 hours, how much money will I earn in 9 hours?, the student does not connect the given rate $15 in 4 hrs to the unit rate, $15/4 in 1 hr or $3.75 per hour and thus, the student cannot generalize the amount earned in 9 hrs. to 9($3.75)].
9	**Extending Rates**	**RN.C. 9.3**	**i.a.** The student expresses a unit rate as a part-to-one relationship, where the ratio is a comparison of fraction: fraction or decimal:decimal. Given ratios in the form *a*/*b*, the student represents the ratio as a unit rate (regardless if *a* and *b* are represented as fractions, decimals, in ratio tables, with strip diagrams, or proportions). **ii.a.** The student identifies additive properties within ratios to answer basic questions (e.g., if the ratio of apples to oranges is 5 to 7, then for every 5 apples added, 7 more oranges are added to maintain the 5 to 7 ratio). **ii.b.** The student uses additive properties with unit rates to solve problems, where the unit rate is a rational number [i.e., if a snail mores 1/2 in. per second, then the student applies additive reasonintg by using repeated addition when calculating the distance the snail has traveled in 4 minutes: (1/2 + 1/2 + 1/2 + 1/2) "four times"].	**i.a. (M)** Cannot identify unit rate given two fractions (i.e., 1/3: 1/5 as 5/3:1). **(M)** Thinks rates are restricted to whole number comparisons (i.e., if the ratio is 5:3, the student cannot identify the unit rate as 5/3:1). **ii.a. (M)** Unable to apply additive properties within ratios that require combining parts from different wholes. **ii.b (E)** Incorrectly uses an additive strategy (i.e., 9/10 = x/8, the student says x = 7 because it is 1 less). **(M)** Can solve proportions by scaling up or down (with addition or subtraction) to find the missing value (i.e, 9/2 = x/8, student says 9 for 2 means 18 for 4, 27 for 6, 36 for 8...). **(E)** Incorrectly uses additive and multiplicative strategy (i.e., 9/2 = x/8, student says 9/2 = 4 R1 so they multiply 8 by 4 and then add 1 to solve for x).
9	**Applications of Ratios and Rates**	**RN.C. 9.4**	**iv.** The student solves problems using a combination of multiplicative and additive reasoning with rates (e.g., If I earn $15 in 4 hours, how much money will I earn in 9 hours? 2($15 in 4 hours) + ($3.75/1 hour).	

APPLICATIONS OF POSITIVE RATIONAL NUMBERS

MULTIPLICATION OF POSITIVE RATIONAL NUMBERS

LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
10	**Multiplication of Rational Numbers**	RN.C. 10.1	**i.** The student finds the product of two fractions (or two decimals) or a decimal and a whole number using common algorithms.	**i. (E)** Unable to take the fraction algorithm and apply it to the multiplication of whole numbers (e.g. 3/4 * 5 = 3/4 * 5/1 = (3 * 4)/(5 * 1) instead of 3/4 *5 = 3/4 * 5/1 = (3 * 5)/(4 * 1). **(E)** When multiplying two fractions, adds, rather than multiplies,the numerators and/or denominators.
10	**Multiplication as Repeated Addition**	RN.C. 10.2	**i.** The student models multiplication between fractions (and decimals) and whole numbers as a repeated addition equation. The student can work flexibly with fractions a/b when a >1 (i.e., 3*4/5 = 4/5 + 4/5 + 4/5 = 12/5).	**i. (E)** Decimal multiplication: Thinks the decimal places have to be lined up (as in addition and subtraction). **(E)** Fraction multiplication: Incorrectly cross-multiplies.
10	**Reasonableness of Products**	RN.C. 10.3	**i.** The student makes the connection that if a whole number is multiplied by a fraction or a decimal that is: less than 1, the product will be less than the original whole number; equal to 1, the product will be that whole number; greater than 1, the product will be greater than the original whole number. **ii.** The student understands that multiplication by any number is a form of scaling (resizing).	**i. (M)** Thinks multiplying always gives a product that is bigger than the factors. **(E)** Thinks the order of multiplication of fractions makes a difference (e.g. they see 5 * 3/4 as something different than 3/4 * 5, and do not connect them). **(M)** Does not always interpret representations of area models correctly. **(E)** Misinterprets the shading that represents the product on an area model. **ii.(E)** Makes errors when scaling (resizing).
10	**Representing Multiplication of Rational Numbers**	RN.C. 10.4	**i.** Given an area model representing fraction multiplication, the student gives the product. **Ii.** The student justifies the algorithm for multiplying two decimals by rewriting decimals as whole numbers times fractional powers of ten and multiplying in a different order.	

Learning Progression RN:
Understanding Positive Rational Numbers

	LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
APPLICATIONS OF POSITIVE RATIONAL NUMBERS					
ADDITION & SUBTRACTION OF POSITIVE RATIONAL NUMBERS	11	**Decimal-Fraction Connections**	**RN.C. 11.1**	**i.** The student represents decimal addition or subtraction with equivalent fraction and number line representations.	**i.(M)** Thinks fractions and decimals cannot be mixed. **(E)** Can add or subtract decimals correctly but when writing the equivalent fraction representation, they make mistakes (e.g., .7 + .2 = .9 but 7/10 + 2/10 = 9/20). **(E)** Incorrectly adds or subtracts decimals. **(M)** Adds the numerators and denominators even though the denominators are not common. **(E)** Does not convert fractions to common, equivalent denominators before adding/subtracting (using the larger of the denominators to form an answer).
	11	**Conversions Between Fractions and Decimals**	**RN.C. 11.2**	**i.** The student converts numbers between fraction and decimal forms in order to determine the most efficient method of solving (e.g., given 0.75 + 1/5, a student may rewrite as 3/4+1/5 or 0.75+0.2 before determining a solution). **ii.** The student identifies examples and non-examples of correctly manipulated fraction and decimal equations, such as 1/3 + 4/3 = 5/3 is a true statement while 1/2+2/3=3/5 is not.	
MODELING ADDITION & SUBTRACTION OF POSITIVE RATIONAL NUMBERS	12	**Preservation of the "Whole"**	**RN.C. 12.1**	**i.** The student partitions the "whole" represented by circles and rectangles into equal regions before adding (or subtracting) fractional parts.	**i.(M)** Can draw a diagram of one fraction but when adding two fractions will count the total number of pieces of both fractions instead of recognizing each diagram has the same whole (even when the denominators are the same). **(E)** Makes errors with addition and subtraction when given a model.
	12	**Modeling Addition and Subtraction with Unlike Denominators**	**RN.C. 12.2**	**i.** The student represents addition or subtraction of fractions with fraction strips or a number line model. The student understands that the only way to represent a sum or difference is with a common unit. If the sum exceeds 1, the student can represent the additional whole on the number line.	**i. (E)** Struggles to find equivalent fractions in order to add or subtract. **(E)** Does not line up the decimals before adding.
	12	**Interpreting Fractions with Sums Greater than One**	**RN.C. 12.3**	**i.** The student uses pictures to bridge the notion that addition and subtraction of fractions involves common denominators.	

Learning Progression RN:
Understanding Positive Rational Numbers

APPLICATIONS OF POSITIVE RATIONAL NUMBERS

DIVISION OF POSITIVE RATIONAL NUMBERS

LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
13	**Division of Positive Rational Numbers**	**RN.C. 13.1**	**i.a.** The student finds the quotient of a whole number divided by a rational number, including in a contextual situation. The student uses groups of unit fractions to determine a solution (e.g., 3 ÷ ¼, becomes how many ¼ groups to get 3?). **i.b.** The student finds the quotient of a rational number divided by a whole number, including in a contextual situation, and uses a diagram to justify the result.	**i.a (E)** Does not write a whole number accurately as a fraction with a 1 in the denominator (i.e., ½ ÷ 5, they might come up with 5/2 - not inverting either fraction before multiplying). **(M)** Incorrect application of the definition of division to the fraction (e.g. does not realize that 3 ÷ ¼ can be understood as how many groups of ¼ are in 3). **(E)** Misapply the invert-and-multiply procedure for dividing fractions: 1. Inverting the wrong fraction before multiplying; 2. Invert both fractions before multiplying. **i.b (E)** Does not know where to place the decimal. **(E)** Only moves the decimal place on one number. **(E)** Does not "add" zeros to whole numbers when moving the decimal.
13	**Conceptual Understanding of Partitioning**	**RN.C. 13.2**	**i.** The student partitions a model to represent fraction division.	**i. (E)** Names the quotient incorrectly within a contextual situation with rational numbers (switches numerator and denominator). (E) Does not partition model correctly based on the two given positive rational numbers.
13	**Justifying Division of Rational Numbers**	**RN.C. 13.3**	**ii.** The student uses the standard algorithm of invert-and-multiply to divide two positive rational numbers, including in a contextual situation. The student justifies the equivalence of dividing by a fraction to multiplying by its reciprocal. **iii.** The student justifies the long division algorithm (e.g. they can explain why moving both decimal places results in an equivalent quotient).	