Learning Progression VE:
Understanding Variables, Expressions, and Equations

UNDERSTANDING VARIABLES AND EXPRESSIONS

	LEVEL	TITLE	SUBLEVEL	SUBLEVEL SYNTHESIZED DESCRIPTION	SUBLEVEL MISCONCEPTIONS (M) or STUDENT ERROR (E)
VARIABLE AS AN UNKNOWN QUANTITY	1	**Interpreting Variables Within Expressions**	**VE.A. 1.1**	**i.** Given a situation with an unknown, the student identifies the unknown but does not necessarily use a "variable" in place of the unknown (e.g., for 10 - 3, the student could write 3 + __ = 10).	**i. (E)** Provides the numerical answer (i.e., solves the equation but does not understand how to represent an unknown). **(M)** Given a situation with an unknown, the student misinterprets the mathematical symbol(s) and the vocabulary in the expression (i.e., thinks 3+a and 3a are the same).
	1	**Variable Usage**	**VE.A. 1.2**	**i.** Given a situation with an unknown, the student identifies the unknown and the situation with an algebraic expression using variables to represent the unknown quantity (replacing the informal blanks or boxes). The student understands that variables represent particular numbers, but not necessarily that variables represent unknown quantities.	**i. (M)** Given a simple context, the student cannot always determine if a "variable" should be used (e.g., may use a variable to describe a "known quantity" (or constant). **(M)** Thinks variables can only represent whole numbers. **(M)** Thinks variables always have one specific value.
	1	**Understanding Variables within Context**	**VE.A. 1.3**	**i.** Given a simple variable expression, the student explains why the expression represents a real-world situation with an unknown (e.g., *e*+5, would represent Elton's friend's age since Elton's friend is 5 years older than Elton).	**i. (M)** Unable to identify the unknown value represented by variable(s) in an expression. Does not distinguish between variables and symbols used to represent units (e.g., 5*m*, product of 5 and m, or 5 m, 5 meters).
	1	**Defining Variables within Context**	**VE.A. 1.4**	**i.** Given a simple variable expression, the student determines if it fits a situation and defines the variable in context (e.g., can define *e* = Elton's age).	
EVALUATE	2	**Order of Operations**	**VE.A. 2.1**	**i.** The student understands how to evaluate a single variable expression (e.g., $\frac{5(x+4)}{3-2}$) by substituting in a number for the variable and simplifying (order of operations).	**i. (E)** Unable to evaluate an expression when a variable appears multiple times in the expression [e.g., 5x+(x+2)]. **(M)** Does not understand that the variable represents the same value throughout the expression. **(E)** Unable to correctly apply the order of operations when simplifying a numeric expression (e.g., solving from left to right regardless of order of operations). **(E)** Does not recognize the notation for multiplication [e.g., ab, a(b), a · b) **(E)** Does not substitute the correct value for the variable in an expression.
	2	**Evaluating Expressions with Variables**	**VE.A. 2.2**	**i.** The student understands how to evaluate a single or multi- variable expression given specific values for each variable [e.g., 5x +(x + 2); or 5L+(W-4), given L and W]. The student correctly applies the order of operations to evaluate an expression. The student recognizes the notation for multiplication of variables, and numbers and variables.	**i. (M)** Does not understand that expressions describe a relationship and can be evaluated with different values for a variable. **(M)** Does not understand that a variable expression represents a relationship (or rule) that is independent of input.
	2	**Generating Multiple Values**	**VE.A. 2.3**	**i.** The student understands how to evaluate single or multi-variable expression and identifies values from a specific set of numbers that will make a related verbal statement true (e.g., which values of x, {1, 2, 3, 4}, is the expression ___ greater than 20?).	**i. (M)** Unable to generate their own numbers to evaluate an expression. **(M)** Thinks that whatever change is made in the substituted variable results in the same change in the expression and/or outcome (i.e., increasing input by 1 increases output by 1). **(E)** Does not consider an expression may have multiple input values (i.e., student finds one value that works, and that must be the only answer).
	2	**Self-Generated Numbers**	**VE.A. 2.4**	**i.** The student understands how to evaluate a single or multi-variable expression and **generates** or identifies numbers that will make a related verbal statement true.	

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VERBAL TRANSLATIONS OF ALGEBRAIC EXPRESSIONS AND EQUATIONS	3	**Algebraic Expressions**	**VE.A. 3.1**	**i.** Given a simple verbal statement (e.g., "subtract *y* from 5"), the student writes a corresponding simple variable expression with a single operation.	**i. (M)** Does not understand that the commutative property does not apply to subtraction and division (e.g., *y*-5 does not necessarily imply 5-*y*). **(M)** Does not understand the commutative property of addition and multiplication (e.g., a number increased by 5, can be written as 5+*y*, as well as *y*+5).
	3	**Algebraic Expressions with Multiple Operations**	**VE.A. 3.2**	**i.** Given a series of operations with a single variable, the student is able to write an algebraic expression (or equation) that correctly represents the series of operations according to the order of operations [e.g., "subtract y from 5, then multiply the result by 2, then add 2" is written: 2(5 - *y*) - 2].	**i. (E)** Does not use parentheses to clarify the order of operations (e.g., w+5•2, instead of (w+5)•2). **(E)** The student does not distinguish between situations requiring an expression and situations requiring an equation.
	3	**Translating Real-World Situations**	**VE.A. 3.3**	**i.** For a given verbal statement or real-world situation, the student writes a single or multi- variable expression (or equation) using the correct order of operations (e.g., the perimeter is 30, 2*l*+2*w* = 30).	**i. (M)** In multi-variable expressions, the student does not understand that different combinations of values may result in the same output (e.g., for 2*L*+2*W*, that *L*=2 & *W*=5, and *L*=1 & *W*=6 both result in 14). **(M)** In multi-variable equations, the student does not understand the connectedness of the variables, that values of one variable will affect the values of the other.
	3	**Making Connections Between Variables in Situations**	**VE.A. 3.4**	**i.** For a given verbal statement or real-world situation, the student writes a single or multi- variable expression or equation using the correct order of operations. With or without creating a table of values, the student understands how the variables are related (i.e., a change in one variable affects the others).	

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SIMPLIFYING EXPRESSIONS	4	**Basic Procedural Ability**	**VE.A. 4.1**	**i.** The student understands the connection of a coefficient to multiplication (i.e., 3*y* is 3 times *y*). The student combines "like terms" (i.e., 2x+3x simplifies to 5*x*, and 3+2x+5 simplifies to 8 + 2*x*).	**i. (M)** Does not understand 3*y* and *y* + *y* + *y* are equivalent expressions. **(E)** Incorrectly combines variables and numbers (i.e., *a* + *b* = *ab*, 2*x* + 3 = 5*x*, 2*x*+3*y* = 5*xy*).
	4	**Decomposition with Whole Number Coefficients**	**VE.A. 4.2**	**i.** The student understands the connection of a coefficient to multiplication and basic decomposition. The student can justify why they can combine "like terms"(e.g., 2x+3x= x + x + x + x + x = 5x). The student combines like terms with whole number coefficients and more commonly used decimals. **ii.** The student distributes over parentheses with whole numbers.	**i. (M)** Does not understand that a single variable such as *n*, has a coefficient of "1" (e.g., *n* = 1*n*, or *n*/2 = 1/2 *n*). **(E)** Does not simplify expressions such as, *a*+3*a*, or incorrectly simplifies to $3a^2$ because the student does not recognize the coefficient of *a* as 1. Similarly, the student does not combine *a*/(2+2*a*) to 2.5*a*. **(E)** Has difficulty simplifying more complex expressions (e.g., [(3+5x)-6x+(2+x)4]). **ii. (M)** Does not always understand when parentheses are no longer necessary, [e.g., (3 + 5x)+ 2(2 - x) = 3 + 5x + 2(2 -x)]. **(M)** Limited understanding of appropriate applications of the distributive Property [i.e., the distributive property is used when combining like terms - 3x + 6*x* = (3+6)x = 9x]. **(E)** Incorrectly applies the distributive property [i.e., does not multiply all of the terms in parentheses as in 3(x + 5) = 3x + 5, 5(3+2x)= 8+7x].
	4	**Rational and Unwritten Coefficients**	**VE.A. 4.3**	**i.** The student combines like terms and recognizes the value of the coefficient (including positive rational coefficients) in all situations. Although the student does not combine unlike terms (e.g., 2x+3*y*), the student cannot provide a mathematical explanation beyond "they are not like terms." **ii.** The student applies the distributive property and explains how the distributive property is applied when combining like terms. The student applies distributive properties within multi-variable expressions. **iii.** The student works backward to factor a constant out of simple expressions [e.g., 4x+8*y* = 2(2x+4*y*)].	**i. (E)** When combining more difficult expressions, the student does not rewrite the expression in the simplest form (i.e., they might leave an expression as 3 + 5*x* + *y* + x/2 instead of 5.5*x* + *y* +3). **(M)** Does not recognize when a variable in a fractional expression is a factor in the numerator or denominator [e.g., thinks (5/6)x = 5x/6 is different from (5/6)x = 5/6x]. **ii. (E)** Does not always distribute a negative over parentheses (e.g., 5-(x+2) =5-x+2). **(M)** Does not always use parentheses correctly as grouping symbols, leading to errors when evaluating expressions. **iii. (E)** Does not always factor the GCF constant [e.g., 12x+18*y* as 2(6x+9*y*), not 6(2x+3*y*)].
	4	**Interpret and Combine Complex Expressions**	**VE.A. 4.4**	**i.** The student combines like terms and simplifies complex expressions using order of operations to write equivalent expressions. **ii.** The student recognizes that applying the distributive property results in an equivalent expression (in the sense that the evaluation for any given x in both expressions would yield the same output). The student correctly applies the distributive property in the case of subtraction (e.g., 5-(x+2)=5-x-2). **iii.** The student factors an expression completely (e.g., 4x+8*y* = 4(x+2*y*)where 4 is the GCF between 4x and 8*y*) and relates "combining like terms" to the distributive property (e.g., *y* + *y* + *y* = *y*(1+1+1)=3*y*). The student interprets coefficients (e.g., a 25% discount is the same as finding *c* -.25*c* = 0.75*c*) and rewrites an expression by factoring or combining like terms (e.g., *a*+0.05*a*=1.05*a*) to better understand the relationship represented.	

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EXPRESSIONS AND EQUATIONS

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UNDERSTANDING RELATIONSHIPS BETWEEN EXPRESSIONS IN EQUATIONS	5	**Meaning of the Equal Sign**	**VE.B. 5.1**	**i.** The students interprets the equal sign as a statement of equivalence or balance between two quantities or expressions. The student understands an expression can equal a number (e.g., 2x+4=12).	**i. (M)** Thinks that the equal sign is a prompt to calculate an answer. **(E)** Does not think two non-identical expressions are equivalent even though there is an equal sign between them.
	5	**Meaning of Equations**	**VE.B. 5.2**	**i.a.** The student understands the equal sign as a symbol for equivalent relationships. The student understands that expressions can have variables on both sides, e.g., 2x + 5=4x -7. The student understands that if the outputs are equal when evaluating both expressions at the same "x" value, then that value is a specific solution for "x." **i.b.** The student recognizes that an equal sign can indicate both an identity (equivalent expressions, i.e., true for all "x"), as well as, a conditional statement (expressions that are equal for only one "x" value). The student recognizes that an identity is true for all values of "x."	**i.a (E)** Uses the equal sign to describe equivalent expressions, but does not recognize the statement is an identity, a statement that has an infinite number of solutions, for x (e.g., 2(x+4)=2x+8, which is true for all x values).
SOLVING SINGLE-STEP EQUATIONS	6	**Single-Step Equations Involving Addition and Subtraction**	**VE.B. 6.1**	**i.a.** The student solves simple single variable equations (e.g., x+5=10, not x+5=10x) by using the properties of equality to manipulate the algebraic equation (i.e., addition property of equality, multiplication property of equality). The student understands in general that manipulating the equation using the properties of equality result in an equivalent equation with the same solution(s). **i.b.** Given single-step equations involving addition or subtraction, the student applies the inverse operation to determine the value of the unknown.	**i.a (M)** Does not know that what is done to one side of the equal sign should also be done to the other side in solving. **(E)** Either forgets to do the operation to the second side or does not use parentheses appropriately when multiplying or dividing. **(M)** Does not use the inverse operation when solving.
	6	**Single-Step Equations Involving Multiplication and Division**	**VE.B. 6.2**	**i.a.** The student solves equations that include fractional coefficients and constants (e.g., they can divide both sides by 4/3 or multiply by 3/4 to solve). **i.b.** Given single-step equations involving multiplication or division, the student applies the inverse operation to determine the value of the unknown. **ii.** Given a real-world situation, the student identifies the unknown in order to write an equation; however, for more complicated equations, they may still reason numerically to solve.	**i.a (E)** Does not always know how to solve equations with a variable in the denominator (e.g., for 5/x=10, they divide by 5 and end up with x = 2). **ii.(E)** Can only solve problems one way; does not see that there may be "multiple" ways to solve the problem.
SOLVING MULTI-STEP EQUATIONS	7	**Multi-Step Equations with a Variable on One Side of the Equal Sign**	**VE.B. 7.1**	**i.** The student solves equations with a variable on one side of the equal sign regardless which side of the equal sign the variable appears in the equation.	**i. (M)** Thinks equations cannot have variables on the right side of the equal sign.
	7	**Multi-Step Equations with Variables on Both Sides of the Equal Sign**	**VE.B. 7.2**	**i.a.** The student solves equations with variables on both sides of the equal sign. The student understands that the solution to the equation is the value that makes the equation true when it is substituted back into the equation. **i.b.** The student concludes there is no solution or an infinite number of solutions during the process of solving and can give examples of each.	**i.a. (E)** Does not always check their answer. **(M)** Does not always know how to solve equations that have variables on both sides (e.g., will solve 2x = x+6 as x = (x/2) +3).
MODELING EQUATIONS	8	**Modeling Equations**	**VE.B. 8.1**	**i.** The student uses or interprets drawings to represent an unknown quantity given a contextual situation.	**i.(E)** Cannot write the algebraic statement from a word problem; reluctant to use their algebraic knowledge in real-world situations.

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SOLVING INEQUALITIES	9	**Modeling and Solving Inequalities**	**VE.B. 9.1**	**i.** The student recognizes and names inequality symbols in expressions. **ii.** The student uses a number line to represent solutions for > or < relationships between expressions. The student understands that inequalities may have infinitely many solutions and distinguishes between solution sets with < or ≤, and > or ≥. The student writes an inequality statement based on a verbal description of a real-world situation. **iii.** The student determines if inequalities hold true for specific values. **iv.** The student writes an inequality from a verbal description of a contextual situation.	**i. (E)** Confuses how the symbols relate to expressions (e.g., Correctly reads less than in $2x<12$ but mistakes the expression $2x$ to be the greater value). **(E)** Unable to determine when to use a symbol in contextual situations (i.e., confuses < and ≤). **ii. (M)** Does not recognize that there will be infinitely many solutions. **(E)** Incorrectly states the solution set to $x > 12$ as whole numbers { 13, 14,15, ... } but does not connect to all real numbers > 12 . **(M)** Does not recognize that the relationship between expressions MUST be exactly one of the following relationships:<, >, ≤, ≥, or =. **(E)** Shades solutions on a number line incorrectly. **(E)** When graphing solutions on a number line, the student confuses when to include the endpoint (shade in the circle) and/or which direction to shade. **iii. (M)** Does not recognize discrete solutions in contextual situations; plausible solution sets (i.e., student writes 1.5 as a solution for representing a person, not realizing it is impossible to have 1.5 of a person).