Intervention for Algebra I

Module 2: Lessons

The Meadows Center
FOR PREVENTING EDUCATIONAL RISK

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FOR PREVENTING EDUCATIONAL RISK
THE UNIVERSITY OF TEXAS AT AUSTIN
COLLEGE OF EDUCATION

Mathematics Institute for Learning Disabilities and Difficulties

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Module 2: Expressions, Equations, and Equivalence
Lesson 1

Lesson 1: Evaluating Numerical Expressions

Lesson Objective	Students will evaluate numerical expressions using addition, subtraction, multiplication, and division. Students will use precise mathematical language to describe and identify the terms in an expression.	
Vocabulary	**expression:** a mathematical phrase that combines numbers and/or variables using the operations of addition, subtraction, multiplication, or division; an expression does not contain an equal sign and it represents 1 single quantity **numerical expression:** an expression in which every value is a constant and a number **algebraic expression:** an expression in which 1 or more terms contains a variable **evaluate:** to find the value of an expression **term (in an expression):** the parts of an expression that are joined by addition or subtraction.	
Reviewed Vocabulary	None	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 1-12) • Overhead/document projector	• Student Booklet (pp. 1-6) • Whiteboard with marker

Engage Prior/Informal Knowledge

To open the lesson, present the *Engage Prior Knowledge Practice Sheet*. Ask questions to activate students' background knowledge and prerequisite skills concerning order of operations.

Give students a whiteboard and marker. Have students describe what variables represent and give an example of a variable.

In some mathematical statements we use variables. What are variables? *(symbols, usually letters, that represent 1 value or a set of values)*

On the whiteboard, write an example of the use of a variable. In 1 minute we will all show our examples of the use of a variable.

Pause for students to work. Write an example of the use of a variable and then have students show their examples.

I wrote "2*b* + 1" as my example of the use of a variable, where *b* is the variable that represents a value or set of values. Show me the example you were able to create.

Check students' work before moving on to the *Engage Prior Knowledge Practice Sheet*.

Have students recall the order of operations and write what each letter represents. Explain to students that the order of operations was created so that everyone follows the same order when solving problems.

When working with mathematical statements, we have to follow a process to solve or simplify. The mnemonic PEMDAS, or Please Excuse My Dear Aunt Sally, is used to remember the order of operations. Each letter stands for a mathematical process and the order in which we use to solve.

What does "P" represent? *(parentheses)*

Write and have students write “parentheses” next to the “P” on the *Engage Prior Knowledge Practice Sheet*.

> **Write “parentheses” next to the “P” on your sheet. Parentheses means that we perform the operation or task inside the parentheses first.**
>
> **What does “E” represent?** *(exponents)*

Write and have students write “exponents” next to the “E” on the *Engage Prior Knowledge Practice Sheet*.

> **Write “exponents” next to the “E” on your sheet. What do M/D, and A/S represent?** *(multiplication/division; addition/subtraction)*

Write and have students write “multiplication/division” next to “M/D” and “addition/subtraction” next to “A/S” on the *Engage Prior Knowledge Practice Sheet*.

> **Write “multiplication/division” next to “M/D” and “addition/subtraction” next to “A/S” on your sheet. If I have a mathematical statement that includes multiplication and division, which operation do I perform first?** *(perform the leftmost operation first; always move left to right)*

Write and have students write “(left to right)” on the sheet next to where they wrote “multiplication/division.” Write “10/2 * 3” on your copy of the *Engage Prior Knowledge Practice Sheet* so students can answer questions about it.

> **Write “(left to right)” next to where you wrote “multiplication/division.” For example, if we have 10/2 * 3, what would we do first?** *(divide 10 by 2)*
>
> **How do you know?** *(division is first, going left to right)*
>
> **If we multiplied the 2 and 3 first, we would have 10/6 as the answer, instead of the correct answer, 6. If I have a mathematical statement that includes addition and**

subtraction, which operation do I perform first? *(always move left to right; perform the leftmost operation first)*

Write and have students write "(left to right)" on the sheet next to "addition/subtraction." Write "1 – 3 + 5" on your copy of the *Engage Prior Knowledge Practice Sheet* so students can answer questions about it.

Write "(left to right)" next to where you wrote "addition/subtraction." For example, if we have 1 − 3 + 5, what would we do first? *(subtract 1 − 3)*

How do you know? *(subtraction is first, going left to right)*

Subtracting 3 first and then adding 5 results in 3. If we add 3 and 5 first, then the result would be -7. Remember, for multiplication/division and addition/subtraction, we move left to right. Following the correct order of operations is very important to achieve the correct result.

Watch For	The mnemonic PEMDAS may imply that the order of operations is to multiply then divide, and then add and subtract. Reinforce that these operations (multiplication/division and addition/subtraction) are to be done from left to right.

Preview

This lesson will build on students' knowledge of evaluating numerical expressions.

Today we will evaluate *numerical expressions* using order of operations. In other words, we will find the value for number phrases by performing math operations in the correct order.

Demonstrate

1. Present the *Demonstration Practice Sheet* to students.

   Using an overhead projector or document projector, display your hard copy of the *Demonstration Practice Sheet* for students to see. Have students look at the Expressions section on the *Demonstration Practice Sheet*. Define expression.

   Look at the Expressions section on your Demonstration Practice Sheet. We are going to work with *expressions*, so we will discuss what an *expression* means.

   Write and have students write examples of expressions in the space provided on the *Demonstration Practice Sheet*. Discuss characteristics of the examples.

   We are going to write some examples of *expressions* and then discuss the characteristics. Write these examples at the top of your Demonstration Practice Sheet.

   - $2x + 4 - 1$
   - $3 + 5 + \frac{10}{2}$
   - $-4y - 6$
   - $3(h - 4) + 7h$
   - $-5 + 9 - 11$

   Looking at these examples we can notice several characteristics. One characteristic is that there is no equal sign. What is another characteristic you notice? *(answers may vary; addition; multiplication; subtraction; variables)*

The use of mathematical operations combines the different values. Also, notice that some of the *expressions* use variables, while others do not.

What is the definition of a variable? *(a symbol, usually a letter, that represents a value or set of values)*

Write and have students write the definition of "expression" in the space provided.

An *expression* is "a mathematical phrase that combines numbers and/or variables using the operations of addition, subtraction, multiplication, or division. An *expression* does not contain an equal sign and it represents one single quantity." Write the definition of *expression* in the space provided on the Demonstration Practice Sheet.

Pause for students to write.

In your own words, what is an *expression*? *(answers will vary)*

Define terms in an expression. Circle and have students circle the terms in the expression "$4x - 5 + 3x + 3$."

An *expression* is made up of *terms*. The *terms* in an *expression* are the parts that are joined by addition or subtraction symbols.

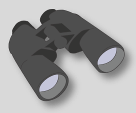
Look at the *expression* "$4x - 5 + 3x + 3$." The *terms* in this *expression* are joined by addition or subtraction symbols. The first *term* is $4x$. Circle the *term* "$4x$." What is the next *term* in this *expression*? *(5)*

Circle the *term* "5." Now circle the rest of the *terms* in this *expression*.

Pause for students to work.

What other terms did you circle? *(3x, 3)*

Watch For



Students may be confused about the inclusion or exclusion of operational symbols when circling terms. The operations give the terms context in relation to each other, but are not part of the term itself.

> **The operations give the *terms* context in relation to each other, but are not part of the term itself. For example, the subtraction sign between 4*x* and 5 describes the relationship between the 2 terms.**

Have students write an expression on their whiteboards and exchange their whiteboards with a neighbor. Have students circle the terms of their neighbors' expression.

> **On your whiteboard, write down an *expression*.**

Pause for students to write. Have students show you their work after circling each term.

> **Exchange your whiteboard with a neighbor and circle all the *terms* in your neighbors' *expression*. When you have finished, show me your work.**

Discuss with students the difference between numerical and algebraic expressions.

> **An *expression* can be either *numerical* or *algebraic*. A *numerical expression* is an *expression* in which every value in the *expression* is a constant and also a number. This means that there are no variables in a *numerical expression*.**

Write and have students write the expression "7(15) – 5" in the Numerical Expressions section on the *Demonstration Practice Sheet*.

One example of a *numerical expression* is 7(15) − 5. This is a *numerical expression* because there are no variables in this expression. Write "7(15) − 5" in the Numerical Expressions box on your Demonstration Practice Sheet.

Have them create their own example of a numerical expression

On the whiteboard, I want you to think of another example of a *numerical expression* and write it down. Remember, an *expression* does not have an equal sign.

Pause for students to work.

Check for understanding by having students show their examples on their whiteboards.

Show me the *numerical expressions* that you created.
(answers will vary; check that students wrote examples that do not contain the equal sign, and that each value in the expression is known)

As students supply their examples, write and have students write them in the Numerical Expressions box on the *Demonstration Practice Sheet*.

We now have a list of *numerical expressions*. Write them in the Numerical Expressions box on your Demonstration Practice Sheet.

The other type of *expression*, *algebraic expression,* is an *expression* in which one or more of the *terms* contains a variable.

Write the expression "$7x - 5$" in the Algebraic Expressions box on the *Demonstration Practice Sheet*.

One example of an *algebraic expression* is $7x - 5$ because one of the terms, $7x$, contains a variable. Write this example in the *Algebraic Expressions* box on your Demonstration Practice Sheet.

Have students come up with their own example of an algebraic expression and write it on their whiteboards.

I want you to think of another example of an *algebraic expression* and write it on your whiteboard. Remember that an *algebraic expression* has at least 1 *term* containing a variable.

Pause for students to work.

Check for understanding by having students show the examples on their whiteboards.

Show me the *algebraic expressions* that you created. *(answers will vary; check that students wrote examples that do not contain the equal sign and have at least 1 term with a variable)*

As students supply their examples, write and have students write them in the Algebraic Expressions box.

We now have a list of *algebraic expressions*. Write them in the Algebraic Expressions box on your Demonstration Practice Sheet.

2. Evaluate numerical expressions.

Define what it means to evaluate an expression and guide students through the process of evaluation in the Evaluating Numerical Expressions section of the *Demonstration Practice Sheet*.

Now we are going to evaluate *numerical expressions*. How can you tell if an *expression* is *numerical*? *(answers may vary; numerical expressions do not contain variables; expressions have only constants or numbers)*

When evaluating *numerical expressions*, we will determine the number that the *expression* is equal to. We do this by performing all of the operations in the *expression*.

Look at the first *expression* in the Evaluating Numerical Expressions section of your Demonstration Practice Sheet. In order to *evaluate* 13 − 2(5), we need to perform the operations that we see in the *expression*. What operations do you see in this *expression*? *(subtraction and multiplication)*

What operation should I perform first? *(multiplication)*

Why? *(multiplication is before subtraction in the order of operations)*

What is 2(5)? *(10)*

Write and have students write "13 – 10" on the *Demonstration Practice Sheet*.

This means we write "13 − 10" below the original *expression* based on the multiplication we performed.

What is 13 − 10? *(3)*

We *evaluated* the *expression* 13 − 2(5), and found that it is equal to 3. Remember, when *evaluating*, all *numerical expressions* are equal to a single value, so 13 − 2(5) is equal to 3.

Have students evaluate the next expression on their whiteboards.

Look at the next *expression*. On your whiteboard, *evaluate* the *expression* 3(2) + 5(11). Remember to use the order of operations to *evaluate* the *expression* correctly.

Pause for students to work. When finished, have them compare their work with a partner.

Now turn to a partner and compare your work. If your answers are different, figure out the correct method for *evaluating*.

Check for understanding by having students show the evaluated expressions on their whiteboards.

When you and your partner have the same value for the *expression*, show me your results. *(61; check students' order of operations to correct any misconceptions)*

Have student pairs explain how they evaluated the expression.

What was the first step for *evaluating* this *expression*? *(multiplication of 3(2) and 5(11))*

What did you write after the multiplication? *(6 + 55)*

How did you get the result of 61? *(added 6 and 55)*

3. Guide students through evaluating the remaining 2 expressions on the *Demonstration Practice Sheet*.

   Use guiding questions to elicit verbal responses from students.

   - What is the first operation?
   - What is the result of performing [insert operation]?

4. Provide a summary of the lesson content.

   Review the key ideas of the lesson by asking questions about expressions and terms.

What is an *expression*? *(a collection of numbers that is combined by operations of addition, subtraction, multiplication, and division)*

How do you know an *expression* is *numeric*? *(the expression has only numbers or constants and no variables)*

How do you know an *expression* is *algebraic*? *(the expression has at least 1 term with a variable)*

What does it mean to *evaluate* an *expression*? *(to find the value of an expression or statement)*

Practice

Guided Practice

1. Have students circle the terms in each expression and determine whether it is a numerical or algebraic expression. Have students evaluate the last 2 numerical expressions.

2. Use probing questions similar to those in the Demonstrate section, such as the following, to guide students through the problem-solving process:

    - What are the terms in this expression?
    - What operations join the terms?
    - How do you know that this is a numerical expression?
    - How do you know that this is an algebraic expression?
    - What is the first operation we need to perform? How do you know?

Pair Practice

1. Have each student in a pair create the numerical expression for each problem.

2. Have partners trade papers and evaluate their partner's expressions.

3. Allow time for students to trade back papers and discuss their answers.

Error Correction Practice

1. Have students examine the work of Students 1-3 and determine why the 3 students arrived at different answers.

2. Have students record their thinking below each example and be prepared to discuss their results.

Independent Practice

1. Have students work independently to match expressions in the first column to the equal value in the second column on the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson and discuss the following questions:

- What are the terms in this expression?
- What operations join terms?
- How do you know that an expression is numerical?
- How do you know that an expression is algebraic?
- What does it mean to evaluate an expression?

Module 2: Expressions, Equations, and Equivalence
Lesson 2

Lesson 2: Testing Numeric Expressions for Equivalence

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students will evaluate and create equivalent numeric expressions.

Students will communicate mathematical thinking about evaluating and equivalent numeric expressions using precise vocabulary coherently and clearly to peers.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">equivalent expressions: two expressions whose values are equal for all replacements of the variable or variables

equation: a math sentence that states that 2 expressions are equivalent</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">evaluate, expression (algebraic/numeric), term</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 13-22)
• Overhead/document projector</td><td>• Student Booklet (pp. 7-11)</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss student responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills about algebraic and numeric expressions.

Give each student a piece of paper. Have students review concepts of algebraic expressions, numeric expressions, and evaluating by writing answers to the following questions, sharing the answers with a partner, and then sharing with the class if called on to respond.

In your own words, what is the definition of an expression? *(answers will vary)*

How can you tell if an expression is numeric? *(the expression has only numbers or constants, and no variables)*

How can you tell if an expression is algebraic? *(the expression has at least 1 variable term)*

What does it mean to evaluate an expression? *(to find the number that the expression is equal to)*

Review answers for problem 1 on the *Cumulative Review Practice Sheet*.

What type of expression is problem 1 on the Cumulative Review Practice Sheet? *(algebraic expression)*

How do you know? *(there is a variable term, 4y)*

Review answers for problem 2 on the *Cumulative Review Practice Sheet*.

When you evaluated the expression, what operation did you perform first? *(multiplied 14 and 2)*

Why? *(order of operations, PEMDAS)*

What was the value of the expression? *(18)*

Preview

This lesson will build on students' knowledge of evaluating and creating equivalent numeric expressions.

Today we will evaluate and create *equivalent* numeric expressions. In other words, we will find the value of a numeric expression and write an expression with the same value.

Demonstrate

1. Present the *Demonstration Practice Sheet* to students.

   Have students look at the Evaluating Expressions section on the *Demonstration Practice Sheet* and examine the 4 expressions listed. Discuss what it means to evaluate an expression and guide students through evaluating the first expression. Use an overhead projector or document projector to display your hard copy of the *Demonstration Practice Sheet* for students to see.

   Look at the Evaluating Expressions section on your Demonstration Practice Sheet. There are 4 expressions that we need to evaluate.

   Today we will work with numeric expressions. How can we tell if an expression is a numeric expression? *(every value is known; it does not contain variables)*

   Remember, when evaluating numeric expressions, we will determine the number that the expression is equal

to. We do this by performing all of the operations in the expression.

In order to evaluate the first expression, 20 − 2, we need to perform the operation of subtraction. What is 20 − 2? *(18)*

Write and have students write "20 – 2 = 18" under the expression 20 – 2.

We evaluated the expression 20 − 2 and found that it has a value of 18. Because the expression 20 − 2 is equal to 18, we can write it as an equation. Write "20 − 2 = 18" on your paper below the expression. I want you to evaluate the next 3 expressions.

Teacher Note
Allow students to use a calculator for calculations as necessary. It is important that the students understand the concept and not get lost in the calculations.

Pause for students to work.

What did you notice about all of the expressions? *(each expression was equal to 18)*

Because each expression was equal to the same value, 18, they are called *equivalent expressions*.

2. Define "equivalent expressions" and write equivalent expressions as an equation.

Write and have students write the definition for equivalent expressions in the space provided.

***Equivalent expressions* are two expressions whose values are equal for all replacements of the variable or**

variables. Write the definition of *equivalent expressions* in the space provided on the Demonstration Practice Sheet.

Pause for students to write.

How do you know that the 4 expressions listed above are equivalent? *(when evaluated, all of the expressions are equal to the same value, 18)*

3. Define "equation" and write equivalent expressions as equations.

   Write "expression = expression" under the definition for equation.

   When 2 expressions are equivalent, they form an *equation*. An *equation* is a math sentence stating that 2 expressions are equivalent. Write the definition in the space provided.

   Pause for students to write

   To remember this, write "expression = expression" underneath your definition of *equation*.

4. Practice writing equivalent expressions as equations. Have students examine the equivalent expressions 11(2) + 4(2) and 2(15). Verify that they are equivalent and write an equal sign between them.

   Look at the expressions 11(2) + 4(2) and 2(15). Evaluate both expressions.

   Pause for students to work.

   What is the value of each expression? *(both equal 30)*

   Because these 2 expressions are both equal to 30 when evaluated, they are equivalent.

When 2 expressions are equivalent, we can write them as an *equation* using the equal sign. Write the equal sign now.

Have students examine the nonequivalent expressions 11(2) + 4(2) and 4(20 – 10). Verify that they are not equivalent and write a not equal sign between them.

Look at the expressions 11(2) + 4(2) and 4(20 − 10). Evaluate both expressions.

Pause for students to work.

What is the value of each expression? *(30 and 40)*

These 2 expressions are not equal, which we can see when they are evaluated. The expression 11(2) + 4(2) is equal to 30, while the expression 4(20 − 10) is equal to 40.

When 2 expressions are not *equivalent*, they do not form an *equation* and we say that they are not equal.

Draw a "not equal" sign between the two expressions.

The sign to indicate that values or expressions are not *equivalent* is called the "not equal" sign. Write the not equal sign now.

5. Determine whether the 2 expressions in problem 1 on page # of the *Demonstration Practice Sheet* are equivalent.

   Have students look at problem 1 in the Are They Equivalent? section. Guide students through the process of determining whether the 2 expressions presented are equivalent.

   Look at the expressions 16 − 2(5) and 36 ÷ 6 − 1. Are these 2 expressions *equivalent*? *(no)*

   How do you know? *(the expressions do not equal the same value when evaluated; the left expression is valued at 6 and the right expression is valued at 5)*

Write and have students write the not equal sign to denote not equivalent.

> **Because the expressions are not *equivalent*, we will write a not equal sign between them. Write the not equal sign now.**

6. Determine whether the 2 expressions in problem 2 are equivalent.

Have students look at problem 2. Guide students through the process of determining whether the 2 expressions presented are equivalent.

> **Look at the expressions 0 − (2)(5)(1) and 14 − 24. Are these 2 expressions *equivalent*?** *(yes)*

> **How do you know?** *(both expressions are equal to -10 when evaluated)*

Write and have students write the equal sign to denote equivalence.

> **Because the expressions are *equivalent*, we will write them as an equation. Write the equation now.**

7. Repeat the process of checking for equivalence in problems 3 and 4.

Have students use an equal sign or not equal sign to show whether the 2 expressions are equivalent. Use specific questions, such as the following to check for student understanding:

- Are the 2 expressions equivalent?
- How do you know?
- What symbol do we write to show the [equivalence/nonequivalence]?

8. Create equivalent expressions that represent the same quantity.

Have students look at the Creating Equivalent Expressions section on the *Demonstration Practice Sheet*.

Look at problem 1 in the Creating Equivalent Expressions section on the Demonstration Practice Sheet. We are asked to create 4 different expressions that represent, or are equal to, the quantity 30. This means that we are to create 4 expressions that are *equivalent* to the value of 30.

At least 1 expression must have 2 different operations, for example $3(5) + (19 - 4)$. This expression uses the operations multiplication, addition, and subtraction. The expression also uses the parentheses to group 19 and 4.

Use questions such as the following to guide students' thinking when creating equivalent expressions.

- Can you think of 2 numbers that, when multiplied, are equal to 30?
- Can you think of 2 numbers that, when added, are equal to 30?
- Can you think of 2 numbers that, when subtracted, are equal to 30?
- Can you think of 2 numbers that, when divided, are equal to 30?

Have students share the expressions they came up with and check that they are all equivalent with a partner's.

Turn to a partner to exchange your expressions. Verify that your partner has created *equivalent* numeric expressions.

Pause for students to work. Call randomly on student pairs to select a few examples for the whole class to review.

> **Because all of the expressions that we came up with are equal to 30, they are all *equivalent*.**

9. Repeat the process of having students create equivalent expressions.

Have students work with a partner to create 4 expressions that are equivalent to the value of 40.

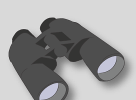
> **Work with a partner to create 4 expressions that are *equivalent* to the value of 40. For example, 52 − (24/2) is equivalent to the value of 40. What are some questions you can ask each other to create *equivalent* expressions?** *(answers will vary; what 2 numbers add up to 40; multiply to 40; when subtracted equal 40; when divided equal 40)*

Pause for student pairs to work. Have student pairs verify that they know the expressions are equivalent because they are all equal to the same value. Have student pairs share with the class.

> **What expressions did you create that are *equivalent* to the value of 40?** *(answers will vary)*

> **How did you know the expression is *equivalent* to 40?** *(answers will vary; when we evaluated the expression it was equal to the value of 40)*

Watch For



Students may not recognize that an expression stands for a value. Use precise mathematical language when describing expressions. For example, instead of saying "9 plus 1," saying "the sum of 9 and 1" may reinforce the idea that the expression "9 + 1" represents 1 quantity.

10. Provide a summary of the lesson content.

Review the key ideas of the lesson by asking questions about equivalent expressions.

How do you know if 2 numeric expressions are *equivalent*? *(when evaluated, the expressions yield the same value)*

What symbol do we use to indicate that 2 expressions are *equivalent*? *(the equal sign)*

What symbol do we use to indicate that 2 expressions are not *equivalent*? *(not equal sign)*

Practice

Pair Practice

1. Have each student in a pair create 2 expressions that are either equivalent or not equivalent.
2. Have the partners trade papers and determine whether their partner's expressions are equivalent or not equivalent. Have students rewrite the expressions with "=" or "≠" to show whether the 2 expressions are equivalent or not.
3. Allow time for students to trade back papers and discuss their answers.

Independent Practice

1. Have students work independently to match the expressions in the first column to the equivalent expression in the second column on the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- How do you know whether 2 expressions are equivalent?
- If 2 expressions are equivalent, what symbol can you use to show that they are equivalent?
- If 2 expressions are not equivalent, what symbol can you use to show that they are not equivalent?

Module 2: Expression, Equation, and Equivalence
Lesson 3

Lesson 3: Evaluating Algebraic Expressions

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students will evaluate algebraic expressions for specific numeric values of given variables.

Students will communicate mathematical thinking for evaluating algebraic expressions using precise vocabulary/steps/strategies coherently and clearly to peers and teachers.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">coefficient: the number in front of the variable that is multiplied by the variable; the coefficient indicates the number of times the variable is added to itself (repeated addition)

substitution: a replacement of 1 mathematical value for another value, usually used to solve</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">evaluate, expression (algebraic and numeric)</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 23-30)
• Overhead/document projector</td><td>• Student Booklet (pp. 13-16)</td></tr>
</table>

Cumulative Review

Have students answer the review questions independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills regarding expressions and equivalence.

Recall the process and definition for "evaluate." Have students look back at previous lessons to recall the definition for "evaluate."

In a previous lesson, we evaluated numeric expressions. For example, what value would we obtain for the numeric expression 12/3 + 2 − 5(2)? *(-4)*

How did you get this answer? *(the order of operations indicates that we work from left to right when completing division and multiplication; division occurs first, then multiplication, 4 + 2 − 10, next perform all addition and subtraction working from left to right)*

What is the definition of "evaluate"? *(to find the value of an expression)*

What value did you find for the expression in problem 1 on the Cumulative Review Practice Sheet? *(22)*

What makes 2 expressions equivalent? *(they have the same value when evaluated)*

Which expression was equivalent to problem 2 on the Cumulative Review Practice Sheet? *(**B**)*

Which expression was equivalent to problem 3? *(**C**)*

Preview

This lesson will build on students' knowledge of evaluating algebraic expressions.

Today we will evaluate algebraic expressions for specific numeric values. In other words, we will replace the variables with specific values to create a list of values for algebraic expressions.

Demonstrate

1. Explain how to use substitution to evaluate algebraic expressions.

   Engage students in a dialogue about the idea of substitution and how it is used in mathematics with algebraic expressions.

   Previously we have worked with numeric expressions. What is a numeric expression? *(an expression in which every value is known and is made up of all numbers)*

   What makes an expression algebraic? *(at least 1 term in the expression has a variable)*

   To evaluate algebraic expressions, we want to find the value of the expression. We use *substitution* to evaluate an algebraic expression.

   If we have a *substitute* in class, what person is the *substitute* replacing? *(teacher)*

   Define substitution.

   The definition of *substitution* is a replacement of 1 mathematical value for another value. In mathematics, we use *substitution* of numeric values, or replacing numeric values for variables, to evaluate an algebraic expression.

2. Guide students through the process of evaluating an algebraic expression for a specific numeric value using the *Demonstration Practice Sheet*.

Write and have students complete the *Demonstration Practice Sheet* as you evaluate an algebraic expression for a given value of x .

The first expression you will evaluate using *substitution* is 3*x* + 2, when *x* = 5. This means we will *substitute* or replace the variable *x* with the value 5. The arrow is pointing to the place where we will write the value "5."

Because we are *substituting* a numeric value for the variable, we must use parentheses to indicate multiplication. Inside the parentheses, write "5".

Now we read this expression as the product of 3 and 5 plus 2. Based on the order of operations, what is the first operation we perform? *(multiply 3 and 5)*

Write and have students write "15 + 2" below the expression 3(5) + 2.

Write "15 + 2" below the expression.

What is the result of the addition? *(17)*

Write and have students write "17" as the final value of the expression.

Write "17" below as the final value of the expression.

If we do not include the parentheses, then this expression would read 35 plus 2. This error would result in a value of 37.

3. Define "coefficient."

Discuss and illustrate the definition of "coefficient" with the expression on the *Demonstration Practice Sheet*.

> **When evaluating algebraic expressions there are *coefficients* for all variables.**
>
> **The definition of *coefficient* is the number in front of a variable that is multiplied by the variable. The *coefficient* indicates the number of times the variable is added to itself (repeated addition).**

Have students determine the coefficient in the algebraic expression in problem 1 with a partner.

> **Look at the algebraic expression in problem 1, $3x + 2$. With your partner, identify the coefficient. You and your partner will have 1 minute to determine the coefficient in the expression. Be prepared to explain your decision.**

Watch For 	**Students may not use parentheses when substituting values into an expression. They may use arithmetic ideas resulting in errors. For example 5 + ½ = 5½, but 4 x 2 ≠ 42.**

Pause for student pairs to work. Call on students randomly to identify the coefficient. Be sure to have student pairs explain their reasoning.

Teacher Note

Some students may need more examples with 2 terms before moving forward to problems with more than one substitution.

4. Guide students through the process of evaluating an algebraic expression for a specific numeric value in problem 2 using the *Demonstration Practice Sheet*.

   Ask guiding questions to lead students to verbalize how they are evaluating the expressions.

   Look at problem 2. Again we have the variable *x* in the expression. How many *x*'s do you see in the algebraic expression? *(2)*

   What is the definition of *coefficient*? *(the number in front of the variable that is multiplied by the variable)*

   If we have the variable *b* multiplied by 1, what is the result? *(b)*

   When we see a variable without a coefficient written in front, then the coefficient is 1. We do not have to write the number "1" because the result of multiplying any variable by 1 is just the variable. Looking at the variables in the expression for problem 2, what are the *coefficients* for each *x*? *(1 and 4)*

   What value are we *substituting* or replacing for *x*? *(2)*

   Because we have 2 *x*'s, we will *substitute* the value 2 into the expression twice. What does it mean to *substitute*? *(answers may vary; replace a value with another value; replace a variable with a numeric value)*

   What symbols do we use to *substitute* a value in an expression? *(parentheses)*

Write down the numeric expression with the value *substituted* for *x*.

Give students time to write.

Why do we use parentheses to *substitute* values for variables? *(the parentheses show multiplication of the coefficient and the value being substituted)*

What did you write down? *((2) − 3 + 4(2))*

Based on the order of operations, what is the first operation we perform to evaluate the numeric expression? *(multiply 4 and 2)*

Complete the multiplication and write down the new expression below.

Pause for students to work.

What numeric expression did you write down? *(2 − 3 + 8)*

Write and have students write "2 − 3 + 8" on the *Demonstration Practice Sheet*.

Write "2 − 3 + 8" on the Demonstration Practice Sheet.

What is the next operation we are to perform based on the order of operations? *(subtract 3 from 2)*

Allow time for students to complete the operations. Write and have students write "-1 + 8" on the *Demonstration Practice Sheet*.

Write "-1 + 8" on the Demonstration Practice Sheet.

What is the value of the algebraic expression when evaluated for x = 2? *(7)*

Write "7" below the expression.

5. Guide students through creating a table of values for the algebraic expression in problem 3 using the *Demonstration Practice Sheet*.

Complete and have students complete the Process column for the table to evaluate the expression for each value of x. Remind students to use parentheses when substituting.

We are continuing to evaluate the algebraic expression for given values of *x,* but now we are creating a table to organize a list of several values for the expression.

Looking at the table, there is a Process column where we will use *substitution* to evaluate for each *x* value.

Looking closely at the expression, what is the *coefficient* for the variable in the expression? *(-1)*

Because there is only a negative sign in front of *x,* the *coefficient* is -1. The expression in problem 3 is read, "opposite of *x* added to 6."

Teacher Note
When expressions have $-x$, it should be read as "opposite of x," rather than "negative x." Students need to focus on "opposite of" so they do not assume all values will be negative.

Write and have students write "-1(-2) + 6" in the Process column on the *Demonstration Practice Sheet*.

When we *substitute* -2 for *x*, we write the expression as "-1(-2) + 6" in the Process column.

Based on the order of operations, what operation do we perform first? *(multiply -1 and -2)*

What is the result of multiplying -1 and -2? *(2)*

When you perform the last operation, what is the result of the expression when evaluated for *x* = -2? *(8)*

Write "8" in the last column on the table.

Let's complete the Process column for the rest of the *x* values.

Guide students through completing the next row in the table using guiding questions to elicit verbal responses.

What is the next value we are to use to evaluate the expression? *(0)*

How do we evaluate the expression for *x* = 0? *(answers may vary; substitute the value 0 in for the variable x)*

How do we write the expression with the value *substituted*? *(-1(0) + 6)*

Write and have students write "-1(0) + 6" in the Process column on the *Demonstration Practice Sheet*.

Write "-1(0) + 6" in the Process column.

How do we evaluate the expression -1(0) + 6? *(multiply -1 and 0, then add 6)*

What is the value of the algebraic expression when evaluated for *x* = 0? *(6)*

Have students work with a partner to complete the last 2 rows of the table.

With a partner, complete the last 2 rows of the table. Use the Process column to help complete the evaluation process.

Give students time to complete the Process column. Ask students what they wrote in the Process column and the result of the evaluation for a given x value.

Teacher Note

Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with student's names on them when selecting students to share their answers or reasoning.

What did you write in the Process column for x = 2? *(-1(2) + 6)*

What is the value of the expression when evaluated for x = 2? *(4)*

What did you write in the Process column for x = 4? *(-1(4) + 6)*

What is the value of the expression when evaluated for x = 4? *(2)*

We have completed a table that lists values for the expression, $-x + 6$.

6. Continue with evaluating the expression for problem 4.

   Guide students through the process to substitute the values for x and evaluating the expression to generate the table of values. Use specific questions, such as the following, to elicit verbal responses from students to check for their understanding.

   - What are the coefficient(s) for the variable(s)?
   - How do you evaluate an algebraic expression?
   - How do you write the expression with the numeric value substituted?
   - What is the process to evaluate the expression [expression]?

- What is the value of the expression?

7. Provide a summary of the lesson content.

   Review the key ideas of the lesson by asking questions about evaluating algebraic expressions.

   When evaluating algebraic expressions, what is the thinking process and questions you ask yourself? *(what are the coefficients for the variables; how do I use parentheses; how do I write the expression with the values substituted; what is the process to evaluate the expression; what operation do I perform first)*

   Why do we use parentheses when *substituting* a value into an expression? *(indicates multiplication; keeps signs straight)*

   What is a *coefficient*? *(the number multiplied by a variable; repeated addition)*

Practice

Based on students' success, the Practice can be guided or a pair practice.

1. Have students work to evaluate the expression for each value of x and then match the correct value. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.

2. Have students present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions, such as the following, to elicit detailed responses:

   - What are the coefficients for the variables?
   - How did you write the expression with the value substituted?

Independent Practice

1. Have students work independently to evaluate the expression for each value of x on the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson and discuss the following questions:

- Why do we use parentheses to substitute a value into an expression?
- What does it mean for a number to be a coefficient?

Lesson 4: Testing for Equivalent Algebraic Expressions, Part I

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students will check for equivalence of algebraic expressions by constructing a pictorial representation of each expression.

Students will use representations to communicate equivalence/nonequivalence of algebraic expressions to peers and teachers.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">algebraic expression, equivalence, expression, numeric expression, terms</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 31-40)
• Overhead/document projector
• Algebra tiles</td><td>• Student Booklet (pp. 17-21)
• Algebra tiles
• Whiteboard with marker</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills regarding expressions (algebraic and numeric) and equivalence.

Have students recall the definitions of expression and equivalence. Have students explain how to determine whether 2 numeric expressions are equivalent.

What is the definition of an expression? *(a collection of numbers and/or variables that are combined using the operations of addition, subtraction, multiplication, or division)*

What is the definition of equivalent expressions? *(2 expressions that are equal for all replacements of the variable or variables)*

How can you tell whether 2 numeric expressions are equivalent? *(when evaluated, the expressions have the same value)*

What is the difference between algebraic and numeric expressions? *(answers will vary; algebraic expressions have variables, while numeric expressions do not)*

Have students explain how they found the equivalent numeric expression in problem 1 on the *Cumulative Review Practice Sheet*.

Which expression is equivalent to problem 1? *(**B**)*

How did you know? *(when evaluated, both expressions are equal to 44)*

Give each student a whiteboard and marker. Have students write an equivalent numeric expression to 3(8 – 2) on their whiteboard. Have them show their equivalent expression to their neighbor.

> **On your whiteboard, create an equivalent numeric expression to 3(8 − 2). When you have written your equivalent expression, show your neighbor and check each other's work.**

Pause for students to work. Randomly call on students to check for their understanding.

Preview

This lesson will build on students' knowledge of equivalent algebraic expressions.

> **Today we will check for equivalence using pictorial representations of algebraic expressions. In other words, we will draw pictures of algebraic expressions to verify that they have the same value.**

Demonstrate

1. Introduce algebra tiles to students.

Explain to students the representations of the various algebra tiles they will be using throughout the lesson. Using an overhead projector or document projector, display your set of algebra tiles and hard copy of the *Demonstration Practice Sheet* to model the use of the algebra tiles.

> **Today we will use and draw algebra tiles to represent algebraic expressions. We need to understand how to use these tiles and what each tiles represents.**

Tell students that each tile represents the area based on the side lengths. Explain to students the positive and negative values that can be represented by the tiles.

Each tile represents an area based on the side lengths. All of these tiles have two colors. The red side of the tiles represents the negative values. The other side of the tiles represents the positive values.

Point to the small square tile and explain the value.

Look at the small square tile. Each edge represents 1 unit of length. If the edges are both 1, what is the area of the small tile? *(1 unit^2)*

Because the area of this tile is 1 unit, we call it the unit tile. One side of the tile represents +1 unit, while the red side represents -1 unit.

Point to the rectangular tile and explain the value.

Look at the rectangular tile. The short edge has a length of 1, the same as the unit tile. Match the short edge with the edge of the unit tile.

The long edge is an unknown length. What do we use to represent an unknown value? *(variable)*

The rectangular tile has dimensions of 1 and x. What is the area of the rectangular tile? *(1*x = x)*

Therefore, the tile represents x. Again, one side of the tile represents x, while the red side represents the opposite of x.

Teacher Note
When expressions have $-x$, it should be read as "opposite of x," rather than "negative x." Students need to focus on "opposite of" so they do not assume all values will be negative.

Point to the large square tile and explain the value.

Look at the large square tile. The lengths of both edges are *x*, the same as the long edge of the *x* tile. So, to find the area of this tile we multiply both edges, *xx*, which equals x^2.**

For today's lesson we will only be working with the 1 and *x* tiles to represent expressions.

2. Practice writing algebraic expressions from algebra tiles.

Present 1 rectangular tile and 3 unit tiles. Write the e

Watch For

Students might write the constant before the variable term. The conventional order for expressions is to write the variable term first. Be sure to stress that expressions can be written in either order because of the commutative property of addition.

n “$x + 3$” for students to see.

Now we will practice writing algebraic expressions from a set of algebra tiles. Looking at the tiles presented, we see there is 1 *x* tile and 3 unit tiles. This means that the algebraic expression the tiles represent is *x* + 3.

Ask students to write an algebraic expression from the algebra tile representation on the whiteboard. Present 3 rectangular tiles and 4 unit tiles.

On your whiteboards, write an algebraic expression from these tiles.

Pause for students to work. Have students turn to their neighbor to check their expression.

Turn to your neighbor and compare algebraic expressions. What did you write? *(3x + 4)*

The tiles represent the algebraic expression 3*x* + 4.

3. Practice representing algebraic expressions with algebra tiles.

Write the algebraic expression "-3x + 4" for students to see and then represent it with algebra tiles. Walk students through the thought process to correctly represent the expression.

Now we are going to represent the expression -3*x* + 4 with algebra tiles. What are the terms in this expression? *(-3x and 4)*

Display all algebra tiles for students to see. Point to each type of tile as you compare it to the algebraic expression.

We need to represent -3*x* with *x* tiles and 4 with the unit tiles. This means that we will use 3 red *x* tiles to represent -3*x* and 4 positive (blue) unit tiles.

Write the expressions "5x" and "2x + 5" on the board and have students create the representation with algebra tiles. Have students turn to a partner to compare their pictorial representation

Now let's practice creating a representation of the algebraic expression with algebra tiles. Create the algebraic expression 5*x*.

Pause for student to work.

Turn to your neighbor and compare your algebra tiles. If they are the same, show me a thumbs up. If your tiles are different, discuss your representation with your neighbor.

How did you represent 5*x* with your algebra tiles? *(used 5 positive x tiles)*

Now, on your own, create a representation of the algebraic expression 2*x* + 5.

Pause for students to work. Have students turn to a neighbor to compare. Call randomly on students to describe and explain their representation.

Turn to your neighbor and compare your representations. I will call on students randomly to describe and explain your representation.

Teacher Note
Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with student's names on them when selecting students to share their answers or reasoning.

Pause for students to compare.

How many of each tile did you use? *(5 unit and 2 x tiles)*

How did you know to use 2 positive *x* tiles? *(because the expression has 2x)*

How did you know to use 5 positive unit tiles? *(the expression has 5)*

Today we will use algebra tiles to compare 2 algebraic expressions to check for equivalence based on the pictorial representations. What is the definition for equivalent expressions? *(two expressions that have the same value for all replacements of the variable or variables)*

Previously, we evaluated numeric expressions. When the expressions were the same value, they were equivalent. Because we are using pictorial representations to compare the expressions, the tiles for each expression must match to be equivalent.

Teacher Note
Students may need more time representing expressions with algebra tiles before testing for equivalence.

4. Students will use algebra tiles to determine equivalence of algebraic expressions.

Guide students through determining whether the algebraic expressions in problem 1 are equivalent with the use of algebra tiles. Represent and have students represent each algebraic expression on the *Demonstration Practice Sheet* with algebra tiles. Students will write "=" or "≠" based on the pictorial representations. Instruct students to write while you are writing.

Look at problem 1. We are going to create pictorial representations of the expressions using algebra tiles. The expressions are $3x$ +2 and $x + 1 + x + 1 + x$. Work with your neighbor to decide how to represent the expressions using algebra tiles.

Pause for student pairs to work. Sketch and have students sketch 3 x tiles and 2 unit tiles on the *Demonstration Practice Sheet*.

> **How did you and your partner represent 3*x* + 2 using algebra tiles?** *(3 x tiles and 2 unit tiles)*
>
> **On the Demonstration Practice Sheet, we will sketch a pictorial representation of the algebra tiles. This means we sketch 3 *x* tiles and then 2 unit tiles under the expression.**

Pause for students to work. Sketch and have students sketch the x tiles and unit tiles in the order of the algebraic expression.

> **How did you and your partner represent *x* + 1 + *x* + 1 + *x* using algebra tiles?** *(x tile, unit tile, x tile, unit tile, and x tile)*
>
> **On the Demonstration Practice Sheet, we will sketch the pictorial representation under the algebraic expression. This means we first sketch 1 *x* tile, then 1 unit tile, next 1 *x* tile, then 1 unit tile, and last 1 *x* tile.**

Pause for student to work.

> **To check for equivalence, we must ask ourselves, "Do the pictorial representations have the same number of each type of tile?" The algebraic expressions are written differently, but if you look at the algebra tiles, they have the same number of *x* tiles and unit tiles. Rearrange the algebra tiles on the right, grouping the *x* tiles together and the unit tiles together.**

Pause for student to work.

> **After grouping the *x* tiles and unit tiles, you can see that the 2 expressions are equivalent because the representations are the same for both expressions.**

Write and have students write "$3x + 2 = x + 1 + x + 1 + x$" in the space provided on the *Demonstration Practice Sheet*.

> **Because the representations are the same for each expression, we know that the algebraic expressions are equivalent. Below the picture representations, we write "3*x* + 2" and "*x* + 1 + *x* + 1 + *x*" in the blank spaces provided. In the box between the expressions, we write "=" to indicate that they are equivalent.**

5. Determine whether the expressions are equivalent in problem 2.

Using algebra tiles, have students represent the algebraic expressions for problem 2 on the *Demonstration Practice Sheet*.

> **We need to determine whether the algebraic expressions are equivalent. How can we compare the expressions?** *(answers may vary; use algebra tiles; sketch a pictorial representation)*

> **Use the algebra tiles to represent both algebraic expressions in problem 2.**

Have students turn to a neighbor to check their representation. Call randomly on students to verify the correct representation.

> **After you have arranged algebra tiles to represent each of the expressions, turn to a neighbor and compare your representations. If you and your neighbor have matching tiles, show me a thumbs up. If you do not match, discuss how to represent the expressions using the algebra tiles. You have 2 minutes to compare and discuss your representations before I call on someone to explain their representations.**

Pause for student pairs to work.

How did you represent each algebraic expression? *(the left expression is 1 x tile, 2 unit tiles, 1 x tile, and 2 unit tiles; the right expression is 2 x tiles and 3 unit tiles)*

Sketch and have students sketch the picture to determine whether the expressions are equivalent.

Sketch the image of the algebra tiles below each expression in the box provided.

Looking at the picture representations, are the 2 algebraic expressions equivalent? *(no)*

How do you know? *(the left expression has 1 more unit tile)*

Write and have students write in the blanks provided "$x + 2 + x + 1 + 1$" and "$2x + 3$." In the box, write and have students write "$\neq$."

In the blanks provided, write each algebraic expression. Because the 2 expressions are not equivalent, we write "$\neq$" in the box.

6. Continue with determining equivalence based on the pictorial representations for problems 3 and 4.

   Guide students through the process to represent each expression pictorially and to determine equivalence. Use specific questions, such as the following, to elicit verbal responses from students to check for their understanding.

   - How would you represent the first algebraic expression pictorially using algebra tiles? The second?
   - Are the pictorial representations equivalent?
   - How do you know?
   - What symbol do we use to represent the equivalence? Non-equivalence?

7. Provide a summary of the lesson content.

   Review key ideas of the lesson by asking questions about equivalent algebraic expression.

   Based on today's lesson, when you are checking for equivalence of algebraic expressions, what should you ask yourself? *(answers may vary; how can I compare the expressions; how do I represent the expressions for comparison pictorially; do the pictorial representations have the same number of each type of tiles)*

   How can we determine whether 2 algebraic expressions are equivalent? *(the pictorial representation will have the same number of each type of tiles)*

   What does it mean when 2 algebraic expressions are equivalent? *(the expressions have the same value and pictorial representation)*

   What symbol do we use to represent equivalent algebraic expressions? *(the equal sign)*

   What symbol do we use to represent non-equivalence? *(the not equal sign)*

Practice

Practice

1. Have students work either as a group or in pairs to answer the questions on the *Practice Sheet*.
2. Have students sketch a pictorial representation for each algebraic expression. Have students write an equivalent algebraic expression from the pictorial representation. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.
3. Have students present their reasoning by explaining their answers to the class. Ask probing questions, such as the following, to elicit detailed responses:

- How did you represent the expression?
- How did you write the equivalent algebraic expressions?

Independent Practice

1. Have students work independently to sketch the algebraic expression and write an equivalent algebraic expressions on the *Independent Practice Sheet*.
2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following question:

- What should you ask yourself when checking for equivalence of algebraic expressions?
- When given a visual representation of an algebraic expression, how do you know they are equivalent?
- What symbol do you use to indicate equivalence? Non-equivalence?

Module 2: Expressions, Equations, and Equivalence
Lesson 5

Lesson 5: Simplifying Algebraic Expressions

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students will combine like terms and distribute to simplify algebraic expressions and create equivalent expressions.

Students will use precise mathematical language to describe the simplification process.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">like terms: when 2 or more terms have the same variable(s); each variable of the same kind being raised to the same power
unlike terms: when terms do not have the same variables
simplify: perform all indicated operations to find an equivalent algebraic expression
distribute: multiply a term over a quantity
zero pair: 2 values that sum to zero</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">algebraic expression, coefficient, equivalent, expression, numeric expression, term</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 41-52)
• Overhead/document projector
• Colored pencils/marker
• Algebra tiles
• Calculator</td><td>• Student Booklet (pp. 23-28)
• Colored pencils/markers
• Algebra tiles
• Calculator</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills about algebraic expressions and equivalence.

Have students recall the definition for algebraic expression.

We have previously discussed algebraic expressions. What is the definition of algebraic expression? *(a collection of numbers and variables that are combined using operations)*

Remember, expressions do not have an equal sign.

Review problem 1 on the *Cumulative Review Practice Sheet*. Have students identify the terms and coefficients in the algebraic expression.

Look at the expression in problem 1 on the Cumulative Review Practice Sheet. I want you to identify the term(s) and the coefficient(s) in this expression. I will give you time to think, then I will call on someone randomly to provide an answer.

Pause for students to identify the terms and coefficients. Call on a student randomly to provide the terms of the expression.

How many terms does this expression have? *(4)*

What are the terms? *(2x, 7, 5x, 3)*

Looking at the expression, we need to identify the coefficients. What is the definition of coefficient? *(the number in front of the variable, multiplied by the variable)*

What are the coefficients in the algebraic expression? *(2 and -5)*

When you evaluated the expression, what answer did you calculate? *(10)*

Review problem 2 on the *Cumulative Review Practice Sheet*. Have students explain how they found the equivalent expression.

Looking at problem 2, which expression is equivalent to the algebraic expression? *(**A**)*

How do you know? *(answers may vary; when sketching the algebraic expression, there were 4 rectangles to represent the y and 6 squares to represent the constants)*

Preview

This lesson will build on students' knowledge of simplifying algebraic expressions using algebra tiles.

Today we will combine *like terms* and *distribute* values to *simplify* algebraic expressions and create equivalent expressions. In other words, we will multiply and combine values in a given expression that are similar to create an equivalent expression.

Demonstrate

1. Present algebra tiles to students and discuss.

Review the meaning of the x unit tile and 1 unit tile with students. Display each tile and ask students to recall what the tile represents.

Display the x tile first, then the 1 unit tile. Discuss the purpose and use of the 2 color tiles.

Previously, we worked with algebra tiles to represent different expressions. What does this rectangular tile represent? *(x)*

Recall that the tiles are 2 colors in order to indicate a positive and a negative value. What does the red rectangular tile represent? *(-x; the opposite of x)*

What does the square tile represent? *(1 unit)*

What does the red square tile represent? *(-1 unit)*

2. Present and discuss the mathematical steps to simplifying algebraic expressions.

   Write and have students write the 2 parts to simplifying algebraic expressions on the *Demonstration Practice Sheet*. Using an overhead projector or document projector, display your copy of the *Demonstration Practice Sheet* for all students to see.

   There are 2 main ideas we use to *simplify* any algebraic expression – *distribution* and combining *like terms*. Write in the space provided on your Demonstration Practice Sheet "*distribute*" for 1 and "collect then combine *like terms*" for 2.

   Pause for students to work.

   There will be times when *simplifying* algebraic expressions that you may not need to *distribute*. If this is the case, then the only step needed will be to combine *like terms*.

3. Discuss combining like terms to simplify algebraic expressions.

   Define "like terms" and "unlike terms." Discuss using examples on the *Demonstration Practice Sheet*.

   We will first discuss combining *like terms*. Looking on the Demonstration Practice Sheet, the example $a + 3 - 5 + 2a - 4a$ has *like terms* and *unlike terms*.

***Like terms* are when 2 or more terms have the same variable or variables. The variables also have the same power.**

***Unlike* terms have different variables, or different powers.**

Teacher Note
Some students will need a reminder about what is a power or exponent. If needed, provide an example of a power such as "b^2" to give students a visual cue.

We are going to use algebra tiles to represent this expression to help us see the *like* and *unlike terms* in the expression.

This means I would use 1 blue rectangle for *a*, 3 blue unit tiles for +3, 5 red unit tiles for -5, 2 blue rectangles for +2*a*, and 4 red rectangles for -4*a*. Gather all the tiles that represent this algebraic expression.

Sketch and have students sketch the tiles in the top box provided under the expression on the *Demonstration Practice Sheet*. Remind students to label the positive and negative tiles.

Sketch the image of the algebra tiles in the top box provided for pictorial representation under the expression. Be sure to label the red tiles in your sketch with "−" and the blue tiles with "+" to indicate positive and negative values. They do not have to be COLORED, only labeled.

Pause for students to work. Circle and have students circle like terms with 2 different colored pencils or markers.

Looking at the algebra tiles, there are 2 types of *like terms*: the unit terms, or square tiles; and the variable terms, or rectangle tiles. Using a colored pencil, circle all of the square unit algebra tiles, both positive and negative. Using a second colored pencil, circle all of the rectangle *a* tiles.

Pause for students to work. Re-sketch and have students re-sketch the algebra tiles in the box below to collect like terms.

We need to collect *like terms*. This means we will re-sketch the algebra tiles in groups in the space labeled "Collected Algebra Tiles." Place all of the variable tiles on the left and the unit tiles on the right. On each side, draw the positive tiles out in 1 row and the negative tiles in a row below.

Pause for students to work. Rewrite and have students rewrite the expression from the algebra tiles.

Now that we have collected *like terms*, let's write the algebraic expression. Looking at the variable tiles, we have 3 positives and 4 negatives. Write "$3a - 4a$" by the tiles.

Pause for students to work.

Looking at the unit tiles, we have 3 positives and 5 negatives. Write "$3 - 5$" by the tiles.

Define zero pairs and give examples.

Next we will combine *like terms*. To do this we must first understand what makes a *zero pair*. An example of a *zero pair* is the numbers -2 and 2 because when added, they sum to zero. If I have the number -8, I would need to add 8 to make a *zero pair*.

A *zero pair* is a pair of values that sum to zero. If I have 5, what number would I need to add to make a *zero pair*? *(-5)*

If I have 3*b*, I would need to add -3*b* to make a *zero pair*.

Explain how to combine like terms using the zero pairs. Draw a line through the zero pair tiles and write “zero” at the bottom to indicate a zero pair. Have students do the same.

What is the definition of a *zero pair*? *(2 values that sum to zero)*

Looking at the sketch of algebra tiles, we can see *zero pairs*. The single positive variable tile with the single negative variable tiles makes a *zero pair*.

We are going to draw a line through the *zero pair* tiles and write “zero” at the bottom. Do this now with the variable tiles.

Pause for students to work.

How many *zero pairs* are there in the variable tiles? *(3)*

Are there any variable tiles left over? *(yes)*

How many variable tiles are left over? *(1)*

Is the tile positive or negative? *(negative)*

This means that when we combine the variable tiles the result is –*a*.

Let’s do the same with the unit tiles. Draw a line through the *zero pair* tiles and write “zero” at the bottom.

Pause for students to work.

How many *zero pairs* are there in the unit tiles? *(3)*

Are there any unit tiles left over? *(yes)*

How many are left over? *(2)*

Are the tiles positive or negative? *(negative)*

This means that when we combine the unit tiles the result is

-2.

Write the algebraic process for simplifying expressions. Connect the algebra tiles to the algebra process. Have students do the same.

Algebra tiles are helpful tools to visually represent the process of *simplifying*. However, algebraic expressions are not always easy to sketch. To *simplify* the expression $a + 3 - 5 + 2a - 4a$, we would first identify *like terms*. Using the same colors that we circled variable tiles in above, draw a vertical rectangle around each of the variable terms a, $2a$, and $-4a$. Once they are all identified, list them in the space below the expression.

Now draw a square around each constant term, 3 and -5, with the same color that you circled the unit tiles. List these in the same space after the variable terms.

Pause for student work.

You should now have written "$a + 2a - 4a + 3 - 5$" below the expression.

Next we combine *like terms*, as we did with the *zero pairs*. This means $a + 2a - 4a$ results in $-a$ and $3 - 5$ results in -2.

Write and have students write the simplified form of the expression on the *Demonstration Practice Sheet*.

Because we have combined *like terms,* we can now write the *simplified* form of the expression. To *simplify* means to perform all indicated operations to find an equivalent algebraic expression.

Write the definition of *simplify* on the Demonstration Practice Sheet in the space provided.

Pause for students to work.

> **This means the *simplified* form of the given expression is -*a* − 2. Write "-*a* − 2" on your Demonstration Practice Sheet. Therefore *a* + 3 − 5 + 2*a* − 4*a* is equivalent to -*a* − 2.**

4. Guide students through simplifying the next 2 algebraic expressions on the *Demonstration Practice Sheet*.

   Use guiding questions to elicit verbal responses from students.

   - Which terms should be identified with the same shape? Why? How do you know they are like terms?
   - How should we rewrite the expression?
   - What is the simplified expression that is equivalent?

5. Guide students through a discussion of how to simplify an algebraic expression.

   Have students verbalize the simplifying process of collecting like terms. Use guiding questions to elicit responses to check for student understanding.

   > **Let's think about the algebraic *simplifying* process so far. What was the first step to *simplify* any of the algebraic expressions algebraically?** *(identify like terms)*
   >
   > **How do you know which are *like terms*?** *(they have the same variables and the variables have the same exponents)*
   >
   > **You have to ask yourself which are *like terms*. Once you have identified *like terms*, what do you do next?** *(combine like terms by adding and subtracting)*
   >
   > **This means that once you have identified *like terms*, you combine *like terms* by adding and/or subtracting, resulting in an equivalent expression.**

6. Use algebra tiles to demonstrate the distributive property.

Define and discuss the term “distribute” using the expression 2(1 + 3).

> **There are times when *simplifying* that we have to do more than just combine *like terms*. In the case of the following problems, we cannot combine *like terms* yet because there are *like terms* in parentheses and *like terms* outside of parentheses. By *distributing* a value to each quantity in parentheses, we eliminate the parentheses and now can combine *like terms*. To *distribute,* we multiply a value over a quantity. The quantity could be a numeric or algebraic expression.**
>
> **For example, we could have the expression 2(1 + 3). The multiplier is 2, while the quantity is (1 + 3). In the example, the 2 is *distributed*, or multiplied, by the 1 and 3.**

Sketch and have students sketch algebra tiles to represent the expression $4 + 2(b - 1)$ on the *Demonstration Practice Sheet*. Define “distribute.”

> **We will use algebra tiles to show the result of distributing a value over a quantity. On your Demonstration Practice Sheet, sketch the tiles to represent the algebraic expression in the pictorial representation section.**

Pause for students to work.

> **Because the 2 is multiplying the quantity $b - 1$, your sketch should have 4 unit tiles, then 1 variable tile and 1 negative unit tile, then another variable tile and negative unit tile because the 2 indicates that we need 2 of the quantity $b - 1$. Make sure your sketch matches my sketch on the Demonstration Practice Sheet.**

Continue simplifying the expression by circling, collecting, and combining like terms. Have students do the same.

Our next step to *simplifying* is to combine *like terms*. Looking at the algebra tiles, circle the like terms with the same color pencil or marker. What terms are the same? *(4 and -2; b and b)*

Pause for students to work. Re-sketch and have students re-sketch the algebra tiles as collected like terms.

Once you have circled the *like terms*, what is the next step to *simplifying*? *(collect like terms)*

Re-sketch the algebra tiles to collect *like terms* in the Collect Algebra Tiles section.

Pause for students to work.

Looking at the collected algebra tiles, write "$b + b$" and "$4 - 2$." What is the next step to *simplifying*? *(combine like terms)*

Combine the algebra tiles and look for *zero pairs* where appropriate.

Pause for students to work.

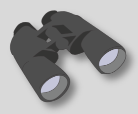
When you combined *like terms* of the algebra tiles, what was the result? *(2b + 2)*

Guide students through the algebraic process of simplifying with distribution. Write and have students write the step-by-step expressions to simplify algebraically.

There are times when using algebra tiles is not the quickest way to *simplify*. Now we will *simplify* this same expression, but only algebraically.

Looking at the example, $4 + 2(b - 1)$, we will *distribute* 2 over the quantity $b - 1$, or multiply each term in the parentheses by 2. Therefore, we write "$4 + 2b - 2$" as the next line to *simplify*.

Watch For



Students may not distribute the multiplier to each term when applying the distributive property for simplifying. For example, 3(x + 2) distributed incompletely will be 3x +2 instead of 3x + 6.

Pause for students to work.

Identify the *like terms* using squares and rectangles.

Pause for students to work.

Write the expression as "2*b* + 4 − 2." What is the next step to *simplify* this expression? *(collect like terms)*

Which are *like terms*? *(4 and -2)*

Combining *like terms* results in the expression 2*b* + 2. Write this in the space provided on the Demonstration Practice Sheet.

We have *simplified* an algebraic expression, which is equivalent to 4 + 2(*b* − 1). In the blank provided, write the expression "2*b* + 2."

7. Guide students through simplifying the next two algebraic expressions on the *Demonstration Practice Sheet*.

   Use guiding questions, such as the following, to elicit verbal responses from students.

   - What values are to be distributed?

- Over what quantity will we distribute [value]?
- Which terms should be underlined with the same color? Why? How do you know they are like terms?
- How should we rewrite the expression?
- What is the simplified expression that is equivalent?

8. Generate a list of questions to guide simplifying algebraic expressions.

Have students generate a list of questions they need to ask to simplify algebraic expressions. Guide students to generate the list through a discussion.

Let's think about the *simplifying* process. We are going to generate a list of questions you should ask yourself to *simplify* algebraic expressions.

What is the first step to *simplify*? *(answers will vary; distribute/multiply a value to all terms in parentheses)*

This means the first question to yourself is, "Is there a value that needs to be *distributed*?"

If there is a value to be *distributed*, then multiply. What if there is no value to be *distributed*? What is the next step? *(identify like terms)*

Therefore, your next question to yourself is, "What are the *like terms*?"

Once you have identified *like terms*, what is the next step? *(collect like terms)*

To collect *like terms*, the next question to yourself is, "How do I collect *like terms*?"

Once you have collected *like terms*, what is the next step? *(combine like terms)*

Thus, to combine *like terms*, the next question to yourself is, "What operation do I perform to combine the *like terms*?"

With these questions you will be able to guide yourself through *simplifying* algebraic expressions.

9. Provide a summary of the key lesson content.

   Review the key ideas of the lesson by asking questions about simplifying.

   What are the steps to *simplifying* an algebra expression? *(distribute if necessary, underline like terms, collect like terms, and combine like terms by adding or subtracting)*

   What does it mean to *simplify*? *(perform all indicated operations to find an equivalent algebraic expression)*

Practice

Pair Practice

1. Have each student in a pair create 1 expression for each problem. Be sure distribution is necessary for at least 1 expression.
2. Have partners trade papers and evaluate their partner's expressions.
3. Allow time for students to trade back papers and discuss their answers.

Teacher Note
Students may find this task difficult. Walk the student pairs through the example to help guide their work. Remind students to look at previous examples to help.

Error Correction Practice

1. Have students examine the 3 solution strategies presented and determine which strategy is correct. Have students justify why the solution strategy they chose is incorrect.

2. Have students share the reasoning for their answers to the class.

Independent Practice

1. Have students work independently to simplify the expressions and match them to the simplified expressions on the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- What are the steps to simplifying an algebra expression?
- What does it mean to simplify?
- What are some of the questions you should ask yourself when simplifying expressions?
- What does it mean for terms to be “like” or “unlike”?

Module 2: Expressions, Equations, and Equivalence
Lesson 6

Lesson 6: Testing for Equivalent Algebraic Expressions, Part II

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students will check for equivalence of algebraic expressions by graphing each expression on a graphing calculator.

Students will create and describe to peers and teachers how to test for equivalent algebraic expressions.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">algebraic expression, distribute, equation, equivalence, evaluated, like terms, simplify</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 53-64)
• Overhead/document projector
• Graphing calculator
• Colored pencils/markers</td><td>• Student Booklet (pp. 29-34)
• Graphing calculator
• 2 Colored pencils/markers</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills, such as equivalent algebraic expressions and simplifying algebraic expressions.

Have students define equivalent expressions and explain how to express whether 2 algebraic expressions are equivalent.

Previously, we used algebra tiles to help visualize whether 2 expressions were equivalent. When we used the algebra tiles to model the algebraic expressions, how did we know expressions were equivalent? *(the expression would have equal numbers of x tiles and 1 unit of algebra tiles)*

Draw an algebra tile depiction of 3x + 1 and x + 3 and ask students to determine whether the 2 are equivalent.

I have drawn the algebra tile representation of 2 expressions on the board. Are they equivalent? *(no, they are not)*

How can you tell? *(answers will vary; the numbers of each type of tile do not match on each side)*

Review problem 1 on the *Cumulative Review Practice Sheet*.

Problem 1 on the Cumulative Review Practice Sheet asked you to draw a pictorial representation of the algebraic expression to find the equivalent expression. Which expression was equivalent? *(D)*

How did you know? *(D was the only choice with 3 h's and 4 units; it was the only expression that match the pictorial representation)*

To write 2 expressions as an equation, the expressions must be equivalent. What does it mean for 2 expressions to be equivalent? *(answers may vary; they have the same value for all replacements of the variable or variables)*

Review problem 2 on the *Cumulative Review Practice Sheet*.

Looking at the Cumulative Review Practice Sheet. Problem 2 asked you to simplify the algebraic expression. How did you simplify the expression? *(answers may vary; distributed 2, then combined like terms)*

How did you distribute? *(multiplied a and -1 by 2)*

What were the like terms that you combined? *(-3a and 2a, -6 and -2)*

What is the algebraic expression that is equivalent to the given expression? *(-a − 8)*

Preview

This lesson will build on students' knowledge of equivalent algebraic expressions.

Today we will graph expressions on a graphing calculator to test for equivalence.

Demonstrate

Teacher Note

Students should have experience plotting ordered pairs on a coordinate plane prior to this lesson for familiarity with graphing.

1. Present and discuss testing for equivalence using a graphing calculator.

Guide students through the process of determining whether algebraic expressions are equivalent using a graphing calculator. Have students complete the *Demonstration Practice Sheet* as you demonstrate the use of the graphing calculator. Using an overhead projector or document projector, display your hard copy of the *Demonstration Practice Sheet* and model the use of the calculator for students.

We are going to use the technology of a graphing calculator to determine whether 2 algebraic expressions are equivalent.

In a previous lesson, we generated a table that listed values for an expression when evaluated. When looking at equivalent expressions, the tables of values for both expressions are equal for all values of the variable.

We can plot the table values as ordered pairs to make a graph. Because the tables of values for equivalent expressions are equal for all values of the variable, the graphs for both expressions are also equal.

Have students graph problem 1 on the *Demonstration Practice Sheet* by typing each expression into Y_1 and Y_2 on the graphing calculator.

To compare the 2 algebraic expressions in problem 1, we will enter each expression into the graphing calculator. We will use Y_1 and Y_2 to label and compare the expressions. To keep your information organized, write "Y_1" above $x - 3 - 3x + 5x + 1$ and "Y_2" above $3x - 2$.

By using Y_1 and Y_2, we can generate a list of ordered pairs for each expression where for a given x value, the y value is the expression evaluated for the specific x.

Watch For	**When entering an equation into the graphing calculator, students may exchange the subtraction sign (−) for the negative sign (-). This will result in a "SYNTAX" error, or incorrect graph.**

Point to the top of the *Demonstration Practice Sheet* for students to see the reference on calculator use.

> **The top of your Demonstration Practice Sheet lists how to enter expressions into your calculator. Enter the first expression into Y_1 and the second expression into Y_2.**

Pause for students to work. Highlight and have students highlight the Y_2 graph to illustrate it graphing on top of the other expression. Use the cursor selection to change the graphing line.

> **To graph different lines for each expression, move the cursor to the slanted line in front of Y_2 and press ENTER. This will result in a bold, slanted line in front of Y_2 and the graph of Y_2 will be bold. When we hit the GRAPH button, the first graph will be for Y_1, while the thicker line will be the graph of Y_2.**
>
> **Now press the GRAPH button.**
>
> **What did you see when you graphed the lines?** *(the thick line traced over the first line)*

Sketch and have students sketch the graphs on the *Demonstration Practice Sheet*. Utilize multiple colors to help visualize the two graphs, one on top of the other.

> **Because the lines graphed are on top of each other, we know that the expressions are of equal value for all *x*-**

values. When the graphs produce the same image, the expressions are equivalent.

On your Demonstration Practice Sheet, sketch the graphs of each expression. Use different colors to indicate each expression.

Pause for students to work. Using the calculator table function, examine and have students examine the matching *y*-values.

Let's look at the table of values to see how the expressions are equal for all *x*-values.

Press 2ND, GRAPH. For every *x*-value listed, the Y_1 and Y_2 values are all the same. List at least 5 *x*-values and their corresponding *y*-values on your Demonstration Practice Sheet to support your determination.

Evaluate and have students evaluate each expression for one of the *x* values from the table. Make connections to the *y* values on the table to the evaluated expressions.

We are going to evaluate each expression for *x* = 3. How do we write the expressions when we are evaluating for a specific value of *x*? *((3) − 3 − 3(3) + 5(3) + 1 and 3(3) − 2)*

Write and have students write "(3) – 3 – 3(3) + 5(3) + 1" and "3(3) – 2" below each expression on the *Demonstration Practice Sheet*.

Write "(3) – 3 – 3(3) + 5(3) + 1" and "3(3) – 2" below each expression and find the values.

Pause for students to work.

What is the value of each expression when *x* = 3? *(7)*

How does that compare with the table of values from the table function in the calculator? *(7 is the same number that is in Y_1 and Y_2 when $x = 3$)*

What symbol do we use to indicate that the 2 expressions are equivalent? *(=)*

Write "=" in the box provided on the Demonstration Practice Sheet to show equivalence.

2. Determine whether the expressions in problem 2 from the *Demonstration Practice Sheet* are equivalent.

Use and have students use the graphing function to determine equivalence for the algebraic expressions in problem 2. Label expressions and use colors to help determine equivalence. Instruct students to do the same.

Look at problem 2 on the Demonstration Practice Sheet. What is the first step in using graphs to determine whether the 2 algebraic expressions are equivalent? *(label one "Y_1" and the other "Y_2")*

Let's keep the order as written. Label the left expression "Y_1" and the right expression "Y_2." Enter the expressions into the calculator.

Pause for students to label and enter expressions into the calculator.

How do we change the line of the graph of the second expression? *(highlight in front of Y_2 and select "ENTER")*

Teacher Note
Students may not need to redo this step. If they accidentally hit "ENTER" again, they should continue pressing "ENTER" to cycle through the graphing options until arriving back at the bold line.

Press GRAPH. What do the graphs look like? *(there are 2 lines that are intersecting)*

Looking at the graph, do you think the 2 expressions are equivalent? *(no)*

How do you know? *(the graphs did not graph one on top of the other; there are two separate graphs, meaning they don't have the same values for all x's)*

Sketch and have students sketch the graphs on the *Demonstration Practice Sheet*.

Sketch the graphs on the Demonstration Practice Sheet. Be sure to use the two different colors to link the expression with the correct graph.

Pause for students to work. Use the table to examine the value of x where the 2 expressions are equal.

Looking at the graphs, the expressions are equal at only 1 *x*-value, the intersection point. Let's look at the table to find the value of *x* where the 2 expressions are equal.

Press 2ND, GRAPH. Using the arrow buttons, scroll to *x* = -3.

Pause for students to adjust their table to see $x = -3$.

Looking at the table, when *x* = -3, the expressions have the same value, -4. List this value, as well as 4 others, that support your determination.

Pause for students to complete the table values.

Because there is only 1 *x*-value where the expressions are equal, the expressions are not equivalent.

Write and have students write "≠" in the box between the 2 expressions.

What symbol do we use to show that 2 expressions are not equivalent? *(≠)*

Write "≠" in the box between the 2 expressions.

3. Continue to determine equivalence based on the graphical representations for problem 3.

   Guide students through the process to represent each expression graphically and determine equivalence. Use specific questions, such as the following, to elicit verbal responses from students to check for understanding.

   - What is the first step to check for equivalence in the calculator?
   - What are you looking for in the graph to check for equivalence?
   - Are the two expressions equivalent?
   - How do you know?
   - Looking at the table, do the Y_1 and Y_2 values match for ALL values of x that you see?
   - What symbol do we use to represent equivalence? Non-equivalence?

4. Read problem 4 to students and utilize the graphing calculator to determine equivalence.

   Guide students through setting up the graphs for each expression in the graphing calculator. Clarify for students the expressions that should be used to test equivalence. Read problem 4 to students while they follow along.

   In the word problem, we are trying to compare 2 expressions to find the profit. What are the two algebraic expressions for profit? *(10x + 3 − (4x + 5) and 6x − 2)*

   Let's use the graph to examine the equivalence of the 2 algebraic expressions. What is the first step to using the graphs to test equivalence? *(label each expression and enter 1 expression into Y_1 and the other into Y_2)*

Let's label $10x + 3 - (4x + 5)$ as "Y_1" and $6x - 2$ as "Y_2." Enter the expressions into your calculator and look at the graph.

Pause for students to work.

In the word problem, Enrique hypothesized that the 2 expressions for profit were equivalent. Was he correct? *(yes)*

How do you know? *(the graphs of the expressions are on top of each other)*

Sketch the graphs on the Demonstration Practice Sheet.

Pause for students to work. Discuss the table of values with the students.

Looking at the table of values, what do you notice? *(for all x-values, the expressions are equal)*

List 5 x-values (t-shirt quantities) from the table and their corresponding y-values (profit amounts) to support your determination.

Because the 2 expressions are equivalent, Henry and Enrique could use either expression to calculate the profit of their t-shirt business. What symbol do we write in the box provided to show equivalence? *(=)*

5. Provide a summary of the lesson content.

Review key ideas from the lesson by asking questions about determining the equivalency of algebraic expressions.

How do you know, graphically, if 2 expressions are equivalent? *(the graphs will be the same; one graph will be on top of the other)*

If the graph shows equivalent expressions, then what should the values in the table look like? *(for all x values, the y values should all be the same or equal)*

What are the steps to using the calculator to check for equivalence? *(label each expression with "Y_1" and "Y_2," enter each expression into Y_1 and Y_2, bold the line of Y_2, press GRAPH, check to make sure the lines graph on top of each other)*

Practice

Practice

1. Have students work either as a whole group or in pairs to answer the questions on the *Practice Sheet*.

2. Have students write "≠" or "=" in the space provided for each set of algebraic expressions. Students should use graphing calculators to determine whether the 2 expressions are equivalent. Students will sketch the graph, as well as list table values that support their determination on the *Practice Sheet*. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.

3. Have students present their reasoning by explaining their answers to the class. Ask probing questions, such as the following, to elicit detailed responses:

   - What is the first step you took to determine equivalence?
   - When using a calculator, how do you know expressions are equivalent?
   - Are the 2 algebraic expressions equivalent? How do you know?

Teacher Note

Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with students' names on them when selecting students to share their answers or reasoning.

Independent Practice

1. Have students work independently to determine whether the set of algebraic expressions are equivalent using graphing calculators.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Teacher Note

It will be helpful to students with lower reading abilities for you to read the word problem aloud. This will ensure that students are working on the intervention content rather than struggling with the reading aspects.

Closure

Review the key ideas. Have students provide examples from the lesson and discuss the following questions:

- Graphically, how do you know that 2 algebraic expressions are equivalent?
- How does the table of values support your answer?

Lesson 7: Solving Algebraic Equations Using Inverse Operations, Part I

Lesson Objectives	Students will use inverse operations and properties of equality to solve one-step algebraic equations. Students will verbalize how to solve one-step algebraic equations using precise mathematical language.	
Vocabulary	**solve:** to find the number or numbers that will replace the variable to make a true statement **inverse operation:** an operation that reverses the effect of another operation.	
Reviewed Vocabulary	algebraic expression, equation, equivalent, expression, simplify, zero pair	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 65-76) • Overhead/document projector • Graphing calculator • Algebra tiles	• Student Booklet (pp. 35-40) • Graphing calculator • Algebra tiles

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present the *Cumulative Review Practice Sheet*. Ask questions to activate students' background knowledge and prerequisite skills about solving algebraic equations.

Have students explain the process to simplify the expression in problem 1 on the *Cumulative Review Practice Sheet*.

We are to simplify the algebraic expression in problem 1. What does it mean to simplify? *(perform all indicated operations to find an equivalent algebraic expression)*

Looking at problem 1 on the Cumulative Review Practice Sheet, what is the first step to simplifying the expression? *(distribute 3, multiply 3 to a and 4)*

How do you know? *(when simplifying you distribute first, then combine like terms)*

What did you write in the blank for the equivalent expression? *(8a + 6)*

What equation did you write? *(5a − 6 + 3(a + 4) = 8a + 6)*

Have students recall the definition of equivalence. Use the definition to guide the discussion of problem 1 on the *Cumulative Review Practice Sheet*.

We have discussed the meaning of equivalence. What does it mean for 2 expressions to be equivalent? *(the expressions have the same value for all replacements of the variable or variables)*

For problem 2 on the Cumulative Review Practice Sheet, we are to graph the expressions and complete

the table to determine whether the algebraic expressions are equivalent. How do we know whether the 2 expressions are equivalent on a graph? *(the expressions' graphs are the same image)*

How does a table support the graph of equivalent expressions? *(for all values of x, the Y_1 and Y_2 values are the same)*

After sketching the graphs and completing the table for problem 2, have you determined that the 2 algebraic expressions are equivalent? *(no)*

What did you circle to indicate your decision? *(not equivalent, not equal sign)*

Preview

This lesson will build on students' knowledge of solving algebraic equations using inverse operations and properties of equality.

Today we will *solve* algebraic equations using *inverse operations* and properties of equality.

This means that we will use operations that "undo" each other to keep an equation balanced and to find the value of the variable that will make the equation true.

Demonstrate

1. Present the *Demonstration Practice Sheet* to students. Determine the properties of equality for addition, subtraction, multiplication, and division.

   Using an overhead projector or document projector, display your hard copy of the *Demonstration Practice Sheet* for students to see. Have students look at the equation in the Properties of Equality and Inverse Operations section and determine whether the 2 expressions form an equation.

Look at the equation in the Properties of Equality and Inverse Operations section on your Demonstration Practice Sheet. What is an equation? *(a math statement showing that two expressions are equivalent or have the same value for all replacements of the variable or variables)*

Numeric expressions 2(5) + 4 and 10 + 4, if equivalent, could be written as an equation. What is the value of the numeric expression 2(5) + 4? *(14)*

What is the value of the numeric expression 10 + 4? *(14)*

What do we write to show that they are equivalent? *(an equal sign)*

Write and have students write an equal sign in the box to demonstrate the equation notation.

Because the expressions are equivalent, we can write the equal sign in the box between the expressions. This forms an equation. We are going to examine what happens if we add the same number to both expressions in an equation.

Have students look at the table in the Properties of Equality and Inverse Operations section. Point to the section that says, "Add the **same** number to both expressions."

Look at the table in the Properties of Equality and Inverse Operations section. Because we are *solving* algebraic equations, we have to understand how operations affect equality. We want to know whether the expressions will remain equivalent if we add a number to both expressions.

Let's pick any number and add it to both expressions in the equation. What number would you like to use? *(answers will vary; for this script we will use 3)*

Write and have students write "+3" next to each expression. Have students determine whether adding the same number to each expression maintains the equivalence of the 2 expressions.

If we add 3 to the expression, 2(5) + 4, what value do we get? *(17)*

If we add 3 to the expression, 10 + 4, what value do we get? *(17)*

Did the expressions remain equivalent? *(yes)*

So, you can add the same number to both expressions in an equation and the expressions will remain equivalent. Because the expressions remain equivalent, what symbol do we write? *(equal sign)*

Repeat the process of adding the same number to both expressions by having students select a different number to add to both expressions.

Have students verify that adding a different number to both sides of an expression does not maintain the equality. Point to the section that says, "Add **different** numbers to both expressions."

Look at the table in the Properties of Equality and Inverse Operations section. We want to know whether the expressions will remain equivalent if we add different numbers to both expressions.

Let's pick 2 different numbers and add it to both expressions in the equation. What 2 numbers would you like to use? *(answers will vary; for this script we will use 2 and 4)*

Write and have students write "+2" next to the first expression and "+4" next to the second expression. Have students determine whether adding different numbers to each expression maintains the equivalence of the 2 expressions.

If we add 2 to the expression, 2(5) + 4, what value do we get? *(16)*

If we add 4 to the expression, 10 + 4, what value do we get? *(18)*

Did the expressions remain equivalent? *(no)*

So, if you add different numbers to both expressions in an equation, the expressions will not remain equivalent. Because the expressions do not remain equivalent, what symbol do we write? *(not equal sign)*

Ask students to think about whether this same property would apply for subtraction. Have students use the Think-Pair-Share process to begin the discussion.

What have you learned about adding the same number to both expressions in an equation? *(the expressions remain equivalent)*

I want you to think about whether this might be the same for subtraction. If you subtract the same number from both expressions, will the expressions still maintain equivalence?

Pause for students to think.

Have students pair with a neighbor and discuss whether subtracting the same number will maintain the equivalence.

Pair with your neighbor to share what you think. Can you subtract the same number from both expressions in an equation and still maintain equivalence?

Pause for students to discuss.

Have student pairs share what they discussed by calling on select student pairs to share.

Teacher Note

Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with students' names on them when selecting students to share their answers or reasoning. The following sentence stem may help students to verbalize their thoughts: "When I subtract the same number from both expressions in an equation, the equation __________."

Write and have students write, "We can add or subtract the same number to both expressions in an equation," in the blank provided for addition and subtraction.

> **What we just learned about is called the Property of Equality for Addition and Subtraction. This means that you can add or subtract the same number to both expressions and the expressions remain equivalent. Write the definition for the Property of Equality for Addition and Subtraction in the space provided for addition and subtraction on your Demonstration Practice Sheet.**

Pause for students to write. Point to the section that says, "Multiply both expressions by the **same number**."

> **Next, we need to determine whether the same is true about multiplication and division. Looking at the table in the Properties of Equality and Inverse Operations section, we are going to multiply both expressions in the equation by the same number. What number would you like to use?** *(answers will vary; for this script we will use 3)*

Write and have students write "3(   )" with parenthesis surrounding each expression. Have students determine whether multiplying the same number by each expression results in the same value.

> **We will use parenthesis to multiply the entire expression by 3. Write "3(2(5) + 4)" and "3(10 + 4)."**
>
> **When we evaluate 3(2(5) + 4), we need to first take care of what is inside the parenthesis based on the order of operations. We already know that 2(5) + 4 equals 14. We can then write "3(14)." What is 3(14)?** *(42)*
>
> **To evaluate 3(10 + 4), what do we perform first?** *(inside the parenthesis, add 10 + 4)*
>
> **Because 10 + 4 equals 14, we write "3(14)." We already know that 3(14) is equal to 42.**
>
> **Did the expressions remain equivalent?** *(yes)*
>
> **So, you can multiply the same number to both expressions in an equation and the expressions will remain equivalent. Because the expressions remain equivalent, what symbol do we write?** *(equal sign)*

Repeat the process of multiplying the same number to both expressions by having students select a different number to multiply to both expressions.

Have students verify that multiplying a different number to both sides of an expression does not maintain equality. Point to the section that says, "Multiply both expressions by **different numbers**."

> **Look at the table in the Properties of Equality and Inverse Operations section. We want to know whether the expressions will remain equivalent if we multiply both expressions by different numbers.**
>
> **Let's pick 2 different numbers and multiply them by each expression in the equation. What 2 numbers would you like to use?** *(answers will vary; for this script we will use 2 and 4)*

Write and have students write "2(2(5) + 4)" and "4(10 + 4)" to demonstrate the expressions being multiplied. Have students determine whether multiplying different numbers by each expression maintains the equivalence of the 2 expressions.

> **If we multiply the expression, 2(5) + 4 by 2, what value do we get?** *(28)*

> **If we multiply the expression, 10 + 4 by 4, what value do we get?** *(56)*

> **Did the expressions remain equivalent?** *(no)*

> **So, if you multiply both expressions in an equation by different numbers, the expressions will not remain equivalent. Because the expressions do not remain equivalent, what symbol do we write?** *(not equal sign)*

Ask students to think about whether this same property would apply for division. Have students use the Think-Pair-Share process to begin the discussion.

> **We have learned that if you multiply both expressions in an equation by the same number, the expressions remain equivalent. I want you to think about whether this might be the same for division. If you divide both expressions by the same number, will the expressions still be equivalent?**

Pause for students to think. Have students pair with a neighbor and discuss whether dividing by the same number will maintain the equivalence.

> **Pair with your neighbor to share what you think. Can each expression in an equation be divided by the same number and still maintain equivalence?**

Pause for students to discuss. Have student pairs share what they discussed by calling on select student pairs to share.

Write and have students write, "We can multiply or divide both expressions in an equation by the same number" in the blank provided for multiplication and division.

> **What we just learned about demonstrates the Property of Equality for Multiplication and Division. This means that you can multiply or divide both expressions by the same number and the expressions remain equivalent. Write the definition of the Property of Equality for Multiplication and Division in the space provided for multiplication and division.**

2. Discuss and define using inverse operations to solve equations.

Discuss with students what it means to solve an equation.

> **We will use the properties of equality to *solve* algebraic equations that contain an unknown variable. To *solve* an equation is to find the number or numbers that will replace the variable to make a true statement.**

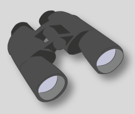
Write the equation "$n + 3 = 5$" on a whiteboard, overhead projector, or document projector.

> **We *solve* the equation $n + 3 = 5$ by finding what value of n makes this equation true. When $n = 1$, the equation is not true because 1 + 3 is not equal to 5. The only value of n that would make this equation true is 2, because 2 + 3 is equal to 5. When $n = 2$, the expressions on the left and right of the equal sign are equivalent and only for this value of n. 2 is called the "solution" of the equation because 2 makes the equation a true statement**

> **We will *solve* equations, or find their solutions, by using *inverse operations*.**

Write and have students write the definition for "inverse operation" in the blank provided.

An *inverse operation* is an operation that reverses the effect of another operation. Write the definition of *inverse operation* in the space provided on your Demonstration Practice Sheet.

Watch For 	Struggling learners may over-generalize aspects of solving equations. For example, students may try to always make a term negative when removing it, even when division is necessary. Clear mathematics language and emphasis on “undoing” what operation has been performed with the variable will help to reinforce correct processes. Focus first on what has happened to the variable (the operations being performed), and then on “undoing” them.

3. Use algebra tiles to represent the solving process of inverse operations.

   Create the pictorial representation of problem 1 on the *Demonstration Practice Sheet* using algebra tiles. Have students create a representation with algebra tiles as well.

Look at problem 1 on the Demonstration Practice Sheet. We are going to use the properties of equality and *inverse operations* to *solve* the algebraic equation $x + 3 = 8$.

We want to find the value of x that makes this equation a true statement. It is sometimes helpful to think of an equation as a balance.

When the 2 expressions are equal, the equation is balanced and is a true statement. When the 2 expressions are not equal, the equation is not balanced and is not a true statement, or not equal.

We will use algebra tiles to represent the equation as a balance. The equal sign goes in the center and the tiles

to represent each expression can be placed on either side. Using the algebra tiles, how do we represent both expressions? *(1 x tile and 3 positive unit tiles; 8 positive unit tiles)*

Place 1 *x* tile, 3 positive unit tiles, and 8 positive unit tiles on your desk. The algebra tiles should look like the picture on your Demonstration Practice Sheet.

Teacher Note
For students who learn by doing, it might be helpful to demonstrate the balance idea by having students "be" the balance. Have students stretch out their arms and put "equivalent expressions" in their hands—put something in the left hand and ask what needs to be in the right hand to be balanced.

Point to the tile illustration in problem 1 on the *Demonstration Practice Sheet* and circle the x-tile on your displayed hardcopy. Add 3 negative unit tiles to demonstrate an inverse operation with algebra tiles. Instruct students to do the same.

Because we are looking for the value of *x*, we need to get the *x* tile by itself on one side of the balance. Right now, there are 3 unit tiles on the same side with *x*. They are being added to *x*, so to "undo" the addition, what operation do we need to perform? *(subtraction)*

To isolate the variable, we will add 3 negative unit tiles. Place 3 negative unit tiles below the 3 positive unit tiles. Looking at unit tiles, the sum of the unit tiles is zero. What do we call 2 numbers that sum to zero? *(a zero pair)*

Draw and have students draw the 3 negative unit tiles below the representation of 3 positive tiles on the *Demonstration*

Practice Sheet. Label the negative unit tiles with a minus sign or color them with red pencil to indicate negative value.

> **Draw on your Demonstration Practice Sheet the 3 negative unit tiles. Inside, write a minus sign or color them red to indicate the negative value.**

Pause for students to draw and label tiles.

> **Adding 3 negative tiles to the 3 positive unit tiles is an *inverse operation*.**

To balance the equation, add 3 negative unit tiles to the other side and instruct students to do the same.

> **If I add 3 negative unit tiles to the left side of the equation, will the equation remain balanced?** *(no)*

> **Then to balance the equation, we need to add 3 negative unit tiles to the right side of the equation. Place 3 negative unit tiles below the 8 positive unit tiles. Remember that to maintain equivalence, we have to add the same amount to both sides of the equation.**

Draw and have students draw the 3 negative unit tiles below the representation of 3 positive tiles on the *Demonstration Practice Sheet*. Label the negative unit tiles with a minus sign or color them with red pencil to indicate negative value.

> **Draw on your Demonstration Practice Sheet the 3 negative unit tiles below the 8 positive tiles. Write or color the tiles to indicate the negative values.**

> **Because we have added the same amount to both sides, we can see the result. On the left side, how many zero pairs do we have?** *(3)*

> **Because the unit tiles sum to zero, we remove the algebra tiles that match to make a zero pair. On the left side, we remove all unit tiles. Do this now.**

> **On the right side, how many zero pairs do we have?** *(3)*

Remove all 3 zero pairs now.

On the Demonstration Practice Sheet, we will cross off the zero pairs. Do this now.

Pause for students to work.

Looking at the algebra tiles and the Demonstration Practice Sheet, we see that the *x*-tile is isolated on the left. The equation is balanced and there is a single value on the right side. The algebra tiles and picture show that *x* must equal 5. The solution to the equation is $x = 5$ because that is the only value that makes the equation a true statement.

Illustrate the algebraic manipulation by writing "- 3" beneath the + 3 on the variable side of the equation, and again beneath the 8 on the other side. Point to the zero pair on the variable side and the net of 5 on the other side that result in the last step of "$x = 5$."

We wrote the Property of Equality for Addition and Subtraction above. Now we will write the actual operation in this example by subtracting 3 from each expression in the equation. Looking at the left side of the equation, we created a zero pair by - 3 from + 3. What is a zero pair? *(2 numbers that sum to zero)*

Addition and subtraction are *inverse operations*. Undoing the addition isolates the *x* on the left side. On the right side, subtracting 3 from 8 results in 5. The solution to the equation is $x = 5$.

4. Guide students through solving problem 2 on the *Demonstration Practice Sheet*.

   Use algebra tile representations of each expression in the equation before sketching on the *Demonstration Practice Sheet*. Instruct students to do the same.

Look at problem 2 on the Demonstration Practice Sheet. We are going to use the properties of equality and *inverse operations* to *solve* the algebraic equation, $n - 2 = 10$.

Again, we will use algebra tiles to represent the equation. While the rectangular tile represents the variable *x,* we will use this tile to represent the variable *n* in the equation. How do we represent the left side of the equation? *(1 x tile and 2 negative unit tiles)*

How do we represent the right side of the equation? *(10 positive unit tiles)*

Set up algebra tiles to represent the equation now.

Draw out the algebra tile representation of each expression on the corresponding sides of the balance. The unit tiles on the variable side of the equation will need to be colored and labeled to indicate that they are negative instead of positive. Instruct students to do the same.

Draw a picture representing the algebra tiles on the balance on the Demonstration Practice Sheet. The equal sign goes in the center and the tiles to represent each expression should be drawn on either side.

When drawing the tiles, we must show the positive and negative values. How can we represent the different tiles? *(answers will vary; indicate the two negatives by coloring or shading them differently; draw a plus sign in the positive tiles and minus sign in the negative tiles)*

Because we have two negative units on the left side of the equation, we will need to indicate that they are negative values by coloring them in and writing the negative sign in the tile drawing. Color and label your draw like mine now.

Pause for students to work.

Create zero pairs by adding 2 positive unit tiles. Instruct students to do the same.

> **Once again, we are looking for the value of the variable, *n*. We need to get the *x*-tile by itself on the left side of the balance. How can we isolate the variable in this problem?** *(answers will vary; undo minus 2; inverse operation; add 2 unit tiles)*
>
> **Because the operation performed on *n* is subtraction, what is the inverse operation?** *(addition)*
>
> **We will have to create zero pairs like the last problem. We want to eliminate all unit tiles on the variable side. What type and number of tiles do we add to the variable side to create zero pairs?** *(add 2 positive unit tiles)*
>
> **Place 2 positive unit tiles below the 2 negative unit tiles.**
>
> **What is the result of adding 2 positive unit tiles to the variable side?** *(answers may vary; 2 zero pairs; sum of zero; isolated variable)*
>
> **Once we add 2 positive unit tiles to the left side, what should we do to make the equation balanced?** *(add 2 unit tiles to the other side)*
>
> **Place 2 positive unit tiles below the 10 positive tiles.**

Draw and cross out the zero pair tiles on the variable side of the equation. Instruct students to do the same. Notice and point out that there are no zero pairs on the other side of the equation.

> **Draw and label the 2 positive unit tiles on both sides of the equation on the Demonstration Practice Sheet.**

Pause for student to draw.

> **Because we have created zero pairs, we can cross out the unit tiles to show the sum of zero, which has no**

effect on the variable. Do we have any zero pairs on the right side of the equation? *(no)*

Now that the *x*-tile is isolated, the algebra tiles and balance picture make it easy to see that *n* must equal 12.

Add 2 to each side of the equation by writing "+ 2" underneath the "– 2" and the "10." Demonstrate the sum of zero or zero pair on the variable side and the sum of 12 on the other to arrive at "$n = 12$."

Because you know that addition and subtraction are *inverse operations*, it might be easier to *solve* algebraically instead of using tile balances. We can simply write "+ 2" on each side of the equation. Write "− 2 + 2 equals zero" and "10 + 2 equals 12" to show that "*n* = 12."

5. Guide students through solving problem 3 on the *Demonstration Practice Sheet*.

You will want to remind students that algebra tiles can be cumbersome and are not always the easiest method for solving. Prepare students to work the next two problems without the tiles and by simply using inverse operations.

What do you notice about this problem that is different from the first 2 problems? *(multiplication is used instead of addition or subtraction)*

This problem uses multiplication instead of addition or subtraction. In this case, algebra tiles will not be as simple to use. Because algebra tiles are not always easy or practical to use when *solving*, we will want to work solely with *inverse operations*.

You will fill in the blanks on your sheet as we work, just as in the first 2 problems.

Write and have students write "multiply by 14" in the first blank provided in problem 3.

What operation is being performed with the variable *y*? *(multiplication)*

Because *y* is being multiplied by 14, we write "multiply by 14" in the blank.

Write and have students write "divide by 14" in the second blank provided in problem 3.

What is the *inverse operation* for multiplication? *(division)*

Because division is the *inverse operation* for multiplication, write "divide by 14" in the next blank.

Pause for students to write. Perform the inverse operation to both sides of the equation on the *Demonstration Practice Sheet*. Instruct students to do the same.

How do you show division when working with an equation? *(answers may vary; draw a line and put the number under it)*

To show that we are dividing by 14, we will draw a line beneath 28 and beneath 14*y* and write "14" underneath each line.

Pause for students to work.

What is 28 divided by 14? *(2)*

So, we know the *solution* is *y* = 2.

6. Guide students through solving problem 4 on the *Demonstration Practice Sheet*.

Discuss with students the operation and the inverse to solve the equation. Write and have students write "divide by 8" in the space provided. Instruct students to write as you do.

Looking at problem 4, again we are trying to *solve* the equation to find the value for *m*. What operation is being performed with the variable *m*? *(division, divide by 8)*

Write "divide by 8" in the blank provided.

Pause for students to work.

What is the *inverse operation* of division? *(multiplication)*

Write "multiply by 8" in the blank provided.

Pause for students to work. Write and have students write "$8(m/8) = 7(8)$" under the equation to show the inverse operation.

How can we show multiplication when working with each expression in the equation? *(answers will vary; use parenthesis; a solid dot; an asterisk)*

We will use parenthesis to show multiplication in our equation. Put each expression inside parenthesis and write an "8" on the outside of each.

Pause for students to work.

What is 7 times 8? *(56)*

Now we know that the *solution* is *m* = 56.

7. Provide a summary of the lesson content.

Review the key ideas of the lesson by asking questions about solving algebraic equations.

What does it mean to *solve* an equation? *(find the value(s) of a variable that makes the equation true)*

What is an example of *inverse operations*? *(answers will vary; addition and subtraction; multiplication and division)*

How do we use the properties of equality to *solve* equations? *(answer will vary; use the same operation and same number on each expression in the equation to isolate the variable)*

What questions should you be asking yourself as you *solve* equations? *(what operations are being performed with the variable; what are the needed inverse operations)*

Practice

Guided Practice

1. Have students identify the operation being performed and fill in the blank for each item.
2. Have students complete the Guided Practice problems. Use probing questions, such as the following, to guide students through the problem-solving process.
    - What does it mean to solve an equation?
    - What are inverse operations?
    - What operation is being performed with the variable?
    - What is that operation's inverse operation?
    - How do you show that operation when working on paper?

Teacher Note

Students may struggle with addition being the operation in problem 2 because it is 150 + x. A discussion of how the positive number is adding to the variable x will help.

Error Correction Practice

1. Have each student in a pair analyze and correct the sampled work. Have student pairs justify the correction on the work.
2. Have partners use the Think-Pair-Share process to discuss their findings and share with the class.

Independent Practice

1. Have students work independently to complete the *Independent Practice Sheet*.
2. Have students share their answers and their reasoning with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- What does it mean to solve an equation?
- What is the inverse operation for [operation]?
- What does it mean to use an inverse operation?
- How do you use the properties of equality to solve equations?

Lesson 8: Solving Algebraic Equations Using Inverse Operations, Part II

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students will use inverse operations and properties of equality to solve two-step algebraic equations.

Students will verbalize how to solve two-step algebraic equations using precise mathematical language.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">algebraic expression, equation, equivalent, inverse operation, simplify, solve, zero pair</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 77-88)
• Overhead/document projector
• Calculator
• Directions from classroom to front office</td><td>• Student Booklets (pp. 41-46)
• Whiteboard with marker
• Calculator</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present the *Cumulative Review Practice Sheet*. Ask questions to activate students' background knowledge and prerequisite skills about solving algebraic equations.

Have students recall the definition of equivalence. Use the definition to guide the discussion of problem 1 on the *Cumulative Review Practice Sheet*.

We have discussed the meaning of equivalence. What does it mean for 2 expressions to be equivalent? *(the expressions have the same value for all replacements of the variable or variables)*

What do you recall from previous lessons that will allow us to check for equivalence? *(answers will vary; substitution/evaluate; use the calculator to graph and watch the second line trace the first line)*

When you graphed the expressions in problem 1, what did the calculator show? *(1 line; the expressions graphed on top of each other)*

What does it mean for 2 expressions to have the same graph? *(the expressions are equivalent)*

What did you notice about the values in the table? *(for all x values, the Y_1 and Y_2 values were the same)*

For problem 1, are the 2 expressions equivalent? What did you circle? *(equivalent =)*

Be sure to circle "EQUIVALENT =" in problem 1.

Because the 2 expressions are equivalent, we can write them as an equation.

Discuss problem 2 with students.

> **For problem 2, we are to solve the equation. What does it mean to solve an equation?** *(find the number or numbers that will replace the variable to make a true statement)*
>
> **Looking at problem 2, what operation was performed with *b*?** *(multiply by 3)*
>
> **What inverse operation was needed to isolate the variable?** *(divide by 3)*
>
> **After dividing by 3, what was the result or solution?** *(5; b = 5)*
>
> **So *b* must equal 5 because it is the only value that makes a true statement.**

Give each student a whiteboard and a marker. Instruct students to write questions they should ask themselves when solving equations.

> **On the whiteboard, write what questions you should ask yourself when solving equations.**

Pause for students to write their responses. Have students turn to a neighbor to compare their responses.

> **Turn to a neighbor to compare your questions to ask yourself when solving.**

Have students show their whiteboard responses. Discuss the questions students wrote down. Make sure students understand that they should be asking, "What operation is performed with the variable?" and "What inverse operation will be needed to isolate the variable?"

Teacher Note

Be sure that all of your students can identify these methods for verifying equivalence of expressions. Rather than just calling on a couple of students to provide answers, you may want to have them write their ideas out and collect them to see whether they all answer correctly.

Summarize single-step solving and discuss using substitution for checking work.

In previous lessons, we used several methods for verifying equivalence of expressions. Which ones do you recall? *(graphing; tables; substitution/evaluation)*

These same methods for verifying equivalence will also work for checking your work when solving for specific values. Today we will practice using substitution as a method for checking our work.

Write and have students write "3(5) = 15" on the *Cumulative Review Practice Sheet* to verify the answer.

Looking back at problem 2, we believe that *b* is equal to 5. To check this answer, we will substitute 5 into the original equation. Write "3(5)=15."

Pause for students to write.

Now we will evaluate each side of the equation to verify that it remains equivalent.

What is 3 times 5? *(15)*

Because 3 times 5 is equal to 15, I can replace that side of the equation and write "15 = 15." This is a true statement.

Because this is a true statement, I know that I correctly solved when I found *b* equal to 5.

Preview

This lesson will build on students' knowledge of solving multi-step equations using inverse operations.

> **In our last lesson, we solved 1-step algebraic equations using inverse operations and properties of equality.**
>
> **Today we will solve two-step algebraic equations using inverse operations and properties of equality.**
>
> **We will continue our work with solving by "undoing" math operations done to the variable in order to find a value that makes the statement true or to find a solution.**

Demonstrate

1. Present the *Solving Equations* foldable sheet to students.

Teacher Note
To maximize instructional time, cut and fold the foldable before teaching the lesson.

The foldable should be pre-cut and folded for each student. Explain that this foldable will be used over the next few lessons to compile information about solving that will be used later by students in their work.

> **The Solving Equations foldable will be used in the next several lessons. We will be able to compile helpful information and reminders about solving equations to use in your work later.**

Using an overhead projector or document projector, display your hard copy of the *Solving Equations* foldable sheet for students to see. Point to the Order of Operations section and list reminders about what order to evaluate expressions and what order to use inverse operations. From top to bottom on the left hand side of the fold, write the letters “P,” “E,” “M/D,” and “A/S,” along with a downward arrow, and label it “working or evaluating.” Instruct students to do the same.

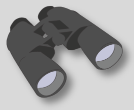
> **Look at the Order of Operations section of the Solving Equations foldable sheet. Recall the order of operations that is used when evaluating expressions. We typically list the order of operations vertically, so on the left hand side of the fold write, from top to bottom “P,” “E,” “M/D,” “A/S.”**

Pause for students to work.

> **Draw an arrow that runs from the top to the bottom of your list and write the words “simplifying or evaluating” next to it.**

Remind students about the use of inverse operations and the meaning of inverse. Write the order of operations down on the right side of the fold, along with an arrow that runs from bottom to top and the words “to ‘undo,’ invert the order.” Instruct students to do the same.

> **In Lesson 7, we discussed using inverse operations. What does inverse operation mean?** *(an operation that reverses the effect of another operation)*
>
> **Give me an example of an operation and its inverse.** *(answers will vary; addition and subtraction; multiplication and division)*
>
> **Because inverse operations “undo” the operations being performed, we will need to reverse the order as well. On the right side of the fold, list the letters from top to bottom, “P,” “E,” “M/D,” “A/S,” and draw an arrow that runs from bottom to top. Label this side with the words “to ‘undo,’ reverse the order.”**

Watch For 	**Struggling learners may over-generalize aspects of solving equations. For example, students may try to always make a term negative when removing it, even when division is necessary. Clear language and emphasis on "undoing" what operation has been performed on the variable will help to reinforce correct processes. Focus first on what has happened to the variable (the operations being performed) and then on "undoing" them.**

2. Prior to this lesson, generate step-by-step and turn-by-turn directions from your classroom to the front office of your school.

   Display the directions from your classroom to the front office now. Have students review the directions and confirm whether the directions are correct; some changes may be needed. Then, instruct students to work with a partner and use their whiteboards to generate precise directions, including turns, from the front office back to the classroom.

   In order to better understand why we must also reverse the order when solving, we will compare the process to reversing directions.

   These are the instructions, including turns, for how to get from this classroom to the front office. With your partner, write the set of directions, including turns, that someone could use to get from the front office back to this classroom.

   Pause for student partners to work.

   Select groups to share their results. Discuss as a class the order in which instructions will need to be followed. Compare the order of the directions and turns both to and from the office.

What do you notice about the way you are writing your directions back to the classroom? What happens to left and right turns? *(they switch to the opposite direction, left becomes right, etc)*

What changed about the order of the steps that you listed? *(it is the opposite order as well, 1-2-3 becomes 3-2-1)*

This change is true for solving equations, as well. We will make a list of the operations that happen to a variable in the equation in the order in which they occur. Then, we will write a list of steps that are inverse operations, in reverse order, that we can use to solve the equation.

3. Guide students through the process of solving problem 1 using inverse operations.

On the *Demonstration Practice Sheet*, write and have students write the operations and the order to solve the equation in problems 1.

As we work on the problems on the Demonstration Practice Sheet, it is important that you write exactly what I write and in the same way that I write it, so that as we discuss our work, we are all communicating in the same way.

Recall that an equation is a math statement that says that 2 expressions are equal, or have the same value. In problem 1, the equation states that 2 times $x + 3$ is equal to 11. We will solve for the value of x to make this a true statement.

Begin by listing the operations that are performed with x on the variable side of the equation. Refer to the Order of Operations list in your foldable if you are unsure about which operation comes first.

Write and have students write "multiply by 2" in the space labeled "1st."

What is the first operation being performed with the variable *x*? *(multiply by 2)*

Write "multiply by 2" in the space labeled "1st."

Write and have students write "add 3" in the space labeled "2nd."

What is the second operation being performed with the variable side of the equation? *(add 3)*

Write "add 3" in the space labeled "2nd."

What operations will be needed to "undo" each operation? *(division "undoes" multiplication, subtraction "undoes" addition)*

Write and have students write "subtract 3" in the first blank and "divide by 2" in the second blank for inverse operations.

According to your foldable notes, what order will we need to apply these operations? *(subtraction first, then division)*

Write "subtract 3" in the space labeled "1st," and "divide by 2" in the space labeled "2nd," to list the order and operation to solve.

Pause for students to write.

Let's perform the operations to solve the equation. First subtract 3 from both sides.

Pause for students to work.

When we see addition and subtraction of the same number, it creates a zero pair. What do you recall about zero pairs in expressions? *(answers will vary; they do not change the value of the expression; numbers can be cancelled out; sum to zero)*

Where do you see a zero pair in your work? *(+ 3, − 3 is a zero pair on the side with the variable)*

The subtraction of 3 is the inverse operation of adding 3. Therefore, we are left with the variable term and results, 8, on the other side of the equal sign. After performing the first inverse operation, what does the equation now state? *(2x = 8)*

Write and have students write "$2x = 8$" on the *Demonstration Practice Sheet*.

Be sure to write "2*x* = 8" under the equation to show the result of the first inverse operation.

Based on the list of inverse operations, we will divide both sides by 2 next.

Pause for students to work. Illustrate dividing both sides by 2. Clarify for students that the result of $2x$ divided by 2 is $1x$, or just x.

When we divide 2*x* by 2, the result is 1*x*. We write *x* as the result because 1 multiplied by *x* is equal to *x*. 8 divided by 2 is equal to 4. This means we know that *x* = 4.

Using substitution, check and have students check the solution, $x = 4$. Write and have students write "$2(4) + 3 = 11$" on the *Demonstration Practice Sheet*.

We can check our work using substitution. Write "2(4) + 3 = 11" and evaluate the equation as we did at the beginning of the lesson. Remember that because we are evaluating the equation, we perform the operation based on the order of operations.

Pause for students to work.

Because we end up with the true statement "11 = 11," we know that we have correctly solved the equation.

Watch For	**Some students can solve algebraic equations without understanding what a solution is. Students do not realize that when an incorrect solution is substituted into an equation, it will produce different values for each side of the equation. Asking students whether the value results in a true statement will help to reinforce the definition of solution.**

4. Guide students through problem 2 on the *Demonstration Practice Sheet*.

   Write and have students write the inverse operations to solve problem 2 algebraically. Ask students specific guiding questions to elicit verbal responses.

   > **Looking at problem 2, we need to solve the equation. This means we have to isolate the variable *a* to find the solution to the equation. We must ask ourselves, "What operations are performed with the variable?" What operation is performed with the variable, *a,* first?** *(division)*
   >
   > **How do you know that division is the first operation?** *(the order of operations tells us to divide before subtracting)*

   Write and have students write "divide by 2" in the space labeled "1st."

   > **Write "divide by 2" in the space labeled "1st" on the Demonstration Practice Sheet. What operation is being performed on *a* second?** *(subtraction)*

   Write and have students write "subtract 4" in the space labeled "2nd" on the *Demonstration Practice Sheet*.

   > **Write "subtract 4" in the space labeled "2nd."**

   Pause for students to work.

What are the inverse operations? *(multiplication and addition)*

Which of the inverse operations will we apply first? *(addition)*

How do you know we are to add first? *(because it is the reverse of the order)*

Write and have students write "add 4" in the space labeled "1st" on the *Demonstration Practice Sheet*.

In problem 2, write "add 4" in the space labeled "1st" on the Demonstration Practice Sheet. What should we write in the space labeled "2nd"? *(multiply by 2)*

Pause for students to work.

Now we will complete the solving process algebraically. What should we write algebraically beneath each side of the equation? *(+4)*

Write "+4" beneath each side of the equation to create the zero pair on the variable side and calculate a value on the other.

Pause for students to work. Write and have students write "$\frac{a}{2}$ = 7" on the *Demonstration Practice Sheet*.

What was the result of adding 4 to both sides? *($\frac{a}{2}$ = 7)*

Now we see that *a* divided by 2 will equal 7. Write "*a*/2 = 7" under the equation to show the result of the inverse operation.

What is the next inverse operation we write on each side of the equation? *(*2)*

Write "*2" on each side of the equation and calculate the value.

Pause for students to work.

What is the result of the inverse operation? *(the solution is a = 14)*

Because 7 times 2 is equal to 14 and $\frac{a}{2}$ times 2 is equal to a, we know that the solution is $a = 14$.

Check and have students check the solution by using substitution.

Again, we want to check our work using substitution.

Pause for students to work.

Is the solution $a = 14$ correct for the equation? *(yes)*

How do you know? *(14/2 = 7 and 7 − 4 = 3; 3 = 3 is a true statement)*

5. Have student partners solve problems 3 and 4 on the *Demonstration Practice Sheet*.

   Remind students to use the previous problems as well as their foldable for guidance. Have student partners verbalize the solving process. Use guiding questions to elicit verbal responses from student partners to check their understanding.

   - What questions do you ask yourself when trying to solve equations?
   - What does it mean to solve an equation?
   - What operations are being performed?
   - What are the inverses of those operations?
   - Is order important? How does it affect what you do?
   - What operation "undoes" [operation]?
   - How will you show the inverse operation when you write it algebraically to solve?

- How can you use substitution to check your work?

6. Provide a summary of the lesson content.

   Review the key ideas of the lesson by asking questions about solving algebraic equations.

   What does it mean to solve an equation? *(find the value(s) of a variable that makes the equation true)*

   What is an example of inverse operations? *(answers will vary; addition and subtraction, multiplication and division)*

   How do you know in what order to perform the inverse operations? *(the order of operations tells us to add/subtract, then multiply/divide, then exponents and last parentheses)*

   How do we use the properties of equality to solve equations? *(answers will vary; using the same operation and same number on each expression in the equation maintains equality)*

   What questions should you be asking yourself as you solve equations? *(what operations are being performed on the variable; what are the needed inverse operations; what order do I need to perform the inverse operations)*

Practice

Pair Practice

1. Have students work with a partner to identify the operations being performed and fill in the blanks for each item.

2. Have student pairs identify the necessary inverse operations and fill in the blanks for each item.

3. Use probing questions similar to those in the Demonstrate Practice section to elicit a detailed explanation of student pair work to the class.

   - What is the inverse operation of [operation]?

- How did you know in what order to perform the inverse operations?

Error Correction Practice

1. Have each student in a pair analyze and correct the sampled work. Then have partners use the Think-Pair-Share strategy to discuss their findings.

2. Have student pairs share their work with the class.

Independent Practice

1. Have students work independently to complete the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- What does it mean to solve an equation?
- What does it mean to use an inverse operation?
- How do you know which inverse operation to perform?
- How do you know what order to perform the inverse operation?
- How do you use the properties of equality to solve equations?

Lesson 9: Solving Algebraic Equations With Variables on Both Sides, Part I

<table>
<tr><td rowspan="3">Lesson Objectives</td><td colspan="2">Students will use inverse operations and properties of equality to solve two-step algebraic equations with variable on both sides.

Students will verbalize how to solve two-step algebraic equations using precise mathematical language.</td></tr>
</table>

Lesson Objectives	Students will use inverse operations and properties of equality to solve two-step algebraic equations with variable on both sides. Students will verbalize how to solve two-step algebraic equations using precise mathematical language.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	coefficient, equation, equivalent, expression, solve	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 89-100) • Overhead/document projector	• Student Booklet (pp. 47-52) • Solving Equations foldable • Calculator (if needed)

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present the *Cumulative Review Practice Sheet*. After students have had the allotted time to work independently, ask questions to activate students' background knowledge and prerequisite skills about solving algebraic equations.

1. Display the *Cumulative Review Practice Sheet*.

   Have students work with a partner to define inverse operation.

   To solve the equation, we must understand certain mathematical processes. For example, what does "inverse operation" mean? Turn to your neighbor and define inverse operation in your own words. I will call on a few pairs to provide a definition.

   Pause for students to work. Call on student pairs to provide a definition for inverse operation. Have student partners discuss the meaning of "property of equality."

   Another mathematical process we use to solve equations is the property of equality. What does the property of equality mean? Again, turn to a neighbor and discuss the meaning of the property of equality. Use an equation to help you discuss with a neighbor.

   Call on students to engage in a discussion of the meaning of the property of equality.

   Discuss and have students discuss their work on problem 1.

   Let's discuss problem 1 on the Cumulative Review Practice Sheet. What operation did you write down?
   (divide by 4)

What is the inverse operation that you wrote down? *(multiply by 4)*

What is your solution after you multiply both sides by 4? *(28 = b)*

How could you check your work? *(substitute the value 28 into the original equation for the variable b)*

Lead a discussion regarding students' work for problem 2.

Looking at problem 2, what operations did you list as being performed on the variable *r*? *(multiply by 6, add 4)*

What inverse operations are needed? *(divide by 6, subtract 4)*

What order did you apply to the inverse operations? *(subtract 4 from both sides first, then divide both sides by 6)*

What is the solution to this equation? *(r = 1)*

2. Display your hardcopy of the *Solving Equations* foldable.

Point to the section labeled "Algebraic Method" in the foldable. Instruct students to complete the entire example and fill in all blanks EXCEPT for the "variables on both sides" section.

Take out your Solving Equations foldable and locate the section about algebraic methods. You will complete this example now. Fill in all of the blanks and show all work. The only empty blanks at this time will be the blanks underneath "variables on BOTH sides." Remember that you want this example to be complete and thorough so that it will be helpful later in your work. You will have 3 minutes to complete this section of the foldable.

Pause for students to work.

Lead a class discussion that reviews what students wrote and fills in any missing details.

Let's go through what you wrote together and make any needed adjustments.

What is the first operation being performed on *x*? *(divide by 3)*

How do you know? *(the order of operations dictates that division happens before subtraction)*

If you have not already, write "divide by 3" in the 1st blank after "list of operations."

Pause for students to write.

What is the second operation? *(subtract 9)*

If you have not already, write "subtract 9" in the 2nd blank.

Pause for students to write.

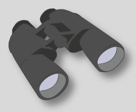
What do we know about the order of the inverse operations? How will they be applied when we work on solving? *(answers will vary; they will happen in the reverse order; "backwards"; they will switch)*

What inverse operation will we need to apply 1st? *(add 9 to both sides)*

How do you know? *(subtraction was 2nd in the list, so it's inverse should be 1st; subtraction and addition "undo" each other)*

If you have not done so already, write "add 9 to both sides" in the 1st blank.

Watch For



Struggling learners may over-generalize aspects of solving equations. For example, students may try to always make a term negative when removing it, even when division is necessary. Clear language and emphasis on "undoing" what operation has been performed on the variable will help to reinforce correct processes. Focus first on what has happened to the variable (the operations being performed) and then on "undoing" them.

Pause for students to write.

What inverse operation will we need to apply 2nd?
(multiply both sides by 3)

How do you know? *(division was first, so multiplication needs to happen second; multiplication and division "undo" each other)*

If you have not done so already, write "multiply both sides by 3" in the 2nd blank.

Pause for students to write. Have students review their work for solving for x.

Now let's look at your work in solving for *x*. Did you apply the inverse operations in the correct order? Did you calculate correctly? Review your work on the Solving Equations foldable.

Pause for students to review.

What was your solution for *x*? *(x = 27)*

If you have not done so already, use substitution to verify that 27 is the correct solution for *x*.

Pause for students to write.

Is 27 divided by 3 minus 9 equal to 0? *(yes)*

What does this mean about our solution? *(it is a true statement and 27 is correct)*

The review of the procedures for solving 1-step equations from the previous lesson and this example on the foldable will help to guide you in solving 2-step equations.

Preview

This lesson will build on students' knowledge of solving algebraic equations.

Today we will solve algebraic equations with variables on both sides using inverse operations and properties of equality.

This means that we will continue to solve by the "undo" math operation, keeping equations balanced to find values of the variable that will make the equations true.

Demonstrate

1. Guide students through solving problem 1 on the *Demonstration Practice Sheet*.

   Display and write on your hardcopy, using an overhead or document projector for students to see. Have students discuss the similarities and differences between problem 1 and the equations from previous lessons.

Look at problem 1 on your Demonstration Practice Sheet. What do you see that is the same about this example and the ones we have worked on previously? *(it has variables (letters), constants (numbers), and an equal sign)*

What do you notice that is different? *(there are variables on BOTH sides of the equal sign)*

Have students recall the definition and goals of solving equations.

Recall what it means to solve an equation and what the goal is of solving an equation.

Pause for students to think.

What is the goal of solving an equation? *(to find the value of the variable that makes the statement true)*

How do we find the value of a variable? *(answers will vary; by isolating the variable; getting the variable by itself)*

When we isolate the variable, we are trying to use inverse operations to get the variable by itself. In previous lessons, all we had to do was add or subtract the constant to begin isolating the variable because there was only one expression that contained a variable in the equation. Today, we will look at what it takes to isolate a variable when it appears in both expressions of the equation.

Use the equation "$2x + 3 = 5$" to discuss solving and collecting terms. Project the equation as you discuss with students. In the discussion, make a connection between collecting constant terms and collecting variable terms as the use of addition and/or subtraction.

Before now, we have been adding or subtracting the constants from both sides of the equation to start isolating the variable. In the equation $2x + 3 = 5$, what are the constant terms? *(the constant terms are 3 and 5)*

What is the coefficient? *(2)*

How would we solve the equation? *(subtract 3 from both sides, then divide by the coefficient 2)*

In problem 1, how many variable and constant terms do we have? *(2 variable and 1 constant)*

Can you think of a way that we could collect the variable terms on 1 side of the equation? *(use inverse operations; add or subtract 1 variable term to/from the other sides)*

Because 1 variable is already on its own on 1 side of the equation, we will collect the variable terms to 1 side by using inverse operations.

Guide students through solving problems with variables on both sides of the equal sign.

Because the $5x$ is already by itself in the equation, we need to use an inverse operation on the $2x$ term to collect the variable terms on the same side. Is the $2x$ positive or negative? *(positive)*

Because $2x$ is positive, we can think of it as addition. What inverse operation will be needed? *(subtraction)*

Write and have students write "subtracting $2x$" in the blank labeled "Collect the variables on one side by" on the *Demonstration Practice Sheet*.

Write "subtracting $2x$" in the space labeled "Collect the variables on one side by" on the Demonstration Practice Sheet.

Perform and have students perform this operation on the equation in problem 1. Write and have students write the resulting equation, "$3 = 3x$" under the equation.

Complete the subtraction from each side on the equation. Notice that a zero pair is created on the left side of the equation and 3x on the right side. What is the result of subtracting $2x$ from both sides? *(3 = 3x)*

Write "$3 = 3x$" under the equation to show the result of subtracting $2x$ from both sides.

Pause for students to write.

Now the equation states 3 = 3x. The problem now looks exactly like the equations we practiced in Lesson 7. There is only 1 variable and it is already isolated.

Write and have students write the operations and their inverses in the corresponding blanks provided on the *Demonstration Practice Sheet*.

What is the operation on the variable *x*? *(multiply by 3)*

Write "multiply by 3" in the blank.

What is the inverse of "multiply by 3"? *(divide by 3)*

Write "divide by 3" in the blank.

Divide both sides by 3 to isolate the *x* on one side. This operation generates 3 divided by 3, which is equal to 1 on the other side. Therefore, we know the solution is *x* equals 1.

Write "*x* = 1" on your Demonstration Practice Sheet.

Pause for students to write. Write and have students write the equation using substitution to check their work.

We can check our work by substituting the value of 1 in for *x* in the original equation. This gives us the equation 2(1) + 3 = 5(1). Write "1" in the parenthesis to show substitution.

What is 2(1) + 3? *(5)*

What is 5(1)? *(5)*

So, we can write "5 = 5."

Pause for students to write.

Is 5 = 5 a true statement? *(yes)*

What does 5 = 5 tell us about our solution? *(that x = 1 is correct)*

Watch For	Some students can solve algebraic equations without understanding what a solution is. Students do not realize that when an incorrect solution is substituted into an equation, it will produce different values for each side of the equation. Asking students if the value results in a true statement will help to reinforce the definition of solution.

2. Instruct students to fill in the blanks under "Questions to ask when solving."

   Students will ask these questions of themselves when they are stuck or need help with solving because the fill-in-the-blank level of scaffolding is being removed from the practice problems. Write and have students write, "What operations are being performed on the variable?" in the first blank on the *Demonstration Practice Sheet*.

   > **We will now fill in the blanks for "Questions to ask when solving" so that you have a reference if you forget the steps for solving.**
   >
   > **What is the first question we have been asking as we have started to solve equations?** *(answers will vary; what happened to the variable; what operations are being performed)*
   >
   > **The first question we have asked in each problem is, "What operations are being performed on the variable?" Write that in the first blank.**

   Pause for students to write.

   > **What is the second question we have been asking as we have started to solve equations?** *(answers will vary; what are the inverse operations; how do you "undo" it)*

Write and have students write, "What inverse operations are needed?" in the second blank, and "What order do we perform the inverse operations?" in the last blank.

> **The second question we have asked in each problem is, "What inverse operations are needed?" Write this in the second blank.**
>
> **The last question to ask is, "What order do we perform the inverse operations?" Write this in the third blank.**

Pause for students to write.

3. Guide students through the solving equations process for problem 2 on the *Demonstration Practice Sheet*.

Instruct students to refer back to the questions they just wrote as needed. Discuss with students which variable term should be moved to solve the equation. Use think-alouds for students to understand why we choose to move certain variable terms.

> **Looking at problem 2, we notice that it also has a variable on each side of the equal sign. We need to collect the variable terms in the equation on the same side. The most logical way to collect the variable terms is to isolate the constant term.**
>
> **On which side of the equation will we collect the variable terms?** *(the left side)*
>
> **By collecting the variable terms on the left side, we will be isolating the constant. Remember that we want to isolate the variable in the end to find a solution.**
>
> **How will we collect the variable terms?** *(add 7y to both sides)*

Write and have students write "add $7y$" in the blank labeled "Collect the variables on one side by."

Write "add 7*y*" in the blank provided for "Collect the variable on 1 side by."

Write and have students write "+ 7*y*" on both sides of the equation.

Write "+ 7*y*" on both sides of the equation and perform the operation. This process created a zero pair. What terms are the zero pair? *(7y and -7y)*

How do you know this is a zero pair? *(7y and -7y sum to zero)*

Pause for students to work. Write and have students write the resulting equation of adding 7*y*, "-2*y* = 4."

What is the result of adding 7y to both sides? *(-2y = 4)*

Write "-2*y* = 4" under the equation to show the result of adding 7*y*.

Discuss with students how to isolate the variable.

The variable term has been isolated to 1 side of the equal sign, but there are no more blanks for us to fill in as there were in previous problems – this is why we wrote our questions above this problem. These will be the questions we ask ourselves as we work.

What is the first question you should ask yourself? *(what operations are being performed on the variable?)*

What are the operations being performed on the variable? *(multiplication; *-2)*

What is the second question your should ask? *(what inverse operations are needed?)*

What inverse operation is needed to isolate the variable *y*? *(division)*

Divide both sides by -2 to find the solution to the equation.

Write and have students write "$1y = -2$" on the *Demonstration Practice Sheet*. Perform the operation and check the solution. Instruct students to do the same.

Perform the operation to solve for the value of *y*.

Pause for students to work.

What does *y* equal? *(-2)*

We can use substitution to check our work and verify the solution.

Pause for students to work.

What is -9(-2)? *(18)*

What is the value of -7(-2) + 4? *(18)*

Based on this work, is the solution *y* = -2 correct for this equation? *(yes)*

We know *y* = -2 is a solution because 18 = 18 is a true statement.

4. Guide students through the solving equations process for problem 3 on the *Demonstration Practice Sheet*.

   Discuss with students which variable term should be moved to solve the equation. Use specific questions to elicit verbal responses to check for student understanding and reasoning.

   Looking at problem 3, on which side of the equation do we collect the variable terms? *(the left side)*

   Why should we collect the variable terms on the left side? *(so that the constant term and variable terms are isolated)*

   What operation will you use to collect 2*m* and 6*m* on the left side? *(subtract 6m from both sides)*

Write and have students write "subtract 6m" in the space provided on the *Demonstration Practice Sheet*.

Write "subtract 6*m*" in the blank provided. Perform this operation to the equation.

Pause for students to work.

Notice that when we subtract 6*m* from 2*m*, the result is a negative number because 2*m* − 6*m* is equal to -4*m*. It is important to remember that this does not change the process or indicate that you have made any mistakes.

Write and have students write "-4m = 4" under the equation to show the result of subtracting 6m.

What is the resulting equation? *(-4m = 4)*

Write "-4*m* = 4" under the equation to show the result of subtracting 6*m*. Use the questions you wrote down to determine the operation that you will perform to solve.

Pause for students to think.

What operation do we perform to isolate *m*? *(divide both sides by -4)*

Perform the division of -4 to each side of the equation.

Pause for students to work.

The result of dividing 4 by -4 is -1. This means that *m* = -1. We now have to check the solution by using substitution.

Use substitution to check the solution. Write and have students write their work on the *Demonstration Practice Sheet*.

Write down the work to check your solution on the Demonstration Practice Sheet.

What was the result of substituting -1 for the variable *m*? *(-2 = -2)*

Because -2 = -2 is a true statement, we know that our solution *m* = -1 is correct.

5. Guide students through problem 4 on the *Demonstration Practice Sheet*.

   Remind students to use the previous examples for guidance. Use guiding questions to elicit verbal responses from students to check for their understanding.

   - On which side of the equation should we collect the variable terms? Why?
   - What inverse operation do we need to perform to collect the variable terms?
   - What questions should you be asking yourself as you solve equations?
   - What operation "undoes" [operation]?
   - How will you show this when you write it down to solve?
   - How can you use substitution to check your work?

6. Provide a summary of lesson content.

   Review the key ideas of the lesson by asking questions about solving equations with variable terms on both sides of the equation.

How do you solve an equation with variable terms on both sides? *(first collect the variable terms on 1 side, then use the inverse operation to isolate the variable)*

How do you check your solution? *(substitute the value into the equation, if it makes a true statement then the solution is correct)*

What questions should you be asking yourself as you solve equations? *(what operations are being performed on the variable; what are the needed inverse operations; what order do we perform the inverse operation)*

Practice

Pair Practice

1. Have students work in pairs to solve each equation. Encourage students to discuss the solving process and checking answers, using mathematical language from the lesson.

2. Use probing questions similar to those in the Demonstrate section, such as the following, to guide students through the problem-solving process.

    - What was your first step to solving?
    - How did you isolate the variable?
    - Did you check your work? How did you check your work?

Error Correction Practice

1. Have each student in a pair analyze and correct the sampled work. Then, have partners use the Think-Pair-Share routine to discuss their findings.

2. Have student pairs present their thinking to the class.

Independent Practice

1. Have students work independently to complete the *Independent Practice Sheet*.
2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- Why do we collect the variable terms first?
- How do we collect the variable terms?
- What questions should we ask ourselves when solving equations?
- How can we check solutions?

Lesson 10: Solving Algebraic Equations With Variables on Both Sides, Part II

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students will use inverse operations and properties of equality to solve multi-step algebraic equations with variables and constants on both sides.

Students will verbalize how to solve multi-step algebraic equations using precise mathematical language.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">coefficient, constant, equation, equivalent, expression, like terms, solve, zero pair</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 101-112)
• Overhead/document projector</td><td>• Student Booklet (pp. 53-58)
• Solving Equations foldable
• Calculator (if needed)</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present the *Cumulative Review Practice Sheet*. After students have had the allotted time to work independently, ask questions to activate students' background knowledge and prerequisite skills about solving algebraic equations.

1. Discuss the questions students should ask when solving equations.

   Have students recall from the previous lesson the questions they should ask when solving equations.

   In lesson 9, we listed 3 questions to ask when solving equations. On the Cumulative Review Practice Sheet, write the 3 questions you are to ask when solving equations.

   Pause for student to work. Call on students randomly to provide 1 question at a time.

2. Display the *Cumulative Review Practice Sheet*.

   Discuss and have students discuss their work on problem 1.

   Problem 1 wanted you to find the solution to the equation. What does it mean to solve an equation? *(find the number or number that when replacing the variable generates a true statement)*

   Let's discuss problem 1 on the Cumulative Review Practice Sheet. What operations did you write down? *(1st: multiplication, multiply by 7; 2nd: subtraction, subtract 18)*

What are the inverse operations that you wrote down? *(1st: addition, add 18; 2nd: division, divide by 7)*

What is your solution? *(4, x = 4)*

Is your solution correct? Did you check your work? *(yes)*

How did you check your work? *(substitution)*

When you substituted the solution in the equation, what was the result? *(10 = 10)*

Discuss with students their work on problem 2.

Looking at problem 2, how did you collect your variable terms? *(answers may vary; collect all variables on the left side; subtract 3x)*

What is the result of subtracting 3*x* from both sides? *(-7x = 14)*

What is the solution to this equation? *(x = -2)*

When you checked your work, was it correct? *(yes)*

How did you know? *(when substituted, I got 8 = 8)*

3. Display your hardcopy of the *Solving Equations* foldable. Point to the section labeled "Algebraic Method." Instruct students to complete the part of the section labeled "if there are variables on BOTH sides."

Take out your Solving Equations foldable and locate the section about algebraic methods. You will now fill in the part labeled "if there are variables on BOTH sides." In this space you will write a reminder about how to proceed if the equation you are given has the same variable on each side of the equation. Write in your own words the first step to do when variables are on BOTH sides.

Pause for students to work.

Lead a class discussion that reviews what students wrote and fills in any missing details.

Let's go through what you wrote together and make any needed adjustments. In the previous lesson we had equations with variable terms on both sides of the equation.

What did we do if there were variables on both sides? *(tried to get the variable terms on the same side by collecting terms)*

How will you collect the variables onto the same side of this equation? *(use the inverse operation by adding or subtracting both sides of the equation)*

If you have not already done so, write all aspects of what we just discussed here in this space. For example, I will write down "collect variable terms together using either addition or subtraction to both sides of the equation."

Pause for students to write.

Preview

This lesson will build on students' knowledge of solving algebraic equations.

Today we will solve algebraic equations using inverse operations and properties of equality for equations with variable and constant terms on both sides of the equation.

In other words, we will continue solving by "undoing" math operations and keeping equations balanced to find the values of the variables that will make the equations true.

Demonstrate

1. Guide students through problem 1 on the *Demonstration Practice Sheet*.

   Display and write on your hardcopy using an overhead or document projector for students to see. Have students follow along and write on their copies.

   Look at problem 1 on your Demonstration Practice Sheet.

   Discuss and have students discuss the similarities and differences between this problem and the others from previous lessons.

   What are the terms in the equation? *(2b, 16, 6b, and 8)*

   What do you see that is the same in this problem and in the others we have worked on previously? *(it has variable terms (letters), constant terms (numbers), and an equal sign)*

   What do you notice that is different about the terms? *(there are variables AND constants on BOTH sides of the equal sign)*

   In Lesson 9, we looked at equations that already had a variable term isolated on one side. Today, we will look at equations that have constants and variables on BOTH sides of the equal sign.

   Discuss with students how to complete the process to solve equations. Make your thinking visible to students using think-alouds throughout the process.

   Because there are both variables and constants on each side of the equation, we have a choice in how we collect like terms. What terms in the equation are like terms? *(2b and 6b, 16 and -8)*

   Which terms do we collect first? *(variable terms, 2b and 6b)*

What are the coefficients in the equation? *(2 and 6)*

Recall that coefficients are the numbers in front of the variables or the numbers multiplied by the variables. We can collect the variable terms by either subtracting 2*b* from both sides, or subtracting 6*b* from both sides.

We have a choice to make. I would prefer to subtract 2*b* from 6*b* because the result is a positive coefficient. Write "subtracting 2*b*" in the blank for "Collect the variable on 1 side by" on your Demonstration Practice Sheet.

Write and have students write "– 2*b*" on both sides of the equation on the *Demonstration Practice Sheet* to collect variable terms.

Perform that operation, − 2*b*, on your equation. What is the result? *(16 = 4b − 8)*

Write in the blanks on your hardcopy the operations on *b* in the resulting equation. Pause for students to write.

What have we created on the left side of the equation? *(a zero pair on the left side with 2b and − 2b)*

On the right side of the equation, what was created? *(4b)*

This leaves 16 = 4*b* − 8. What is the next step to solve the equation? *(we need to list the operations being performed on the variable)*

What happens to *b* first? *(multiply by 4)*

What happens to *b* second? *(subtract 8)*

Write the operations in the space provided.

Write on your hardcopy the inverse operations to solve for *b*. Pause for students to write.

Think about what inverse operations will be needed.

Pause for students to think.

What is the first inverse operation? *(add 8 to both sides)*

What is the second inverse operation? *(divide both sides by 4)*

Write the inverse operations in the space provided.

Apply and have students apply the inverse operation to the equation to solve for *b*. Write and have students write on the *Demonstration Practice Sheet*.

We use the inverse operation to solve the equation. Remember what it means to solve an equation? *(to find the value or number that will replace the variable to make a true statement)*

Let's apply these inverse operations to solve the equation.

Add 8 to each side of the equation. Adding 8 created a zero pair on the right and 16 + 8 equals 24 on the left.

Pause for students to write.

Next, we will divide both sides by 4. This isolates the variable *b* on the right and results in 6 on the left.

The solution we get is *b* = 6.

What can we use to verify whether the answer is correct? *(substitution)*

Substitute 6 into the original equation in the section labeled "Check using substitution."

Pause for students to write.

We are looking at the equation 2(6) + 16 = 6(6) − 8. What is the value of 2(6) + 16? *(28)*

What is the value of 6(6) − 8? *(28)*

This means we write "28 = 28." What does this tell us about our solution? *(b = 6 is correct because 28 = 28 is a true statement)*

Watch For	Some students can solve algebraic equations without understanding what a solution is. Students do not realize that when an incorrect solution is substituted into an equation, it will produce different values for each side of the equation. Asking students if the value results in a true statement will help to reinforce the definition of solution.

2. Guide students through solving problem 2 on the *Demonstration Practice Sheet*.

Problems 1 and 2 are the same equation. Guide students through the solving process of moving $6b$ and discuss the similarities and differences.

Looking at problem 2, notice that it is the same equation as problem 1. The reason we are working with this problem again is because we had to decide which side of the equation we collected the variable term.

When we solved the equation the first time, we decided to subtract $2b$ when we could have also subtracted $6b$. Think about what would have changed if we had made this choice instead.

Pause for students to think. Have students turn to a neighbor to share their thinking about what would have changed.

Turn to a neighbor and discuss what you think would have changed.

Pause for students to discuss. Have students share their thoughts by randomly calling on students.

Teacher Note

Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with students' names on them when selecting students to share their answers or reasoning.

> **What did you and your neighbor think would have changed?** *(answers may vary; it would have been -4b, instead of 4b, to start)*

> **This time we will work the problem by subtracting 6*b* first and see what – if anything – is different. Write "subtracting 6*b*" in the space provided on the Demonstration Practice Sheet.**

Write and have students write "– 6*b*" on both sides of the equation on the *Demonstration Practice Sheet* to collect variable terms.

> **Perform that operation, − 6*b*, on your equation. What is the result?** *(-4b +16 = − 8)*

Pause for students to write.

> **This time, we created a zero pair on the right by combining 6*b* and −6b and the result on the left was -4*b*. We now have the equation -4*b* + 16 = -8.**

> **What is the first question we should ask, now that we have variable terms combined?** *(what operations are being performed on the variable?)*

> **We will list them on the side on your paper. Use the space to the right of problem 2 to write "multiply by -4 and add 16."**

Pause for students to write.

Write on your hardcopy the inverse operations to solve for *b*. Pause for students to write.

> **What is the second question you should ask?** *(what inverse operations are needed?)*

Pause for students to think.

> **Write the inverse operations "subtract 16 and divide by -4" near the list you already made for problem 2.**

Pause for students to write.

Apply and have students apply the inverse operations to the equation to solve for *b*. Write and have students write on the *Demonstration Practice Sheet*.

> **Let's apply these operations to our equation. What is the result of subtracting 16 from both sides of the equation?** *(answers may vary; -4b on the left and -24 on the right; -4b = -24)*

> **The result will be -4b = -24. What is the result of dividing both sides by -4?** *(b = 6)*

> **Use substitution to verify *b* = 6 in the original equation.**

Pause for students to work.

> **Now we are looking at 2(6) + 16 = 6(6) − 6. What is the value of 2(6) + 16?** *(28)*

> **What is the value of 6(6) − 8?** *(28)*

> **We can write 28 = 28. What does this tell us about our solution?** *(it is correct)*

At this point, lead students in a discussion about how the two approaches result in the same correct answer and that they will need to consider the approach they chose each time in order to make the process as efficient and error-free as possible.

Both approaches resulted in the same correct answer. Think about what you feel are the differences in the steps we completed in each one.

Pause for students to think.

What differences did you notice? *(answers will vary; the second solving process had more negatives)*

Which one seemed easier? *(the first one)*

Why do you think that is the case? *(there were fewer negatives to work with)*

When solving equations, sign errors are a common error. To reduce the possibility of sign errors, we need to choose carefully on which side of the equation to collect the variable terms.

Display the *Solving Equations* foldable and direct students to the Algebraic Methods section. Write and have students write "*Try to keep it positive!*" on the foldable.

Look back at your Solving Equations foldable in the Algebraic Methods section. We need to write ourselves a reminder about trying to make the variable collection result in a positive coefficient. Write down, "*Try to keep it positive!*"

Pause for students to write.

3. Guide students through the solving process for problem 3 on the *Demonstration Practice Sheet*.

   Using specific questions, elicit verbal responses from students to check for their understand of the solving process with variables on both sides of the equation.

Looking at problem 3, again we have variable terms on both sides. What is the sign on each of these terms? *(positive)*

What do we do first to solve the equation? *(collect like terms on the same side of the equation using inverse operation; collect the variable terms on the same side of the equation by using inverse operation)*

What inverse operation would be used to collect the variable terms on the same side of the equation? *(subtraction)*

Think for a moment about on which side of the equation you want to collect the variable terms. Should we subtract 5*a* or 4*a* from both sides of the equation and why?

Pause for students to think. Have students turn to a neighbor and discuss which variable term to subtract and why.

Turn to your neighbor and discuss which variable term you choose to subtract and give your reason.

Pause for students to discuss with their neighbor.

Which term do we want to move? *(4a)*

Why do we want to move 4*a*? *(because subtracting 4a would result in a positive coefficient for the variable term)*

Write "– 4a" on your hardcopy and work through that step of solving. Instruct students to do the same.

Write "– 4*a*" on both sides of the equation and then perform that operation.

Pause for students to work.

What does subtracting 4*a* from each side create? *(-6 on the left and 1a + 21 on the right; -6 = 1a + 21)*

Discuss with students the fact that 1*a* is the equivalent to writing "*a*." When the coefficient is 1, we know that a more direct approach to solving indicates simply writing "*a*" instead of "1*a*." Explain this so that students do not feel obligated to divide by 1 as an extra step. Write "-6 = *a* + 21" on your hardcopy as the next step in solving.

> **What is the resulting equation?** *(-6 = 1a + 21)*
>
> **Looking at -6 = 1*a* + 21, what do you know about multiplying anything by 1?** *(the value stays the same)*
>
> **This means that we can rewrite the equation as "-6 = *a* + 21." Write this as the next step of your solving work.**

Pause for students to work.

> **Notice that without that coefficient of 1, it is easier to see that the equation is a 1-step equation. Therefore, we will only have to perform 1 inverse operation.**
>
> **What operation is being performed on *a*?** *(add 21)*
>
> **What inverse operation will be needed?** *(subtract 21 from both sides)*

Write and have students write "– 21" on both sides of the equation to solve.

> **Write "− 21" on both sides of the equation.**

Pause for students to work.

> **What is the result of the subtraction?** *(creates a zero pair of 21, – 21 and a on the right, and -6 − 21 leaves -27 on the left)*
>
> **What is the result or solution to the equation?** *(a = -27)*

Use substitution to check your work. Substitute the value of -27 into the spaces provided on problem 3 and evaluate to verify that both sides are equal. Instruct students to do the same.

How will you check your work? *(substitute -27 into the variable spaces provided)*

Substitute the values now.

Pause for students to work.

Did the substitution result in a true statement? *(yes, -114 = -114)*

What does this tell us about our solution? *(that a = -27 is correct)*

4. Guide students through problem 4 on the *Demonstration Practice Sheet*.

   Remind students to use the previous examples for guidance. Use guiding questions to elicit verbal responses from students to check for their understanding.

   - What is the coefficient on a variable term when there isn't one written down?
   - What is the first step to solving the equation?
   - Which variable term should be subtracted? Why?
   - What questions should you ask yourself when solving equations?
   - How do you check your solution?

5. Provide a summary of the lesson's content.

   Review key ideas of the lesson by asking questions about solving with variable and constant terms on both sides of the equation.

When there are variable and constant terms on both sides of the equation, what should be the first step to solve the equation? *(subtract the smaller variable term so that the result of adding or subtracting is a positive coefficient variable term)*

What questions should you be asking yourself as you solve equations? *(what operations are being performed on the variable; what are the needed inverse operations; what order do I perform the inverse operations)*

How do you know your answer is correct? *(when I substitute the value for the variable, the result is a true statement)*

Teacher Note
Extension: To increase content practice and difficulty, use fractional coefficients with common denominators for students to practice inverse operation of fractions and division.

Practice

Pair Practice

1. Have students work in pairs to solve the equations on the *Practice Sheet*. Students will alternate each step to solve the equations. Each student will be labeled as "A" and "B."

2. For problems on the *Practice Sheet*, have students complete the row that matches their label. They will alternate each row for both problems.

3. Use probing questions similar to those in the *Demonstrate Practice* section to guide students through the problem-solving process.

    - What is the first step to solving equations?
    - How did you isolate the variable?
    - How did you know which inverse operation to perform and the order to perform it in?
    - How did you check your answer? Was it correct? How did you know?

Error Correction Practice

1. Have student pairs examine the solving process presented and determine where the error occurred and why.
2. Have partners share the reasoning for their findings to the class.

Independent Practice

1. Have students work independently to complete the *Independent Practice Sheet*.
2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- When solving equations with variable and constant terms on both sides, what is the first step?
- Describe how to determine if the solution to an equation is correct.
- What questions should we ask ourselves when solving equations?

Lesson 11: Solving Algebraic Equations Using Tables and Graphs

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students will use the graph and table functions of the graphing calculator and properties of equality to solve algebraic equations.

Students will verbalize how to use the graphing and table functions of the graphing calculator to solve equations using precise mathematical language.</td></tr>
<tr><td>Vocabulary</td><td colspan="2">No new words are introduced.</td></tr>
<tr><td>Reviewed Vocabulary</td><td colspan="2">equation, equivalent, expression, solve</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters (pp. 113-125)
• Overhead/document projector
• Calculator</td><td>• Student Booklet (pp. 59-64)
• Calculator</td></tr>
</table>

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present the *Cumulative Review Practice Sheet*. After students have had the allotted time to work independently, ask questions to activate students' background knowledge and prerequisite skills about solving algebraic equations using tables and graphs.

1. Discuss the answers on the *Cumulative Review Practice Sheet*.

   Discuss the solution to problem 1. Have students define "solving" in their own words and write at the top of the *Cumulative Review Practice Sheet*.

   At the top of the Cumulative Review Practice Sheet, write in your own words what it means to solve an equation. *(answers will vary)*

   When solving an equation, what questions do you ask yourself to solve? *(answers will vary; are variable terms collected on 1 side; what operations are performed on the variable; what are the inverse operations needed to isolate the variable)*

   Problem 1 has variable terms on both sides. How did you solve the equation? *(answers may vary; subtracted 6x and then divided by 2)*

   What is your solution? *(x = -8)*

   How did you know *x* = -8 was the correct solution? *(when substituted into the equation, the result was -64 = -64, which is a true statement)*

   Discuss how to find the solution to problem 2.

Looking at problem 2, how did you collect variable terms? *(answers may vary; subtracting 3v is acceptable, subtracting v from both sides is preferred)*

Once the variable terms were collected on 1 side of the equation, what operations were performed on the variable? *(for the purpose of this script we will assume v was subtracted from 3v; multiply by 2, subtract 4)*

Why is it best to subtract *v* from 3*v*? *(the result is a positive coefficient)*

Remember that there is less of a chance of sign error when collecting the variable terms results in a positive coefficient.

What inverse operations did you use to solve? *(add 4, then divide by 2 to both sides)*

What is the solution to this equation? *(v = 6)*

Preview

This lesson will build on students' knowledge of solving algebraic equations using a calculator.

Today we will solve algebraic equations using the graph and table functions in the graphing calculator and properties of equality.

Demonstrate

1. Guide students through the process of solving problem 1 using tables and graphs on the *Demonstration Practice Sheet*.

   Lead a discussion that guides students to recall the graphing method of verifying expression equivalence. Explain that it can be used to solve equations as well.

   Recall the way we used the calculator to determine if 2 expressions were equivalent.

Pause for students to think.

What do you remember about graphing 2 equivalent expressions? *(if they were equivalent, they graphed right on top of each other)*

We know 2 expressions are equivalent if they graph exactly on top of each other. In other words, they have all of the same input and output values or all of the same (x, y) points.

What did the graphs look like when the expressions were NOT equivalent? *(answers will vary; they were not exactly the same; they crossed; they didn't match)*

The point where the graphs intersect is the solution to the equation when we use graphs to solve equations. It is the place that both expressions yield the same output (y-value) for a specified input (x-value).

Let's look at how that works on your Demonstration Practice Sheet.

Display your hardcopy of the *Demonstration Practice Sheet*. Have students look at problem 1. Circle each expression in the example and label 1 expression "Y_1" and the other "Y_2." Fill in the blanks to the right with the corresponding expressions. Instruct students to do the same.

Look at problem 1 on the Demonstration Practice Sheet. We will treat this just like we were checking for equivalence of expressions. Circle each of the 2 expressions in the equation.

Pause for students to work.

It doesn't matter which expression you choose when you are working on your own, but so that we can stay together, let's all label the left-hand expression "Y_1" and then write that expression in the blank next to "Y_1 = ___."

Pause for students to work.

> **Now we will label the right-side expression with "Y_2" and write it in the blank next to "Y_2 = _____."**

Pause for students to work.

Using your calculator with the document or overhead projector, have students also enter the expressions into the *y*= menu. Once entered, direct students to graph their expressions and sketch what they see in the graphic on the *Demonstration Practice Sheet*.

> **Now we will enter these expressions into the corresponding rows in the *y*= window of the calculator.**

Display your calculator as you work and pause to allow students to work.

> **Verify that yours, your neighbor's, and mine all match before we move on.**

Pause to allow students to check.

> **Now press GRAPH on your calculator.**

Watch For 	**Students may have issues when graphing. The calculator may respond with ERR: SYNTAX or produce a different graph. This could indicate the use of [–] subtraction sign, instead of [(-)] negative.**

Display your calculator as you work and pause to allow students to work.

> **Describe to me what you see.** *(two lines that intersect at one point)*

Sketch what you see in the coordinate plane on your paper. Remember that a sketch is just a reasonable representation and does not have to be a perfect image, but be sure that your graphs are reasonably placed.

Have students work with a partner to speculate about the ordered pair that represents the intersection of the graphs and then calculate the intersection using the feature in the calculator. Explain that they will lean more about calculating intersections later, but for now, it is a valuable tool to calculate solutions to equations. Display your hardcopy and calculator using an overhead or a projector.

You will learn more about intersections of graphs and equations later in your algebra work. For now, we can use the calculator as a tool for solving equations.

Work with your neighbor to inspect the graphs displayed. The point where the 2 graphs intersect is the solution to the equation, or the *x* value where the 2 expressions are equal. Together, estimate the ordered pair where the two graphs intersect. We know that the expressions are equal at this point. When you and your partner feel you both have a reasonable estimate, write the estimate in the blank provided.

Pause to allow student pairs to work. Ask a few student pairs what their estimation point is and write it down to compare after calculations.

What point did you and your partner estimate? *(answers will vary)*

Explain the procedures for calculating the intersection point with the Calculate menu and lead students through calculating the intersection point. Display your calculator and hardcopy using a document or overhead projector for students to see.

We can check our estimates and have the calculator provide the actual ordered pair using the Calculate menu.

There are instructions on your paper. You can follow along as I display my calculator.

Looking at the point where the two graphs intersect, we can use the Calculate menu to determine the exact intersection.

Point to the TRACE button on the calculator for students to follow.

On your calculator, notice the word CALC above the TRACE button. Because it is the same color as the "2nd" button, we need to first press the "2nd" button to access this option. CALC is short for "Calculate." To access the Calculate menu, press 2nd and TRACE now.

Pause for students to work.

Check that your calculator is displaying the same image as mine.

We are looking at a list of the various attributes that your calculator can solve.

Which item in this list do you suppose that we will need today? *(5:intersect)*

You may either press the number 5 key or arrow down to highlight "Intersect" and press ENTER. Make your selection now.

Point to "First curve?" and "$Y_1 = x - 6$" on the screen as you tell students.

You have returned to the graph screen. Across the bottom there is the question "First curve?" and the top of the screen displays "$Y_1 = x - 6$."

Your calculator is asking if this is the line and equation that you want to use to calculate the intersection. Pressing ENTER is the equivalent of saying "yes." Press ENTER now.

Pause for students to work. Again point to "Second curve?" and "$Y_2 = 3x - 4$" on the screen as you explain to students.

Now you see "Second curve?" at the bottom of the screen and "$Y_2 = 3x - 4$" at the top. Again, press ENTER to verify that this is the second equation and the line that you wish to use to calculate the intersection.

Your calculator is asking "Guess?" which means that it is asking you if you want to calculate, as exactly as possible, the ordered pair that represents the intersection. Pressing ENTER again is a response of "yes." Press ENTER now.

Pause for students to work. Have students compare their graph image with yours.

Check to make sure your graph display is the same as mine.

Pause for students to check. Point to the displays as you tell students.

At this point, your calculator displays "Intersection," along with "$x = -1$" and "$y = -7$."

That means that the calculated ordered pair is (-1, -7). Write this in the blank on your paper now. Remember that we are trying to find the value for x that is the solution to the equation. This means that $x = -1$ is the solution.

Pause for students to write.

Let's look at how close the estimates were.

Lead a discussion that allows students to discern why this method works and why the solution is the value for x in the ordered pair, and not the y. Start by investigating the table in the calculator and finding the point where the y values are both equal to indicate that the expressions are equal. Have students write the solution as “$x = -1$,” and then use substitution to check.

To understand why this works, let’s look at the table. Access the table by pressing “2nd” and then GRAPH. Once you are looking at the table, write the values for the *y*-columns onto your paper for each of the given *x*-values.

Pause for students to write.

Compare the two columns. Pay careful attention to the way the *y*-values in each row compare to each other. Which one is greater? Are they ever equal? Take a minute to analyze the data and write down what you notice.

Pause for students to think and write.

Guide students through the analysis of the table. They need to notice that for x-values less than -1, the Y_1 values are greater than those of Y_2. For x-values greater than -1, the values of Y_1 are less than those of Y_2. This is the numeric representation of what is happening graphically. When the lines intersect at the ordered pair (-1, -7), the table reflects it by showing that for $x = -1$, Y_1 and Y_2 are equal. Because we labeled each expression as “Y_1” and “Y_2,” we know that for the expressions to be equal, x must equal -1.

What did you write down? *(answers will vary)*

Let’s look at the first 2 *y*-values in each column. What do you notice when you compare the values for Y_1 to those of Y_2? *(they are greater, higher)*

What about the last 2 values in the columns? *(they are less, lower)*

What is happening when x = -1? *(they are the same, equal)*

Write and have students write the summary of the relationship between the 2 expressions.

Write on the space provided "When $x = -1$, Y_1 is equal to Y_2. Or, if $x = -1$, $Y_1 = Y_2$."

Pause for students to write.

Because we labeled the expressions "Y_1" and "Y_2" in our equation, we can write a sentence to say, "When $x = -1$, then $x - 6 = 3x - 4$."

Write and have students write the algebraic check by substituting $x = -1$ into the equation.

Use substitution to verify the solution algebraically.

When you substituted -1 for x, what was the result? *(-7 = -7)*

The result, -7 = -7, matches the graph and the table values we examined. What does this tell us, algebraically, about our solution, $x = -1$? *(it is correct)*

Be sure to write your solution in the box provided.

Teacher Note
Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with students' names on them when selecting students to share their answers or reasoning.

2. Guide students through the solving process for problem 2 on the *Demonstration Practice Sheet* using a calculator.

   Use the calculator to find the solution to the equation in problem 2 from a graph and a table. Be sure to display your calculator for students to see and follow along. Discuss with students the use of the variable x for all equations when entering into the calculator.

   > **The calculator only allows us to use the variable *x* in the Y= section. Looking at problem 2, how will that impact our work?** *(answers will vary; type x instead of b; replace all b variables with x when typing in the calculator)*

   > **When we type into the calculator, we will use the *x* variable instead of *b*. The use of *x* instead of *b* does not affect the value of the solution. The calculator only uses the variable *x*. Remember that we are trying to find the value of the variable that make the expressions equal.**

   Write and have students write the expressions that will be typed into the calculator in the space provided, "$-2x + 8$" and "$6x – 8$."

   > **Circle each of the 2 expressions and label them "Y_1" and "Y_2." Write the expressions that we will type into the calculator on the blanks on your paper. This means we write "-2x + 8" for Y_1 and "6x − 8" for Y_2.**

   Pause for students to write. Have students check their graph with a neighbor.

   > **Enter each expression into the Y= screen and press GRAPH. Before sketching your graphs on your paper, verify with your partner that your graphs match. If they do not, discuss with your partner why they do not and make them both correct before sketching on your paper.**

Pause for students to check with their neighbor and sketch. Have students write an estimated point of intersection on their *Demonstration Practice Sheet*.

Where is the solution we are looking for on the graph of the 2 lines? *(the place where the expressions intersect; where expressions are equal)*

Estimate an ordered pair for the intersection before you calculate it. Write the estimation on your Demonstration Practice Sheet.

Pause for students to write.

We will use the steps we followed from problem 1 to calculate the intersection point.

How do we access the Calculate menu? *(press 2nd and TRACE)*

Press 2nd and TRACE. Which option do we select from the Calculate menu? *(5:intersect)*

Select option 5. How should we answer each of the questions that the calculator asks? *(press ENTER each time)*

When do you know that we have the answer to problem 2? *(when the calculator screen says "Intersection" and lists x = and y =)*

According to the intersect function on the calculator, what is the ordered pair of the intersection? *((2, 4))*

Write and have students write the calculated point of intersection in the blank provided.

Write the calculated point of intersection in the space provided.

Complete the table to show the point of intersection and circle the row that shows this relationship. Instruct students to do the same.

We need to complete the table to show the solution to the equation. How do we get to the table function on the calculator? *(press 2nd, GRAPH)*

Looking at the table, fill in the values for Y_1 and Y_2 for the given *x* values.

Pause for students to complete the table.

How do we find the solution in a table? *(we have to find the x value that produces the same y values)*

Which row in the table should we circle? *(the row where $Y_1 = Y_2$)*

Is the row of values the same as the intersection point? *(yes)*

What is the solution to the equation? *($x = 2$)*

Write and have students write "2" in the blanks for "In the graph and the table, x = ___, therefore b must equal: ___"

Fill in the blanks on your paper. What do we write in the blanks "*x* = ___" and "therefore *b* must equal: ___?" *(2)*

Pause for students to write. Verify and have students verify that the solution is $b = 2$ using substitution.

Verify *b* = 2 using substitution.

Pause for students to work.

When you used substitution, what was your result? *($4 = 4$)*

What does 4 = 4 mean for this problem? *(the solution $b = 2$ is true)*

3. Guide students through problem 3 on the *Demonstration Practice Sheet*.

   Remind students to use the previous examples for guidance. The prompts in the questions will also be helpful. Use guiding questions to elicit verbal responses from students to check for their understanding.

   - How do you enter a variable that isn't x in Y= of the calculator?
   - Where is the solution we are looking for in the graph of the 2 lines?
   - Where is our solution in the table?
   - Do we use the x-value or the y-value as the solution?

4. Provide a summary of the lesson concepts.

   Review key ideas of the lesson by asking questions about solving with a graph or table.

   On a graph, where do we find the solution? *(the x-value where the graphs intersect)*

   How do you use a calculator to find the solution graphically? *(2nd, TRACE; Intersection and then answer the questions)*

   On a table, where do we find the solution? *(the x-value where the Y values are the same)*

   How do you get to the table on the calculator? *(2nd, GRAPH)*

Practice

Guided Practice

1. As a class, work through the problems on the *Practice Sheet*. Display your hardcopy and calculator using an overhead or document projector.

2. Use specific questions to guide students through the problem-solving process similar to those on the *Demonstration Practice Sheet*. Use the mathematically precise language from the lesson.

    - How do we enter the equation in the calculator?
    - How do we access the table to find the solution?
    - Looking at the table, how do we find the solution?
    - How do we find the solution on the graph?
    - How can we check our solution?

Pair Practice

1. Have students work with a partner to complete the Pair Practice problems. Encourage students to use mathematically precise language when discussing the problems.

2. Student pairs must be prepared to justify their answers and share with the class. Use the following questions to elicit detailed responses from the student pairs:

    - What were the steps you and your partner took to find the solution using a table?
    - What were the steps you and your partner took to find the solution using a graph?
    - How do you know $u = [\quad]$?
    - How do you know $w = [\quad]$?

Independent Practice

1. Have students work independently to complete the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- What does it mean to find a solution?
- Where can we find a solution to an equation when we graph each expression?
- How can we find a solution to an equation by analyzing tables?

Module 2: Expressions, Equations, and Equivalence
Lesson 12

Lesson 12: Solving Algebraic Equations: Choosing Appropriate Methods for Solving Algebraic Equations

Lesson Objectives	Students will choose appropriate methods for solving algebraic equations using graphs, tables, inverse operations, and properties of equality. Students will verbalize how to use the graphing and table functions of the graphing calculator to solve equations using precise mathematical language.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	coefficient, equation, equivalent, expression, solve	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 125-136) • Overhead/document projector • Calculator	• Student Booklet (pp. 65-70) • Calculator

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet*.

Engage Prior/Informal Knowledge

To open the lesson, present the *Cumulative Review Practice Sheet*. After students have had the allotted time to work independently, ask questions to activate students' background knowledge and prerequisite skills about solving algebraic equations.

1. Display the *Cumulative Review Practice Sheet*.

   Discuss with students their work on problem 1.

   Looking at problem 1 on the Cumulative Review Practice Sheet, how did you decide to collect the variables? *(answers may vary; for the script we will subtract 2r from both sides)*

   While we can subtract either variable term, why is it best to subtract 2*r* from both sides? *(the result is a positive coefficient)*

   What inverse operations did you perform to isolate the variable *r*? *(subtract 19 from both sides, then divide both sides by 6)*

   What is your solution? *(-2)*

   When you checked your solution, what was the result? *(3 = 3)*

   Lead a discussion regarding students' work on problem 2.

   Looking at problem 2, which expression did you use as Y_1? *(answers may vary; most will use the left-hand expression)*

   Which expression did you use as Y_2? *(answers may vary; most will use the right-hand expression)*

In which row does $Y_1 = Y_2$? *(the row for x = -8)*

What is the solution to this equation? *(p = -8)*

Lead a discussion regarding students' work on problem 3.

Looking at problem 3, how did you solve the equation using a graph? *(answers may vary; entered each expression into the calculator; graphed; found the point of intersection and the x-value is the solution)*

At what ordered pair did the 2 lines intersect? *((-1, 2))*

What is the solution to this equation? *(q = -1)*

2. Complete the last 2 sections of the *Solving Equations* foldable.

Display the *Solving Equations* foldable using a document or overhead projector for students to see. Guide students to complete the sections for graphs and tables with the steps for using the calculator features to solve equations. Write on your hardcopy and instruct students to do the same.

Now look at the Graphs section of your Solving Equations foldable. We will fill in the steps required to solve using the calculator here so that you may refer to it later in your work.

Think about the steps you used to solve equations using graphs in the calculator.

Pause for students to think.

What was the very first thing we did to solve with graphs? *(labeled each expression "Y_1" and "Y_2")*

Write "label each expression 'Y_1' and 'Y_2'" next to the number 1 in the graphs section.

Pause for students to write.

After labeling each expression with Y_1 and Y_2, what must we do next? *(type them into y=)*

Write "type each expression into Y=" next to the number 2.

Pause for students to write.

Once the expressions are carefully typed in their corresponding rows in the Y= screen, we are ready to press GRAPH and check to see if the lines intersect. This is step number 3. Write down "press GRAPH and look for the intersection."

Pause for students to write.

Once we can see the intersection, we can estimate the ordered pair, but we want to write down the keys to calculate the exact ordered pair of intersection.

Next to step 4, write "2^{nd}, TRACE 5," and "respond to questions with ENTER until the exact intersection is displayed."

Pause for students to write.

Which member of the ordered pair represents the solution to your equation? *(the x-value)*

After step 4, write a reminder that the solution you are looking for is the x-value only.

Pause for students to write.

We need to also update the Tables section of your foldable.

The first 2 steps are the same for both tables and graphing. Copy 1 and 2 from your Graphs section to the Tables section now.

Pause for students to write.

Once the expressions have been typed into the Y= screen, we can go directly to the table to look for the place where $Y_1 = Y_2$. For step 3, write "find the row in the table where $Y_1 = Y_2$."

Pause for students to do the same.

Again, what value is the solution to the equation? *(the x-value)*

Write that reminder here as well.

Pause for students to write.

We will continue adding to these sections in your foldable as we work today.

Preview

This lesson will build on students' knowledge of the solving algebraic equations.

Today we will choose appropriate methods for solving algebraic equations using graphs, tables, inverse operations and properties of equality.

We will make decisions about what method is the most efficient for solving different types of equations.

Demonstrate

1. Guide students in a brainstorm of the benefits and drawbacks to the various methods of solving equations.

Teacher Note
Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with students' names on them when selecting students to share their answers or reasoning.

Have students brainstorm about the different methods of solving equations and what they see as the benefits and drawbacks of each. Have students turn to a neighbor to discuss.

What are the 3 methods we have used to solve equations? *(algebraic methods, graphs and tables)*

Think about the benefits and drawbacks to each method. Ask yourself, "Which method was easiest and why? Does the type of equation make it easier?"

Pause for students to think.

Turn to a neighbor to discuss your thoughts. Be prepared to share some ideas.

Pause for students to discuss with a neighbor.

What do you think are the benefits or drawbacks to each method? *(write student responses on the board in groups by method; answers will vary; for example, algebraic method may be confusing sometimes, graphing is easy, but the steps may be hard to remember, etc)*

Today we will look at ways to identify which method is the most direct in various examples. Remember you can use any method to solve an equation, but today we are focusing on which method is the best or most direct way to find the answer.

2. Guide students through the solving and reasoning process on problem 1 on the *Demonstration Practice* sheet.

Display your hardcopy using an overhead or document projector as well as your *Solving Equations* foldable. Demonstrate as you work with students on how to refer back to the foldable as you solve algebraically, graphically, and then with a table.

Looking at problem 1 on your Demonstration Practice Sheet, we will attempt to use all 3 methods in order to determine a rule for which method will be best for future problems.

First we will solve algebraically. Look at the Algebraic Methods section of your Solving Equations foldable.

What does our example reminds us to do first? *(list the operations being performed on the variable)*

What are the operations? *(multiply by 2, add 3)*

After identifying the operation, what do we do next to solve? *(we need to list the inverse operations to solve)*

What are the inverse operations for this problem? *(subtract 3 from both sides, then divide both sides by 2)*

How do we know the order in which to apply the inverse operations? *(to solve, we reverse the order of operations, add/subtract first, then multiply/divide, etc.)*

Perform those operations now to arrive at a solution.

Pause for students to work.

After performing the inverse operations, what is the solution value for *x*? *(x = 3)*

Next we will solve graphically. Move over to the Use Graphs part of problem 1.

Check the Graphs section of your foldable to find out what we need to do first.

Pause for students to look up the information.

What is the first step when solving with graphs? *(label each with "Y_1" and "Y_2")*

Label the expressions now and then enter each expression into the Y= screen on your calculator.

Perform those steps using your calculator and hardcopy, while projecting for students to see. Pause for students to do the same.

Once they are correctly typed into the Y= screen, what is the next step? *(press GRAPH and look to see if they intersect)*

Press GRAPH now.

Watch For

Students may have issues when graphing. The calculator may respond with ERR: SYNTAX or produce a different graph. This could indicate the use of [–] subtraction sign instead of [(-)] negative.

Pause for students to work.

Do you see the graphs intersect? *(yes)*

How do we calculate the exact intersection point using the calculator? *(press 2^{nd}, TRACE, 5:intersect, and ENTER to answer the questions)*

Perform the steps on your calculator and pause for students to work.

What is the ordered pair of the intersection? *((3,9))*

Write the ordered pair "(3,9)" on the Demonstration Practice Sheet.

Looking at the ordered pair that describes the intersection, which value is the solution to the equation? *(the x value, x = 3)*

Now we will solve using the table. The first steps for working with the table are the same as working with graphs. How will you access the table information in your calculator? *(press 2nd, GRAPH)*

Press 2nd, GRAPH now.

Pause for students to work while you do the same.

In which row is the solution found? *(the row for x = 3)*

How do you know? *(that is the row where $Y_1 = Y_2$)*

Write and have students write the values in the table from the calculator to the solution on the *Demonstration Practice Sheet*.

Write the values in the table from the calculator to show the solution.

What do you notice about the solution in all 3 methods? *(the solution is the same, x = 3)*

Check the solution, *x* = 3, using substitution.

Pause for students to work while you do the same.

When we substitute the value of 3 into the equation, we arrive at the statement 9 = 9. What do you notice about this result that connects back to the 3 methods above? *(answers will vary; the 9 is the y-value of the intersection point; the*

9 = 9 is like the two 9s in the table; when we substitute 3 into the equation, we get 9)

We have just verified that all 3 methods arrive at the same solution. Which method seems the easiest with problems like this equation? *(answers will vary; the algebraic method is the most direct)*

Use think-alouds to make the reasoning process of selecting a method for solving visible to students. Write "When the variable appears only on 1 side, the algebraic method will be the most direct," on the *Demonstration Practice Sheet* to explain reasoning.

In order to make it easier to analyze the differences and figure out which method is more direct, let's count the number of steps involved in each method.

For the algebraic method, we would count 1 step for subtracting 3 and then another for dividing by 2, for a total of 2 required steps.

According to your foldable, there are 4 steps required for graphing and 3 required for using tables.

In this example, the method with the fewest required steps to solve is the algebraic method.

In the space provided on your Demonstration Practice Sheet, write down a reminder, "When the variable appears only on 1 side of the equation, the algebraic method is the most direct."

Remember when you are solving to always have 3 methods to use. You must choose which method makes the most sense to you.

Pause while students write.

3. Guide students through the solving and reasoning process for problem 2 on the *Demonstration Practice Sheet*.

Have students look at problem 2 and notice the difference between problems 1 and 2.

Looking at problem 2, what do you notice that is different about it from problem 1? *(problem 2 has variables on both sides of the equal sign)*

How does having variables on both sides of the equal sign change our work? *(we will have to collect the variables first)*

Using your notes in your foldable, what does the Algebraic Method say regarding variables on both sides? *(variables should be collected to 1 side of the equation first, and we should try to keep the resulting coefficient positive)*

On which side of the equation should we collect the like variable terms? *(the left side)*

How will we collect the variable terms? *(subtract b from both sides)*

What is the coefficient for the variable term *b*? *(1)*

Write "– b" on each side of the equal sign and instruct students to do the same.

Subtract *b* from each side of the equal sign.

Pause for students to work.

What is the result of subtracting *b* from both sides? *($3b - 21 = 9$)*

Now we are looking at $3b - 21 = 9$. What operations were performed on *b*? *(multiply by 3, then subtract 21)*

What inverse operations will be required? *(add 21, then divide by 3)*

How do you know you are to add 21 first? *(use reverse order of operations when solving)*

Perform these operations to solve on your own paper while I do the same on mine.

Pause for students to work.

What is the result of adding 21 to both sides? *($3b = 30$)*

What is the result of dividing both sides by 3? *($b = 10$)*

Now we need to find the solution in both the graph and the table.

Recall the steps for using graphs or look them up in your foldable.

Pause for students to think or look up steps. Remind students that they will need to replace the variable b with x when typing into the calculator.

What should we do first to solve using the graph? *(label each expression "Y_1" and "Y_2")*

What is the next step to solve using the graph? *(enter the equations into Y= and press GRAPH)*

What are you looking for on the graph of the equations? *(the point of intersection)*

Remember our calculators only use the variable *x*. This means we will replace the variable *b* with *x* when typing into the calculator.

Display your calculator as you work and pause to allow students to work also.

Verify that yours, your neighbor's and mine all match before we move on.

Pause to allow students to check.

What is different about these graphs? *(we cannot see where they intersect)*

With a partner, discuss where on the graph you both think the lines will intersect. You will have 2 minutes to make a prediction.

Wait 2 minutes for student pairs to make a prediction about the intersection of the lines. Call on several student pairs and list their prediction on the board for the class to see.

Where do you and your partner predict the graphs will intersect? *(answers will vary)*

By looking at this graph, we can see that the point where they will intersect will be greater than 5. Because we can see 1 line crossing the x-axis at $x = 5$, we know the point of intersection must be greater than 5.

Sketch what you see in the graph window on your paper.

Pause to allow students to work.

In the blank for Intersection, write down "greater than $x = 5$."

Pause for students to write.

We may not be able to see the intersection point in the graph window, but because we know approximately where to look, we can use the tables to find the solution.

How do we access the table for the equation? *(press 2nd, GRAPH)*

Because we know that the solution happens at an x-value greater than 5, we can scroll through the table until we find our solution.

How will you know when you have found the solution? *(Y_1 and Y_2 will be equal)*

Scroll through the table now until you find the solution to the equation.

Pause for students to find the solution on their calculator. Write and have students write the values from the calculator in the table on the *Demonstration Practice Sheet* that show the solution to the equation.

In what row did you find the solution? *(the row for x = 10)*

What is the solution to the equation? *(b = 10)*

Copy the rows of values into the table on your paper to completely fill in the table. Make sure that the solution is listed in the table of values.

Pause for students to write.

This agrees with our solution from solving algebraically. Why couldn't we see it when we graphed? *(because the window was too small, it was off of the graph window)*

There is a way to adjust your window settings, but that just adds to the number of steps that would be required to use the graphing method to solve.

Before making any final decisions, use substitution to check your solution of $x = 10$.

Pause to allow students to check the solution using substitution.

Because $10 + 9 = 19$ and $4(10) - 21 = 19$, we know our solution is correct. The 19s are again mirrored in both the tables and the equation.

Variables on both sides of the equation add at least one step of work to the algebraic method. This means that the algebraic method requires 3 steps and solving with tables also requires 3 steps. Which of the 2 do you prefer? Think about which will be the most direct.

Pause for students to think.

> **What are your impressions about which method is preferable?** *(answers will vary; if students are comfortable with algebraic manipulation, solving is most efficient; if not, it's easier to look it up in the table; since both methods have the same number of steps, it is up to the individual)*
>
> **Because some of you are more comfortable with solving algebraically, that may continue to be your preferred method. Those of you who are not comfortable with algebraic solving will prefer to use tables when there are variables on both sides, or graphs when appropriate.**
>
> **Checking the graph first will help when you look through your table for the solution. Getting an estimated point of intersection first will also indicate whether you could simply use the intersection method or if it's better to look through the tables.**
>
> **Write yourself a reminder here about what you believe to be the best course of action when dealing with equations that have variable on both sides. Remember, you choose which methods make the most sense to you to solve.**

Pause for students to work. Write and have students write under "When to Use," in the graph section of the foldable, "when variables are on both sides and I can see the lines intersect in the graph."

> **We need to update our foldable. In the graphs section, under "When to use," we can write, "when variables are on both sides and I can see the lines intersect in the graph."**
>
> **Write in your foldable while I write in mine.**

Pause for students to write. Write and have students write under "When to use" in the table section of the foldable, "when variables are on both sides and I have an estimated x-value to start looking up values."

> **In the table section, under "When to use," we can write "when variables are on both sides and I have an estimated *x*-value to start looking up values."**
>
> **Write in your foldable while I write in mine.**

Pause for students to do the same.

4. Guide students through problem 3 on the *Demonstration Practice Sheet*.

   Remind students to use the previous examples for guidance. The prompts in the questions will also be helpful. Use guiding questions to elicit verbal responses from students to check for their understanding.

   - What method do you think will be the most direct?
   - Where is the solution we are looking for in the graph of the 2 lines?
   - Where is our solution in the table? How do you know?
   - Do we use the x-value or the y-value?
   - How can you verify that your solution is correct?
   - When typing expressions into the calculator, what variable will you use (regardless of the letter used in the problem)?

5. Provide a summary of the lesson content.

   Review key ideas of the lesson by asking questions about solving equations and the best method reasoning.

   > **What does it mean to find a solution?** *(find a value or values of the variable that make a true statement)*

Where can we find a solution to an equation when we graph each expression? *(x-value(s) of the point(s) of intersection of the graphs is the solution)*

How can we find a solution to an equation in the table? *(the matching y-values in the table indicate that the corresponding x-value is the solution)*

How can you determine which method is most appropriate? *(by looking at the equation – if 1 side has a variable, algebraic;,if 2 sides have variables, calculator or algebraic)*

Practice

Guided Practice

1. As a class, work through the questions on the *Practice Sheet*. Discuss as a class the best method to solve each equation and write the reasoning.
2. Elicit verbal responses with guiding questions similar to those in the Demonstration section.
    - Looking at this equation, what do you think is the best method to solve the equation?
    - How do you know?
    - Why would you use the algebraic method for this equation?
    - Why would you use the graphing method for this equation?
    - Why would you use the table method for this equation?

Pair Practice

1. Have students work with a partner to complete the Pair Practice problems.

2. Have student pairs use the Guided Practice sections results to guide the problem-solving process.

3. Student pairs will discuss their analysis and solutions using the specified methods, then share their results with the class. Encourage students to use mathematical language from the lesson.

Independent Practice

1. Have students work independently to complete the *Independent Practice Sheet*.

2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- What does it mean to find a solution?
- Where can we find a solution to an equation when we graph each expression?
- How can we find a solution to an equation when we use each expression in tables?
- How can you determine which method is most appropriate?