ESTAR INTERVENTION

Tier 2 Mathematics Intervention

Module: *Fraction Models (FM)*

Teacher Lesson Booklet

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

The Meadows Center

FOR PREVENTING EDUCATIONAL RISK

THE UNIVERSITY OF TEXAS AT AUSTIN

COLLEGE OF EDUCATION

Mathematics Institute for Learning Disabilities and Difficulties

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 1

Equal Sharing and Unit Fractions: Halves, Fourths, and Eighths

Lesson Objectives	• The student will partition sets of objects into fractional equal shares. • The student will apply the term "equal share" to the partitioning of quantities into equal parts and use fraction names (e.g., one-fourth) to describe resulting fraction quantities.	
Vocabulary	**equal share:** the breaking of quantities apart so that everyone gets the same amount; the resulting amount of the action of equal sharing **one-fourth (or fourths):** the equal share received when a whole is shared among 4 people **one-eighth (or eighths):** the equal share received when a whole is shared among 8 people	
Reviewed Vocabulary	one-half	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 1 – 10) • Whiteboard with marker • 10 rectangles or other counters • Fraction bars: 1 whole, 1 set of $\frac{1}{2}$ • 2 small paper plates	• Student Booklet (pp. 1 – 5) • Fraction bars: 1 whole, 1 set of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ (1 set per student) • Talk and Share Cards: halves, fourths, and eighths

Preview

Say: **Today we will use what we know about *equal sharing* to create and name fractional amounts.**

Engage Prior/Informal Knowledge — Time: 3 min

Explore equal sharing with non-fractional shares.

Place 10 objects and 2 plates in front of students.

Say: **Imagine these rectangles are carrot sticks. Juanita wants to share these carrot sticks with a friend so that each of them gets the same amount. How would you suggest she divide them?**

Have student discuss how to divide the group of rectangles.

Say: **We can give each person 1 carrot stick until there are no more to share and each person has the same amount. Let's try it.**

Have students take turns putting the rectangles on the plates.

Say: **How many carrot sticks did each person receive?** *(5 carrot sticks)*

Each person received 5 carrot sticks. When we distribute a quantity so that each person gets the same amount, we call it an *equal share*.

Did each person get the same amount? *(yes)* **Then each person got an *equal share* of the carrot sticks.**

Modeled Practice Time: 8 min

1. Model the concept of equal shares – 2 friends share 1 chocolate bar.

   Have students turn to ***Modeled Practice Sheet #1***. Give each student 1 whole fraction bar. Display 2 plates or draw circles to represent the plates and 1 whole fraction bar.

Say: **Let's read the first problem together. Ready, read: "Jasai and Markesha are sharing 1 chocolate bar. If they share the chocolate bar equally, how much will each of them receive?"**

Remember that an *equal share* is the amount each person receives when we share an amount equally. Earlier, we divided 10 carrot sticks equally between 2 people. This time, we have 2 people and only 1 chocolate bar. What should we do?

Give students a chance to speculate about how to share the chocolate bar.

Say: **This whole fraction bar represents 1 chocolate bar.**

How many friends are sharing the chocolate bar? *(2)* **I will draw 2 circles and on your sheet Jasai and Markesha are holding plates. My 2 circles represent the friends.**

We need to equally share the chocolate bar between 2 friends. What does this mean? *(you have to give the same amount of chocolate to each child)*

What can you use to represent the chocolate bar? *(the whole fraction bar)*

How can you *equally share* the 1 chocolate bar between 2 friends? *(divide it in half)*

To *equally share* the chocolate bar, you need to divide it equally in half. Trade your whole fraction bar for 2 halves.

Trade in the whole fraction bar while students do the same. Place a $\frac{1}{2}$ part on each plate on the ***Modeled Practice Sheet #1*** while students do the same.

Say: **What do you do with the 2 halves?** *(give one to each of the children)*

Place each share on the plate to represent the amount each friend will receive.

How much is the *equal share*? *(one-half)*

When 1 chocolate bar is shared between 2 friends the *equal share* is one-half of a chocolate bar. Write "one-half of a chocolate bar" on the "*Equal Share*" line.

2. Model the concept of equal shares – 4 friends share 1 chocolate bar.

Have students turn to ***Modeled Practice Sheet #2***. Use the whiteboard to draw the fraction bar.

Say: **2 friends join Jasai and Markesha.**

Let's read the problem together. Ready, read: "4 friends want to share 1 chocolate bar so that each friend receives the same amount. What is the *equal share* that each friend will receive?

How many chocolate bars? *(1)*

How many friends are sharing? *(4)*

We need to *equally share* the chocolate bar among 4 friends. What does this mean? *(you have to give the same amount of chocolate to each child)*

I will draw a picture while you use fraction bars and the pictures of the friends to equally share the chocolate bar. What can you use to represent the chocolate bar? *(the whole fraction bar)*

Draw a rectangle on the whiteboard. Draw 4 circles below the rectangle on the whiteboard to represent the plates.

Say: **How can you *equally share* 1 chocolate bar among 4 friends?** *(divide it into 4 parts)*

Is it possible to break a chocolate bar into 4 parts? *(yes)*

To equally share the chocolate bar you need to divide it equally into 4 parts. When you divide a whole into 4 equal parts, each part is called *one-fourth*. What do you call 1 part of a whole divided into 4 equal parts? *(one-fourth)*

Trade your whole fraction bar for 4 *fourths*.

Divide the rectangle on the whiteboard into 4 equal parts.

Say: **What do you do with the 4 *fourths*?** *(give one to each of the friends)*

Place each share on the plate to represent the amount each friend will receive.

Draw a line from each part to one of the plates to represent the sharing process.

Say: **How much is the *equal share*?** *(one-fourth)*

The *equal share* is *one-fourth* of a chocolate bar. Write it on the "*Equal Share*" line.

Write "one-fourth of a chocolate bar" on the whiteboard.

Say: **When 4 friends share 1 chocolate bar each friend receives *one-fourth* of a chocolate bar.**

What would happen to the pieces of the chocolate bar if 4 more friends wanted to share? How do you think you would divide the chocolate bar to share among 8 friends?

When would you get more of the chocolate bar, when you share among 4 people or among 8 people? Why? *(allow for a variety of answers)*

Have students discuss how to divide the chocolate bar to share with 8 friends.

Say: **If we shared the chocolate bar among 8 friends, how would we divide it?** *(into 8 equal parts)*

If an *equal share* with 4 friends is called *one-fourth*, what do you think an *equal share* among 8 friends is called? *(answers may vary; one-eighth)*

An *equal share* when 1 whole is shared among 8 friends is called *one-eighth*. *One-eighth* is 1 part of a whole divided equally into 8 parts.

Have students model sharing with 8 people in the first problem on the *Practice Sheet*.

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 3. Students will work with a math partner and use fraction bars to complete the activity.

Say: **Solve the first problem with your math partner. After we discuss the answer, you will work on your own to solve the second problem.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How do you divide the whole rectangle equally? How many parts are there?
- How many parts does each friend get?
- What is an equal part called?

- What is the equal share?

Activity 2: In pairs, each student draws a ***Talk and Share Card: halves, fourths, eighths*** that describes a sharing scenario. Students will use fraction bars to solve each sharing scenario. Students can use circles to represent the number of friends on a piece of paper or a whiteboard. Students should verbalize their reasoning as they physically share to model the problem solutions. Observe and listen as a means of formative assessment.

Ask questions such as:

- How many friends are sharing?
- How will you divide the fraction bar?
- What is the *equal share*?

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 2

Equal Sharing and Unit Fractions: Thirds and Sixths

Lesson Objectives	• The student will partition sets of objects into fractional equal shares. • The student will apply the term "equal share" to the partitioning of quantities into equal parts and use fraction names (e.g., one-sixth) to describe the resulting fractional quantities.	
Vocabulary	**one-third:** the equal share received when a whole is shared among 3 people **one-sixth:** the equal share received when a whole is shared among 6 people	
Reviewed Vocabulary	equal share, one-eighth, one-fourth, one-half	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 11 – 28	• Student Booklet (pp. 6 – 12) • Fraction bars: 1 whole, 1 set of $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ (1 set per student) • Talk and Share Cards*:* thirds and sixths

Preview

Say: **Today we will find equal shares when sharing among 3 and 6 people. We will name the equal shares we create.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** with fraction bars to practice sharing among 4 people.

Ask questions such as:

- How many children are sharing? *(4)*
- How do you divide the whole? *(into 4 equal parts)* How do you know? *(have to share among 4 children)*
- What is the equal share called? *(one-fourth)*

Ask follow-up questions such as:

- If 2 people are equally sharing 1 sandwich, what is the equal share called? *(one-half)*
- If 8 people are equally sharing 1 sandwich, what is the equal share called? *(one-eighth)*
- What happens to each size of the share as more children join the party? *(smaller parts, less share of the sandwich)*

Modeled Practice — Time: 8 min

1. Model the concept of equal shares – 3 friends share 1 sandwich.

   Use the ***Modeled Practice Display #1***. Have students turn to ***Modeled Practice Sheet #1***. Use fraction bars to help solve.

Say: **Read the question. Ready, read: "On a field trip, 3 friends shared 1 sandwich equally. How much of the sandwich did each friend eat?"**

How many sandwiches are being shared? *(1)*

How many friends are sharing the sandwich? *(3)*

We can use the fraction bars to help understand the problem. What tool can we use to show the whole or the 1 sandwich? *(the whole fraction bar)* **What can represent the friends sharing?** *(pictures of friends holding plates or circles)*

We will use our model to show the equal sharing.

Previously, we divided a whole into halves, fourths, and eighths to share equally. Can you predict how we can share this 1 sandwich among 3 friends so each friend gets an equal amount? *(cut the sandwich into 3 equal parts)*

When we share something equally between 3 friends, each of those shares, or parts, is called *one-third*. What do we call each share of the sandwich? *(one-third)*

Trade your whole sandwich or fraction bar for 3 thirds. I will divide the rectangle on the display into 3 equal parts.

Divide the rectangle into 3 equal parts. Draw a line from each part to a plate representing the equal share.

Say: **Share the thirds from the whole sandwich to each plate or friend. How much does each friend receive?** *(one-third)*

Does each person have an equal share? *(yes)* **How do you know?** *(each person has the same amount)*

How much is each equal share? *(one-third of a sandwich)* **Write "*one-third* of a sandwich" on the "Equal Share" line.**

2. Model the concept of equal shares – 6 friends share 1 sandwich.

Use ***Modeled Practice Display #2***. Have students turn to ***Modeled Practice Sheet #2***.

Say: **What if 3 more friends wanted to share 1 sandwich? How much of the sandwich does each friend receive?**

How many sandwiches are being shared? *(1)*

How many friends are sharing the sandwich now? *(6)*

Show me how many fraction bars and friends with plates are needed to represent this problem. *(1 fraction bar and 6 plates)* **How do you know?** *(1 sandwich and 6 friends)*

How we can share this sandwich among 6 friends so each friend gets an equal amount? *(cut the sandwich into 6 equal parts)*

When we share something equally between 3 friends, each of those shares, or parts, is called *one-third*. What do you think it is called when we share something equally between 6 friends? *(one-sixth)*

The name for 1 part of a whole divided into 6 equal parts is *one-sixth*.

Trade your whole sandwich for 6 sixths, and I will divide the rectangle on the display into 6 equal parts.

Divide the rectangle into 6 equal parts. Draw a line from each part to a plate representing the equal share.

Say: **Distribute the *one-sixth* parts to each plate. How much does each friend receive?** *(one-sixth)*

Does each person have an equal share? *(yes)* **How do you know?** *(each person has the same amount)*

How much is each equal share? *(one-sixth of a sandwich)*

Write "one-sixth of a sandwich" on the "Equal Share" line.

Say: **When 6 friends share 1 sandwich equally, each friend receives *one-sixth* of the sandwich.**

When 3 friends shared 1 sandwich equally, each friend received *one-third* of the sandwich. When would you get more, or a bigger part, of the sandwich? *(when you share with 3 friends)*

Teacher Note
If students have difficulty with the answer, have them compare a one-third part to a one-sixth part using fraction bars.

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 9. Students will work with a math partner and use fraction bars to complete the activity.

Say: Solve the first problem with a math partner. After we discuss the answer, you will work on your own to solve the rest of the problems.

Ask questions such as:

- How do you divide the rectangle equally? How many parts are there?
- How many parts does each monkey get?
- What is an equal part called?
- What is the equal share?

Activity 2: In pairs, each student will draw a ***Talk and Share Cards: thirds and sixths*** **card** that describes a sharing scenario. Students will use fraction bars to solve each sharing scenario. Students can draw circles to represent the number of friends on a whiteboard or piece of paper. Students should verbalize their reasoning as they physically share to model the problem solutions. Observe and listen as a means of formative assessment.

Ask the same questions as above.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 3

Unit Fractions: Area Model

Lesson Objectives	• The student will name unit fractions in equal sharing situations. • The student will create area models that represent unit fractional shares. • The student will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.	
Vocabulary	**unit fraction:** a fraction that is 1 part of a whole divided into equal parts	
Reviewed Vocabulary	area models, equal share, one-eighth, one-fourth, one-half, one-third, one-sixth	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 29 – 38) • Whiteboard with marker • Fraction Words Mat • Fraction bars: 1 whole, 1 set of $\frac{1}{3}, \frac{1}{8}$	• Student Booklet (pp. 13 – 17) • Whiteboard with marker (1 per student) • Fraction bars: 1 whole, 1 set of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ (1 set per student) • Different colored markers or pencils (2 per student) • Talk and Share Cards: area models (2 per pair of students)

Preview

Say: **Today we will draw area models that show equal shares as *unit fractions*.**

Engage Prior/Informal Knowledge — Time: 3 min

Draw a large rectangle on the whiteboard. Divide the rectangle into 4 unequal parts.

Say: **I divided the rectangle into 4 parts. Do you think these parts are equal?** *(no)* **Why?** *(the parts are different sizes)*

Erase the divisions inside the rectangle.

Say: **How can I cut the rectangle into [2, 3, 4, 6, 8] equal parts?** *(answers will vary, ensure students recognize that the quantities produced by the divisions must show equal amounts)*

Give each student a whiteboard and have students divide rectangles in halves, thirds, fourths, sixths, and eighths with a marker. Show students how to divide the rectangle first in thirds, and then each third in half, to get more accurate divisions.

Modeled Practice — Time: 8 min

1. Model the concept of equal shares – 3 friends share 1 sandwich.

Have students turn to *Modeled Practice Sheet #1*.

Say: **Let's read the problem together. Ready, read: "3 friends share 1 sandwich equally. How much of a sandwich does each friend get?"**

How many sandwiches do we have? *(1)*

How many friends are sharing the sandwich? *(3)*

We will draw a picture to help us understand the problem. What does the rectangle represent? *(the sandwich being shared)*

Teacher Note
Students may need to be reminded that since the model is hand drawn, it will not be perfect. The students should assume all the parts are equal unless otherwise specified.

What do the 3 plates represent? *(the 3 friends sharing the sandwich)*

What can we do with the sandwich so we can share it equally between the 3 friends? *(divide it equally into 3 parts)* **Divide the rectangle representing the sandwich into 3 equal parts.**

Assist students in doing the same.

Say: **What do we do with the 3 equal parts of the sandwich?** *(give 1 to each friend)* **What do we call each share, or part, of a sandwich when we share it equally between 3 friends?** *(one-third)* **Draw a line from each part of the sandwich to a plate to show the sandwich being shared.**

Does each person have an equal share? *(yes)* **How do you know?** *(each person has the same amount)*

Look at the rectangle next to the words "Equal Share." This rectangle is going to represent the part of the sandwich that each friend has on a plate. How much of a sandwich does 1 friend receive? *(one-third of a sandwich)*

How can you show how much of the sandwich 1 friend receives? *(divide the rectangle into thirds and shade 1 part of it)* **Do it.**

What is the equal share when 3 friends share 1 sandwich? *(one-third of a sandwich)* **So each friend gets to eat one-third of the sandwich. Write "one-third" on the "Equal Share" line.**

Another way to write one-third using numbers is like this: "or $\frac{1}{3}$." Write it on the "Equal Share" line.

This means the equal share is 1 part out of 3 equal parts of the whole sandwich. When you read it, you say "one-third," just like when reading the words. One third is a *unit fraction* because it is 1 part of a whole divided into equal parts. In this case, it is 1 part of 3 parts.

What is a *unit fraction*? *(a fraction that is 1 part of a whole divided into equal parts)*

What is the name of the whole? What is the thing we are sharing? *(a sandwich)* **Write "of a sandwich" on the "Equal Share" line.**

What is the equal share when 3 friends share 1 sandwich equally? *(one-third of a sandwich)*

2. Model the concept of equal shares – 2 friends share 1 pizza.

Have students turn to *Modeled Practice Sheet #2*.

Say: **Let's read the problem together. Ready, read: "2 friends share 1 pizza. If the pizza is equally shared, how much does each friend receive?"**

How many pizzas do we have? *(1)*

How many friends are sharing the pizza? *(2)*

In previous lessons, we have shared things that are shaped like a rectangle. Since most pizzas are round, we will use a circle to represent the whole pizza.

We will draw a picture to help us understand the problem. What does the circle represent? *(the pizza being shared)*

What do the 2 plates represent? *(the 2 friends sharing the pizza)*

What can we do with the pizza so we can share it equally with 2 friends? *(divide it equally into 2 parts)* **Trace the line that breaks the pizza into 2 equal parts.**

What do we do with the 2 equal parts of pizza? *(give 1 to each friend)* **What do we call each share, or part, of the pizza when we share it equally between 2 friends?** *(one-half)* **Draw a line from each part of the pizza to a plate to show the pizza being shared.**

Does each person have an equal share? *(yes)* **How do you know?** *(each person has the same amount)*

Is one-half a *unit fraction*? *(yes)* **How do you know?** *(because it is 1 part out of a whole divided into 2 equal parts)*

One-half is a *unit fraction* because it represents 1 part of 2 equal parts in a whole.

Look at the pizza by the equal share line. Shade the equal share each person receives, or how much pizza 1 friend will eat. How much does 1 friend receive? *(one-half of a pizza)*

What is the equal share when 2 friends share 1 pizza? *(one-half of a pizza)* **Write "one-half" on the "Equal Share" line.**

Another way to write one-half using numbers is like this: "or $\frac{1}{2}$." Write it on the "Equal Share" line.

This means the equal share is 1 part out of 2 equal parts in the whole pizza. When you read it, you say "one-half."

What is the whole? What is the thing we are sharing? *(a pizza)* **Write "of a pizza" on the "Equal Share" line.**

Teacher Note
If students struggle to write out fraction words, have them refer to the Fraction Words Mat for assistance.

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 15. Students will work with a math partner to complete the activity.

Say: Work with a partner to solve the first problem. After we discuss the different models, you will work on your own to solve the remaining problems on the practice sheet.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How do you divide the circle equally? How many parts are there?
- How many parts does each student get?
- What is an equal part called?
- What is the equal share?

Activity 2: In pairs, students will practice drawing models to represent fractional shares. Each student will draw a ***Talk and Share Cards: Area models card*** that shows a sharing scenario with the equal share provided. Students will draw a model on a whiteboard or piece of paper to show the equal share. When they finish, students trade drawings and write what fractional share is shown in words (one-third) and as a fraction ($\frac{1}{3}$).

Ask questions such as:

- How many equal parts are shown in your model?
- How many parts are shaded?
- What is the equal share?

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 4

Non-unit Fractions

Lesson Objectives	• The student will partition sets of objects into fractional equal shares. • The student will use fraction names (e.g., two-thirds) to describe resulting non-unit fractional quantities and identify the number sentence that describes the fraction (e.g., $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$). • The student will organize and verbalize steps in solving to explain mathematical thinking through communication with peers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	equal share, one-eighth, one-fourth, one-sixth, one-third, unit fraction	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 39 – 52) • Fraction bars: 3 wholes, 3 sets of $\frac{1}{6}$ • 6 small paper plates	• Student Booklet (pp. 18 – 24) • Fraction bars: 3 wholes, 3 sets of $\frac{1}{6}$, 2 sets of $\frac{1}{4}$, 3 sets of $\frac{1}{2}$ (per student) • 6 small paper plates • Talk and Share Cards: non-unit fractions

Preview

Say: **Today we will equally share more than one food item among friends. These equal shares will not be unit fractions.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice finding an equal share resulting in a unit fractional part.

Ask questions such as:

- How many equal parts in the whole? *(4)*
- How do you know when each person has an equal share? *(each person gets the same amount)*
- What is the equal share called? *(one-fourth)*

Ask students to recall the names of other equal shares (8 people share 1 sandwich, 6 people share 1 pizza, etc.)

Modeled Practice — Time: 8 min

1. Model the concept of equal shares – 6 share 2 sandwiches.

   Use the ***Modeled Practice Display #1***. Use fraction bars to solve.

Say: **Read the problem with me. Ready? Read: 6 workers share 2 sandwiches equally. How much of a sandwich does each worker receive?**

How many sandwiches are being shared? *(2)*

How many workers are sharing the sandwiches? *(6)*

We can use fraction bars to help us understand the problem. Tell me how to use fraction bars and plates to show the sandwiches and the workers in this problem.

Invite a student to explain. If students have trouble, suggest they use 2 whole fraction bars to represent the sandwiches and the 6 plates for the workers.

Say: **What do the whole fraction bars represent?** *(the 2 sandwiches)* **What do the 6 plates represent?** *(the workers sharing the sandwiches)* **How can we use this model for equal sharing?** *(divide the sandwiches into six pieces, then give one from each sandwich to each worker)*

When we share a sandwich equally among 6 workers, what is each equal piece called? *(one-sixth)* **What kind of fraction is one-sixth?** *(a unit fraction)* **Why?** *(because it is 1 part out of a whole divided into equal parts)*

Each worker will get a one-sixth piece from the first sandwich. Trade your first fraction bar for 6 sixths.

Exchange one whole for 6 sixths.

Say: **Distribute the sixths to each worker.**

Distribute the sixths, 1 piece per plate.

Say: **So far, how much of a sandwich has each worker received?** *(one-sixth)*

Say: **How do we share the second sandwich?** *(divide it into sixths, then give each worker a sixth)*

Trade in your second whole fraction bar and distribute the sixths to the plates representing the workers.

Give students a chance to share the third sandwich on their own. Have students remove the pieces from 1 plate while you do the same, then count the pieces together.

Say: **How much is each equal share? Let's count: one-sixth, two-sixths. What is the equal share?** *(two-sixths of a sandwich)***Write "two-sixths" on the Equal Share line.**

Another way to write two-sixths using numbers is like this: "or $\frac{2}{6}$." Write using numbers on the "Equal Share" line.

What is the whole, or label? What did we share with the workers? *(a sandwich)* **Write "of a sandwich" on the "Equal Share" line.**

What is the equal share when 6 workers share 2 sandwiches equally? *(two-sixths of a sandwich)*

2. Model the concept of equal shares – 4 share 2 candy bars.

Use the ***Modeled Practice Sheet #2***.

Say: **Read the problem. Ready, read: "4 friends share 2 candy bars equally. How much does each friend receive?"**

How many candy bars are being shared? *(2)*

How many friends are sharing the candy bars? *(4)*

We will use a picture to help us understand the problem. You will share the picture of the whole, or the rectangle to represent the sharing.

What does the rectangles represent? *(the 2 candy bars)* **How can we use the model for equal sharing?** *(divide rectangle into fourths, then give them to each friend)*

Divide each rectangle equally into fourths. When we share a candy bar equally among 4 friends, what is each equal piece called? *(one-fourth)*

Each friend will get a one-fourth piece from the first candy bar.

Draw a line from each the first candy bar to the plates.

After we shared the first candy bar, how much did each friend receive? *(one-fourth)*

How do we share the second candy bar? *(divide it into fourths, then give each friend a fourth)*

Divide the second rectangle, or candy bar into four eqaul pieces. Draw a line from each equal fourth to the 4 plates.

How much is the equal share? Let's count: one-fourth, two-fourths. What is the equal share? *(two-fourths of a candy bar)* **Write "two-fourths" on the "Equal Share" line.**

Another way to write three-fourths using numbers is like this: "or $\frac{2}{4}$. Write it on the "Equal Share" line.

This means the equal share is 2 pieces out of 4 equal pieces in a whole candy bar. When you read it, you say "two-fourths" just like when reading the words.

What is the whole, or label? *(a candy bar)* **Write "of a candy bar" on the "Equal Share" line.**

What is the equal share when 4 friends share 2 candy bars equally? *(two-fourths of a candy bar)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 21.

Say: **Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How many are being shared? How many are sharing?
- How should the whole be divided?

- How much does each person receive from each whole? What is the part called?
- What is the equal share?

Activity 2: In pairs, each student draws a ***Talk and Share Cards: non-unit fraction card*** that shows a sharing scenario. Instead of modeling the situation with fraction bars, students will verbalize the steps for solving the problem to their partner, who cannot see the card. The partner will use the fraction bars to solve the problem that is being read. Then students switch roles and solve the next problem.

Students will need to pool their fraction bars to model some of the problems.

Ask questions such as:

- How many wholes does your partner need?
- How should they trade in the wholes?
- How much does each person get from each whole?
- (to the partner) What is the equal share?

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 5

Non-unit Fractions with Number Sentences

Lesson Objectives	• The student will partition sets of objects into fractional equal shares. • The student will apply the term "equal share" to the partitioning of quantities into equal parts and use fraction names (e.g., two-thirds) to describe resulting non-unit fractional quantities.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	equal share, one-eighth, one-fourth, one-sixth, one-third	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 53 – 64) • 3 different colored pencils or markers	• Student Booklet (pp. 25 – 30) • 3 different colored pencils or markers (per student) • Matching Cards: non-unit fraction (1 set per pair)

Preview

Say: **Today we will equally share quantities that give each person a fractional amount.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice finding non-unit fractional parts through equal sharing.

Ask questions such as:

- How many objects are being shared? *(2)*
- How many people are sharing? *(4)*
- How much of a share does a person get from the first object? The second? *(one-fourth, one-fourth)*
- What is the equal share? *(two-fourths of a cupcake)*

Modeled Practice — Time: 8 min

1. Model the concept of equal shares – 3 share 2 brownie bars.

   Have students turn to ***Modeled Practice Sheet #1***. Complete each step on the sheet along with the students.

Say: **Read the problem with me. Ready, read: "3 friends share 2 brownie bars equally. How much does each friend receive?"**

How many brownie bars are being shared? *(2)*

How many friends are sharing the brownie bars? *(3)*

We can use a picture to help us understand the problem.

Teacher Note
If students have difficulty using pictures to model the sharing situations, use fraction bars to make the connection.

Each rectangle represents a brownie. We will shade each brownie a different color to help us keep track of how we shared.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **How should we divide the first brownie so we can share it equally?** *(into thirds, then give them to each friend)* **Why into thirds?** *(sharing with 3 friends)* **Divide the first rectangle into 3 equal parts.**

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **When we share a brownie bar equally between 3 friends, what is each equal part called?** *(one-third)*

Draw a fraction bar for each friend to represent the amount each friend will receive. We know each friend will get at least one-third of a brownie. Divide each friend's fraction bar into thirds.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **Each friend will get one-third from the first brownie. To show how the first brownie is shared, draw a line using the colored pencil from each brownie bar part to each friend's fraction bar.**

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **So far, how much of a brownie bar has each friend received?** *(one-third)*

Shade the first third of each friend's fraction bar to represent the first shared brownie bar part.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **How do we share the second brownie bar?** *(divide it into thirds, then give each friend a third)*

Give students a chance to share the second brownie on their own, drawing the lines from the second rectangle to each friend's fraction bar in the coordinating color. Then shade another third of each friend's fraction bar.

Say: **How much is the equal share? Let's count: one-third, two-thirds. What is the equal share?** *(two-thirds of a brownie)*

We can write a number sentence to show the parts of the equal share. How much of the first brownie bar did each friend receive? *(one-third)*

Write "$\frac{1}{3}$" on the "Equal Share" line.

Say: **How much of the second brownie bar did each friend receive?** *(one-third)*

Write "$+ \frac{1}{3}$" on the "Equal Share" line.

Say: **And what is the total equal share?** *(two-thirds)*

Write "$= \frac{2}{3}$" on the "Equal Share" line.

Say: **One-third plus one-third equals two-thirds, which is the equal share. We can also write the words to show the equal share.**

Write "or two-thirds" on the "Equal Share" line.

Say: **What is the whole, or label?** *(a brownie bar)*

Write "of a brownie bar" on the "Equal Share" line.

Say: **What is the equal share when 3 friends share 2 brownie bars equally?** *(two-thirds of a brownie)*

2. Model the concept of equal shares – 4 share 3 taffy squares.

Use ***Modeled Practice Sheet #2***. Complete each step on the sheet along with the students.

Say: **Read the problem with me. Ready? Read: 4 friends share 3 taffy squares equally. How much does each friend receive?**

How many taffy squares are being shared? *(3)*

How many friends are sharing the taffy squares? *(4)*

We can use a picture to help us understand the problem. In the past, we have shared things that are shaped like a rectangle and a circle. Since we are sharing taffy *squares*, we will use squares to represent them.

Each square represents a taffy square. Shade each of the taffy squares a different color.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **How can we use the model for equal sharing?** *(divide squares into fourths, then give them to each friend)*

When we share a taffy square equally between 4 friends, what is each equal part called? *(one-fourth)*

Divide each taffy squares into fourths.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **Draw fraction bars for each friend to represent the amount each friend will receive. We know each friend will get at least**

one-fourth of a taffy square. Divide each friend's fraction bar into fourths.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **Each friend will get a one-fourth part from the first taffy square.**

Draw lines from the first taffy square to each friends' plate. Then shade.

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **So far, how much of a taffy square has each friend received?** *(one-fourth)*

How do we share the second taffy square? *(divide it into fourths, then give each friend a fourth)*

Wait 3-5 seconds for students to work. Complete the same step on your sheet.

Say: **Draw lines to share the fourths of the second taffy square. Then shade in each friend's fraction bar with the second fourth.**

Give students a chance to share the second taffy square on their own, drawing the lines from the second square to the plates in the coordinating color.

Say: **Draw lines to share the third taffy square. Shade the amount each friend receives.**

Give students a chance to draw lines and shade the equal share.

Say: **How much is the equal share? Let's count: one-fourth, two-fourths, three-fourths. What is the equal share?** *(three-fourths of a taffy square)*

We can write a number sentence to show the parts of the equal share. How much of the first taffy square did each friend

receive? *(one-fourth)* **How much of the second taffy square?** *(one-fourth)* **How much of the third taffy square?** *(one-fourth)*

Write "$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$" on the "Equal Share" line.

Say: **And what is the total equal share?** *(three-fourths)*

Write "$= \frac{3}{4}$" on the "Equal Share" line.

Say: **Read the number sentence together: one-fourth plus one-fourth plus one-fourth equals three-fourths. We can also write the words to show the equal share.**

Write "or three-fourths" on the "Equal Share" line.

Say: **What is the whole, or label?** *(a taffy square)*

Write "of a taffy square" on the "Equal Share" line.

Say: **What is the equal share when 4 friends share 3 taffy squares equally?** *(three-fourths of a taffy square)*

Practice **Time: 8 min**

Activity 1: Have students turn to the ***Practice Sheet*** on page 28.

Say: **Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How many objects are being shared? How many are sharing?
- What is an equal share from the first [second, third…] object?
- What is the number sentence to represent the equal share?

- What is the equal share?

Activity 2: Students play in pairs. Starting with all ***Non-unit Fraction Matching Cards*** face up, students work together to match equal shares written as a fraction to the equivalent number sentence.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 6

Fractional Parts of Set Models

Lesson Objectives	• The student will identify the whole represented by a set. • The student will identify fractional parts of a set using symbols. • Student will recall and recognize mathematical vocabulary to communicate fractional parts.	
Vocabulary	**set:** a group of objects	
Reviewed Vocabulary		
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 65 – 76) • Counting objects such as colored cubes: 6 orange and 6 blue • Colored pencils: orange and blue	• Student Booklet (pp. 31 – 36) • Counting objects such as colored cubes: 6 orange and 6 blue (12 per student) • Colored pencils: orange and blue (1 set per student) • *Describing a Set Matching* Cards (1 set per group)

Preview

Say: **Today we will name fractional parts of groups that contain multiple objects.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice identifying the shaded fractional part of an area model and shading an area model to represent a fraction.

Ask questions such as:

- How many parts are in the whole?
- How many parts are shaded?
- What is the fraction?
- How many parts should be shaded?

Modeled Practice — Time: 8 min

1. Identify the fractional part of a set – 3 orange and 5 blue cubes.

   Have students turn to the *Modeled Practice Sheet*.

Say: **In previous lessons, the whole has been a whole rectangle or fraction bar. In this lesson, we will describe amounts with fractions in which the whole is a *set* of objects.**

A *set* is a group of objects. Our math group is a *set*. We have [insert number of students] in our math group to make the whole group.

How many in our group have brown eyes?

The fractional part of our group that has brown eyes is [number of brown eyed students] out of [number of students in the group] or [state the fraction].

Display 8 cubes, 3 orange and 5 blue, on the table.

Say: **This is a *set*. How many cubes are in the *set*?** *(8)*

Since all 8 of the 8 are cubes, we can say that $\frac{8}{8}$ are cubes. They represent a whole.

Fill in the blanks associated with identifying the whole.

Say: **Look at the colors of the cubes. What do you see?** *(3 are orange and 5 are blue)*

Color your sheet to show the colors of the cubes.

Model how to quickly shade 3 cubes orange. Fill in the blanks that refer to the orange cubes.

Say: **How many of the 8 cubes are orange?** *(3 out of 8 are orange)*

Because 3 of the 8 cubes are orange, we can say that $\frac{3}{8}$ of the cubes are orange. What is the fraction to show the 3 orange cubes? ($\frac{3}{8}$) **The 8 represents the total number of cubes. The 3 represents the number of cubes that are orange.**

Model how to quickly shade 5 cubes blue.

Say: **How many of the 8 cubes are blue?** *(5 of the 8 cubes are blue)*

We can say that 5 out of 8, or $\frac{5}{8}$, of the cubes are blue.

Fill in the blanks that refer to the blue cubes.

Say: **What does the 8 in this fraction represent?** *(the total number of cubes)*

What does the 5 in this fraction represent? *(the number of blue cubes)*

For these fractions, what is the whole? *(8 cubes)*

2. Identify the fractional part of a set – 1 sun and 3 moons.

Say: **Look at the shapes on your sheet. How many shapes are in the *set*?** *(4 shapes)* **4 shapes represent the whole.**

What fraction can we use to say how many objects in the *set* are shapes? *($\frac{4}{4}$)*

Fill in the blanks associated with identifying the whole.

Say: **What shapes do you see in the *set*?** *(1 sun and 3 moons)*

How many of the 4 shapes are suns? *(1 out of 4 are suns)*

What fraction of the *set* are suns? *($\frac{1}{4}$)*

In this fraction, what does the 4 represent? *(the 4 total shapes in the set)*

What does the 1 represent? *(the number of shapes that are suns)*

Fill in the blanks that refer to the suns.

Say: **How many of the 4 shapes are moons?** *(3 of the 4 shapes are moons)*

What fraction of the *set* are moons? *($\frac{3}{4}$)*

Fill in the blanks that refer to the moons.

Say: **What does the 4 in this fraction represent?** *(the total number of shapes)*

What does the 3 in this fraction represent? *(the number of moons)*

For these fractions, what is the whole? *(4 shapes)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 33 and 34.

Say: Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How many [animals, shapes] make up the whole?
- How many of the [animals, shapes] are [chickens, pigs, etc.]?
- What fraction of the [animals, shapes] are [chickens, pigs, etc.]?
- Are there more [chickens, pigs, etc.]? Is that fraction greater than or less than the other fraction?

Activity 2: As a group, students will match fraction cards that describe a set of objects. Use the ***Describing a Set Matching Cards***. Multiple fractions will represent a given set. For example, if given a set of 3 apples and 5 oranges, the fractions $\frac{8}{8}$, $\frac{3}{8}$, and $\frac{5}{8}$ would be appropriate matches. When a student finds a match, he or she must identify the fractional relationship. For example:

"$\frac{8}{8}$ is a match: $\frac{8}{8}$ of the objects are fruits."

"$\frac{3}{8}$ is a match: $\frac{3}{8}$ of the pieces of fruit are apples."

Students must find all possible fractions before choosing another set card. Fraction cards may be reused.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 7

Fractions on the Number Line

Lesson Objectives	• The student will locate and name fractional points on the number line. • The student will create and use representations to record and communicate mathematical ideas to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Eighths, equal share, fourths, fraction, halves, number line, sixths, thirds	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 77 – 90) • Fraction bars: 3 wholes, 3 sets of $\frac{1}{4}$ • 2 different colored pencils or markers	• Student Booklet (pp. 37 – 43) • Fraction bars: 3 wholes, 3 sets of $\frac{1}{4}$ (1 set per student) • Different colored pencils or markers (2 per student) • Matching Cards: equal share (1 set per group)

Preview

Say: **Today we will practice placing fractions on the number line.**

Engage Prior/Informal Knowledge — Time: 3 min

Use the ***Engaged Practice Sheet*** as students practice equal sharing – 4 students share 3 sandwiches.

Say: **4 students want to share 3 sandwiches equally. We want to find out how much of a sandwich each student receives.**

How many wholes do we need? *(3)* **Why?** *(sharing 3 sandwiches)* **How many people are sharing?** *(4)* **How much is 1 part of 1 sandwich shared among 4 people equally?** *(one-fourth)*

How much of the first sandwich does each friend receive? *(one-fourth)* **Trade in the whole for 4 fourths and share.**

How much of the second sandwich? *(one-fourth)* **Trade and share.**

How much of the third sandwich? *(one-fourth)* **Trade and share.**

Allow students time to trade in the whole fraction bars for fourths and distribute the parts.

Say: **How much is an equal share? Let's count the parts: one-fourth, two-fourths, three-fourths. How much is each equal share?** *(three-fourths)*

Instead of the parts we can show each equal share using the area model. How many parts are you going to shade on 1 whole? *(shade 3 of the 4 parts)* **Shade the model.**

Three-fourths of a sandwich is an equal share.

Write "three-fourths, or $\frac{3}{4}$ of a sandwich" on the "Equal Share" line.

Modeled Practice — Time: 8 min

1. Model the placement of a fractional share on the number line using a fraction bar – 4 students share 3 sandwiches.

Continue on the ***Engaged Practice Sheet***.

Say: **What do you notice about the number line? What is the range?** *(from 0 to 1)*

What do you think the hash marks to the left of 1 represent? *(answers vary: numbers less than 1, fractions)* **How do you think you could figure out what numbers go there?** *(answers vary: measure with a fraction bar)*

Put a whole fraction bar along the top of the number line and have students do the same. Have them use ***Modeled Practice Sheet #1***.

Say: **Now we're going to use a number line to show a different way to represent three-fourths. Compare the number line to the whole fraction bar. What do you notice?** *(it is the same length)* **Notice that the length from 0 to 1 is the same length as a whole fraction bar.**

Think about when we used the fraction bar. How would you show one-fourth on the fraction bar? *(trade it for 4 parts, divide the whole into fourths)*

If the length from 0 to 1 represents the same whole, how could you show one-fourth on the number line? *(answers vary: put a one-fourth fraction bar part on the number line)*

Put a one-fourth part from the fourths fraction bars on the number line. Model writing the fractions and shading as the script is taught.

Say: **Take a one-fourth fraction bar part and put it on top of the number line. It is the length of the number line from 0 to the first hash mark. The length from 0 to the first hash mark is one-fourth. Write $\frac{1}{4}$ under the first hash mark to label the length.**

Now shade above the number line from 0 to this mark. The length represents one-fourth.

What is the fraction that shows no one has an equal share when sharing with 4 friends? *($\frac{0}{4}$)* **Write $\frac{0}{4}$ under the 0.**

I want to show two-fourths. Put another fourth part on top of the number line. The length from 0 to the second hash mark is $\frac{2}{4}$.

Where is two-fourths on the number line? *(at the hash mark after the second one-fourth part)*

Watch For 	**Students often think the fraction is a label for the hash mark, not for the distance from 0 *to* the hash mark.** **Emphasize that the fraction is represented by the shaded length from 0 or by the length of the fraction bar above the number line.**

Write $\frac{2}{4}$ under the second hash mark to show the length and shade from 0 to $\frac{2}{4}$ above the number line.

Shade the portion of the number line after $\frac{14}{}$ and label the hash mark "$\frac{2}{4}$."

Say: **How can we find the length representing three-fourths?** *(by going to the next hash mark)* **Place the third part of the fourths fraction bar above the number line.**

Have students put the third part from one of the fourths fraction bars on the number line.

Say: **What is the length of the fourth parts above the number line?** *($\frac{3}{4}$)* **Write $\frac{3}{4}$ under the third hash mark to show the length from**

0 to this part is $\frac{3}{4}$ and shade above the number line. Let's count the parts on the number line: one-fourth, two-fourths, three-fourths. The next hash mark is at 1 whole. What is the fourths fraction to show 1 whole? *($\frac{4}{4}$)* **Write $\frac{4}{4}$ under the 1.**

2. Model the placement of a fractional share on the number line using a fraction bar – 6 friends share 5 ribbons.

Have students turn to *Modeled Practice Sheet #2*.

Say: **6 friends share 5 ribbons equally. The equal share is shown. How much of a ribbon does each student receive?** *(five-sixths of a ribbon)*

Each friend receives five-sixths of a ribbon.

Write "five-sixths, or $\frac{5}{6}$ of a ribbon" on the "Equal Share" line.

Say: **The number line is drawn under the model of the equal share so that 0 is at one end of the fraction bar and 1 is at the other end.**

What do you notice about the hash marks on the number line? *(they line up with the division on the fraction bar)*

When we start at 0 on the number line, how many sixths can we count? *(0 sixths)* **When we are at 0, we can also label this hash mark as $\frac{0}{6}$.**

The first shaded part of the fraction represents what fraction? *(one-sixth)* **What does the length from 0 to the first hash mark represent?** *(one-sixth)*

What is the equal share we have to show on the number line? *($\frac{56}{}$)* **How many parts do we shade to show the length from 0 to $\frac{5}{6}$?** *(shade until the fifth hash mark)*

Shade to show $\frac{5}{6}$. Write the fractional part for each hash mark under the hash mark.

Give students time to shade the number line.

Label the hash marks as you count, from zero-sixths to six-sixths.

Say: **Let's count the length from 0: zero-sixths, one-sixth, … five-sixths.**

What is the label for the next hash mark? How many sixths are the same length as 1? *(six-sixths)*

3. Model the placement of a fractional share on the number line without a fraction bar – 8 friends share 5 licorice ropes.

Say: **8 friends share 5 licorice ropes. We need to locate the equal share, five-eighths of a licorice rope, on the number line.**

Into how many parts was each licorice rope divided? *(8 equal parts)* **Why?** *(sharing with 8 friends)* **Into how many parts is the number line divided?** *(8 equal parts)*

Watch For

Students may count the hash marks instead of the spaces *between* the hash marks, arriving at an answer of 7 or 9 parts.
If students struggle with recognizing the value of the space between each hash mark, have them lightly circle around the length from one hash mark to another as they count.

Say: **Before we shade the equal share, let's label the hash marks on the number line. How many eighths are at 0?** *($\frac{0}{8}$)* **How many eighths are from 0 to the first hash mark?** *($\frac{1}{8}$)* **How many eighths from 0 to the second hash mark?** *($\frac{2}{8}$)*

Label the hash marks through $\frac{8}{8}$ below the number line.

Say: **How many one-eighth sized parts are in each equal share?** *(5)*

Each student would receive five-eighths of a licorice rope. Since 5 parts make an equal share, we shade 5 one-eighth sized parts on the number line.

Shade above the number line, counting each part as you shade.

Say: **Ready, count: one-eighth, two-eighths, … five-eighths. Write $\frac{5}{8}$ below the hash mark to show the length of each equal share of licorice.**

Label "$\frac{5}{8}$" on the number line.

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 39 and 40.

Say: **Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- What is the equal share?
- What are the labels for the hash marks?
- What does the fraction below a hash mark represent?
- How do you know what length to shade on the number line?

Activity 2: As a group, students will use the ***Matching Cards: equal share*** to place into pairs: a sharing scenario and a fraction bar or number line, or a fraction bar and a number line. The group will discuss which cards go together.

Ask questions such as:

- How many people are sharing?
- How many equal parts is the whole divided into?
- How many equal parts does each person receive?
- How can you tell when a fraction bar and a number line represent the same equal share?

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 8

Fractions Equal to 1

Lesson Objectives	• The student will build fractions that are equal to 1 using fraction bars, area models, sets, and number lines. • The student will select, apply, and demonstrate knowledge of fractions equal to 1 whole using different representations and mathematical vocabulary.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	number line, set	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 91 – 106) • Fraction bars: 1 whole, 1 set of $\frac{1}{2}, \frac{1}{4}$ • 4 different colored pencils or markers	• Student Booklet (pp. 44 – 50) • Fraction bars: 1 wholes, 1 set of $\frac{1}{2}, \frac{1}{4}$ (1 set per student) • Different colored pencils or markers (4 per student) • Number Cubes: Fraction Model and Denominator (1 set per pair)

Preview

Say: **Today we will build fractions that are equal to 1.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to review the types of models for fractions they have seen in previous lessons.

Ask questions such as:

- How many parts are in the whole?
- How many parts are shaded?
- How could you represent this fraction with a different model?

Modeled Practice — Time: 8 min

1. Model a fraction equal to 1 with equal shares – 3 friends share 3 sandwiches.

   Have students turn to ***Modeled Practice Sheet #1***. The colors suggested to use below are blue, red, and brown, but you may have students use any three colors available for each sandwich.

Say: **3 friends want to share 3 different sandwiches equally. 1 friend has a tuna fish sandwich, 1 friend has a ham sandwich, and the third friend has a peanut butter sandwich. Each friend wants a part of each sandwich. Let's see what the equal share will look like.**

Think about the tuna fish sandwich. How would you divide it so that each friend got an equal share? *(into 3 equal parts)*

How would you divide the other 2 sandwiches so each friend got an equal share? *(into 3 equal parts)*

We will use the rectangle below to represent the equal share of the 3 sandwiches. Each sandwich will be divided into 3 equal parts before sharing.

Divide the tuna fish rectangle into thirds and shade the sandwich blue.

Divide the ham sandwich rectangle into thirds and shade the sandwich red.

Divide the peanut butter sandwich rectangle into thirds and shade the sandwich brown.

Divide the rectangle into thirds. As the lesson suggests, shade the rectangle using three different colors.

Say: **How much of the tuna fish sandwich will each person get?** *(one-third of a sandwich)*

Shade the first third of the rectangle blue to represent the share or the part from the tuna fish sandwich.

How much of the ham sandwich will each person get? *(one-third)* **Shade another third of the rectangle red to show the share or the part from the ham sandwich.**

How much of the peanut butter sandwich will each person get? *(one-third)* **Shade the last third brown to represent the share from the peanut butter sandwich.**

This rectangle represents the equal share each friend would get when they share the 3 sandwiches equally. How much of a sandwich is an equal share in thirds? *(three-thirds)*

The equal share is three-thirds of a sandwich.

Write "$\frac{3}{3}$ of a sandwich" on the "Equal Share" line.

Say: **What else do you notice about the three-thirds of the different sandwiches?** *(it is 1 whole sandwich)*

It is also the same amount as 1 whole sandwich. Three-thirds of a sandwich is the same amount as 1 whole sandwich. Why couldn't we just give each friend 1 whole sandwich to start

with? *(because they wanted to eat a part of each different kind of sandwich)*

2. Use fraction bars to determine how many parts fit in 1 whole – halves and fourths. Use ***Modeled Practice Display #2***. Students continue on ***Modeled Practice Sheet #1***.

Say: **We just figured out that three-thirds is the same amount as 1 whole. I wonder how many halves are the same as 1 whole? Use your fraction bars to find out.**

Give students time to put one-half parts over the second rectangle.

Say: **How many halves are the same as 1 whole?** *(two-halves)* **Divide and shade the rectangle to show the two halves.**

Divide and shade the rectangle as the students do.

Say: **Two-halves of a sandwich is the same amount as 1 whole sandwich.**

Write "$\frac{2}{}$ of a sandwich" on the line.

Say: **I wonder how many fourths equal 1 whole sandwich. Look at the first 2 rectangles. Can you predict how many fourths equal 1 whole sandwich?** *(four-fourths)*

Teacher Note
If students do not come to this conclusion on their own, have them use the fraction bars to discover the answer.

Say: **Four-fourths of a sandwich is the same amount as 1 whole sandwich. Divide and shade the fourths rectangle in the picture to represent the equal share.**

Divide and shade the picture. Write "$\frac{4}{4}$ of a sandwich" on the line.

Say: **What can you say about the 3 fractions we have modeled that are equal to 1 whole?** *(they have the same number on the top and bottom, all the parts are shaded)*

These fractions all say that for however many equal parts there are in the whole, all of them are shaded. Can you think of another fraction that would be the same as 1 whole? *(answers will vary)*

3. Recognize other models of fractions equal to 1 – sets and the number line.

Have students turn to ***Modeled Practice Sheet #3***.

Say: **One way we model fractions is with fraction bars.**

Let's look at a set of objects. When we work with fraction bars, what represents the whole? *(the whole fraction bar, or the rectangle)* **In this group, what represents the whole?** *(all of the shapes in the group)*

How many shapes are in the set? *(6 shapes)*

Our fraction is going to describe "how many out of 6."

How many of these shapes are squares? *(all 6 of them)*

Teacher Note
Some students may not recognize that all the shapes are squares because they are different sizes. Discuss the attributes that define a square if students are uncertain.

Say: **6 out of the 6 shapes are squares. So what fraction of the shapes are squares?** *(six-sixths)*

Fill in the blanks with the students.

Say: **Six-sixths of the shapes represent 1 whole. In this case, all the shapes in the set represent 1 whole, so the fraction says all shapes in the set are squares.**

What is another way to model fractions? *(on the number line)*

On this number line, how many thirds equal 1 whole? *(three-thirds)* **Shade three-thirds and write the fraction.**

Shade the number line with the students.

Say: **How many eighths equal 1 whole?** *(eight-eighths)* **Shade eight-eighths and write the fraction.**

Shade the number line.

Say: **On a number line, you can see into how many parts the length from 0 to 1 is divided. In this case, there are 8 parts. When they are all shaded, 8 of the 8 parts are shaded. This means the length from 0 to 1 is completely shaded.**

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 47.

Say: **Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How do you know when a fraction is equivalent to 1?
- What are the different ways you can show a fraction is equivalent to 1?

Activity 2: Students will play with a partner using the ***Number Cubes: Fraction Model and Denominator***. Students will take turns rolling the

number cubes for their partner. One number cube contains the 3 models they are familiar with: Fraction Bars, Sets, and Number Lines. The second contains numbers representing the bottom number of the fraction.

The partner must draw the model described by the dice. For example, if the first partner rolls "Set" and "3," the second partner must draw a set model that represents $\frac{3}{3}$.

Independent Practice

Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 9

The Language of Fractions: Naming a Model

Lesson Objectives	• The student will name the fractional parts represented by a model in symbols and words, using numerator and denominator. • The student will develop and utilize associative thinking skills through defining and describing representations.	
Vocabulary	**denominator:** the number of equal parts in the whole **numerator:** the number of parts from the whole being described	
Reviewed Vocabulary	area model, fraction, number line, set model	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 107 – 124) • 2 different colored pencils or markers • Fraction Words Mat	• Student Booklet (pp. 51 – 59) • Different colored pencils or markers (2 per student) • What Fraction Am I? Cards (1 set per pair)

Preview

Say: **Today we will name fractional parts represented by a model.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice identifying the fraction words that go with a fraction.

Ask questions such as:

- How many parts are in the whole?
- How many parts would be shaded?

Modeled Practice — Time: 8 min

1. Name a fraction represented by an area model - $\frac{3}{4}$.

Have students turn to ***Modeled Practice Sheet #1***.

Say: **4 friends are sharing 3 sandwiches equally. An equal share is shown with the area model. We want to name the fraction.**

First, we will talk about how many equal parts are in the whole. The bottom number in the fraction, or the *denominator*, represents the number of equal parts in the whole. What is a *denominator*? *(number of equal parts in the whole)*

How many equal parts are in the whole? *(4)* **What is the *denominator*?** *(4)* **There are 4 equal parts in the whole, so the *denominator* in this fraction is 4.**

Write "4" as the denominator.

Say: **Second, we will look at the parts being described, or the shaded parts. The top number in a fraction, or the *numerator*, represents the number of parts being described. What is a *numerator*?** *(number of parts being described)*

How many parts are shaded? *(3)* **What is the *numerator*?** *(3)* **There are 3 parts shaded, so the *numerator* in this fraction is 3.**

Write "3" as the numerator.

Say: **This fraction says that 3 out of 4 equal parts are shaded. How do we say the fraction?** *(three-fourths)*

Since we are talking about sandwiches, the equal share is three-fourths of a sandwich.

Write "three-fourths of a sandwich" on the line, modeling the use of the ***Fraction Words Mat*** as necessary to confirm spelling.

2. Name a fraction represented by a set model - $\frac{2}{6}$.

Have students turn to ***Modeled Practice Sheet #2***.

Say: **This set model contains circles and triangles. We want to name the fraction that describes the fractional amount of the set that consists of circles.**

First, we want to think about how many equal parts are in the whole or the *denominator*. What is the whole in this set model? *(all of the shapes in the group)* **How many are there?** *(6)*

In a fraction, which position is the total number in the whole written? *(the bottom position/number)* **What is the mathematical word for the total number in the whole?** *(the denominator)*

How many items in the whole set? *(6)* **What is the *denominator* for this set of shapes?** *(6)* **There are 6 items or parts in the whole set, so the *denominator* is 6.**

Write "6" as the denominator.

Say: **Look at the parts being described. Which parts did we want to count?** *(the circles)* **What does the top number in a fraction represent?** *(the number of parts being described)* **What is the mathematical word for the parts being described in this**

fraction? *(the numerator)* **The *numerator* represents the number of parts being described.**

In this set, how many of the shapes are circles? *(2)* **What is the *numerator* for the number of circles in this set?** *(2)*

Write "2" as the numerator.

Say: **This fraction says that 2 out of 6 total shapes are circles. How do we say the fraction?** *(two-sixths)*

Two-sixths of the shapes are circles.

Write "two-sixths of the shapes are circles" on the line.

3. Name a fraction represented by a number line - $\frac{5}{8}$.

Have students turn to *Modeled Practice Sheet #3*.

Say: **The shaded portion of this number line represents a length of rope in feet. The distance from 0 to 1 represents 1 foot. We want to name the fraction that describes the length of the rope.**

First, we want to think about how many equal parts are in the whole, the *denominator*. What is the whole on this number line? *(the length from 0 to 1)* **Run your finger along the number line to show 1 whole.**

Check that students run their finger from 0 to 1 on the number line.

Say: **Count the spaces in between the hash marks, not the hash marks themselves. How many parts are in this whole?** *(8)*

In a fraction, where do you write the total number of equal parts? *(on the bottom)* **What is the mathematical word for the number of equal parts in the whole?** *(the denominator)*

What is the *denominator* for this number line? *(8)* **There are 8 equal parts, so the *denominator* is 8.**

Write "8" as the denominator.

Say: **Look at the parts being described, the *numerator*. We want to count the number of parts that are shaded. What does the top number in a fraction represent?** *(the number of parts being described)* **What is the mathematical word for the number of parts being described?** *(the numerator)*

What is the *numerator* for this number line? *(5)*

Write "5" as the numerator.

Say: **This fraction says that 5 out of 8 total parts are shaded. How do we say the fraction?** *(five-eighths)*

We can say that the rope is five-eighths of a foot long.

Write "the rope is five-eighths of a foot long" on the line.

4. Name a model with a fraction – does the model represent $\frac{8}{6}$?

Have students turn to *Modeled Practice Sheet #4*.

Say: **Amber says this model represents the fraction $\frac{8}{6}$. Is she correct? Why or why not? Let's use what we know about *numerators* and *denominators* to find out if she is correct.**

First, let's look at how many parts are in the whole. What is the word for the number in the fraction that represents the number of equal parts in the whole? *(the denominator)*

How many equal parts are in the whole model? *(8)* **What is the *denominator* in the fraction Amber suggested?** *(6)* **Is this correct?** *(no)* **What should the denominator be?** *(8)* **Why?** *(because there are 8 equal parts in the whole)*

Write "8" as the denominator.

Say: **How many parts are being described, or are shaded?** *(6)* **What is the *numerator* of the fraction Amber suggested?** *(8)* **What should the *numerator* be?** *(6)* **Why?** *(because there are 6 parts shaded)*

Write "6" as the numerator.

Say: **What is the fraction that describes this area model?** *($\frac{6}{8}$)*

Why do you think Amber said the fraction is $\frac{8}{6}$? *(answers vary – she reversed the numerator and denominator)*

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 56.

Say: **Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How many parts are in the whole?
- How many parts are described, or shaded?
- What number is the *denominator*? Which number is the *numerator*?
- How do you say the fraction?

Activity 2: Using the ***What fraction am I?*** **Cards**, the students will take turns reading the numerator and denominator for a fraction to a partner. Partner 1 will say something like: "My denominator is 8 and I have 3 shaded parts." Partner 2 will write the fraction ($\frac{3}{8}$) and read it out loud: "You are three-eighths."

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 10

The Language of Fractions: Create a Model

Lesson Objectives	• The student will create 2 different models to represent a given fraction. • The student will create and use representations to organize, record, and communicate mathematical ideas.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	area model, denominator, fraction, number line, numerator, set model	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 125 – 136) • 2 different colored pencils or markers • Fraction Words Mat	• Student Booklet (pp. 60 – 65) • Different colored pencils or markers (2 per student) • Number Cube: Fraction Model (1 per pair) • *Fraction* Cards (1 set per group)

Preview

Say: **Today we will build models to represent fractions.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice identifying the numerator and denominator of fractional models.

Ask questions such as:

- Which number in the fraction represents the number of equal parts in the whole? *(the bottom number)* What is it called? *(the denominator)*
- Which number in the fraction represents the number of parts being described? *(the top number)* What is it called? *(the numerator)*

Modeled Practice — Time: 8 min

1. Create 2 models for a fraction - $\frac{2}{3}$.

   Have students turn to ***Modeled Practice Sheet #1***. Have students complete the sheet along with you.

Say: **In the previous lesson, we identified fractions represented by models. Now we are going to create our own models to represent a fraction.**

The first fraction we are going to model is two-thirds. What can you tell me about two-thirds? First, what is the denominator? *(3)*

Write "Denominator:" to the right of the "3."

Say: **What does this 3 in the denominator tell us about the amount?** *(there are 3 equal parts in the whole or numbers of items in set)*

Write "parts in the whole" under "Denominator:."

Say: **What is the numerator?** *(2)*

Write "Numerator:" to the right of the "2."

Say: **What does a 2 in the numerator tell us?** *(there are 2 parts being described, 2 parts are shaded)*

Write "parts being described" under "Numerator:."

Say: **The amount we are going to model has 2 parts shaded, or described, out of 3 equal parts in the whole.**

First, we will draw an area model. How many equal parts are in the whole? *(3)* **How should we divide the rectangle?** *(into 3 equal parts)* **How do you know?** *(the denominator is 3)*

Divide the rectangle.

Say: **How many parts are shaded?** *(2)* **Shade the parts.**

Shade 2 parts of the rectangle.

Say: **What fraction does this area model represent?** *(two-thirds)* **Two-thirds of the rectangle is shaded.**

Next we will draw a set model to represent the fraction, two-thirds.

What does the denominator, 3, tell us about the set model we will draw? *(it has 3 total objects in the group)*

There will be 3 total objects in the set. We will draw shapes because we can draw them quickly. What 2 kinds of shapes would you like to draw?

Take 2 suggestions that will be easy to draw. [the script will use squares and triangles]

Say: **Two-thirds of the shapes will be which shape?**

Let students pick 1 of the 2 shapes. [the script will use squares]

Say: **Which number in the fraction tells us how many will be [squares]?** *(the numerator)* **How many will there be?** *(2)*

Draw 2 [squares] in the box.

Say: **Next we need to determine how many [triangles] to draw for the set. Let's think about the denominator. How many shapes should there be in the whole group?** *(3)*

How many shapes have we drawn? *(2)* **How many more do we need to draw?** *(1)*

Draw 1 [triangle].

Say: **We have drawn 2 [squares] and 1 [triangle]. What fraction of the shapes are [squares]?** *(two-thirds of the shapes are squares)*

Write "two-thirds of the shapes are squares" under the model.

Say: **What fraction does this set model represent?** *(two-thirds)* **What other fraction does this set model represent?** *(answers vary: one-third of the shapes are triangles; three-thirds of the objects are shapes)*

2. Create 2 models for a fraction - $\frac{4}{8}$.

Have students turn to ***Modeled Practice Sheet #2***. Have students complete the sheet along with you.

Say: **Read the fraction.** *($\frac{4}{8}$)* **We are going to model four-eighths. What is the denominator?** *(8)*

Write "Denominator:" to the right of the "8."

Say: **What does a denominator of 8 tell us about the amount?** *(there are 8 equal parts in the whole)*

Write "parts in the whole" under "Denominator:."

Say: **What is the numerator?** *(4)*

Write "Numerator:" to the right of the "4."

Say: **What does a numerator of 4 tell us?** *(there are 4 parts being described, 4 parts are shaded)*

Write "parts being described" under "Numerator:."

Say: **The amount we are going to model has 4 parts shaded, or described, out of 8 equal parts in the whole.**

First, we will draw an area model. How many equal parts are in the whole, the denominator? *(8)* **Trace the dotted lines to divide the rectangle.**

How many parts, the numerator, do we need to shade? *(4)* **Shade the parts.**

Shade 4 parts of the rectangle.

Say: **What fraction does this area model represent?** *(four-eighths)* **Four-eighths of the rectangle is shaded.**

Now, let's locate four-eights on the number line. What part of the number line represents 1 whole? *(the distance or length between 0 and 1)*

How many equal parts in the whole, the denominator? *(8)* **The number line is already divided for us. We need to write the fractions that are represented by length from 0 to the hash mark. What is the fraction that is equal to 0?** *($\frac{0}{8}$)* **What is the length from 0 to the first hash mark?** *($\frac{1}{8}$)* **Finish labeling the hash marks. What is the fraction equivalent to 1 whole?** *($\frac{8}{8}$)*

How many parts, the numerator, of the number line will we need to shade? *(4)* **Shade along the number line.**

Shade the number line.

Say: **What fraction of the length between 0 and 1 is shaded?** *(four-eights)*

The same fraction can be represented using many different models. Although the models and the wholes they represent may be different, as long as they have the same number of equal

parts in a whole and the same amount shaded, they represent the same fraction.

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 63.

Say: Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- Which number is the denominator? Which number is the numerator?
- What is the whole?
- How many parts are in the whole?
- How many parts are described, or shaded?

Activity 2: Using the ***Number Cubes: Fraction Model*** students will play with a partner. Students will take turns rolling the number cube for their partner. The number cube contains the 3 models they are familiar with: Fraction Bars, Sets, and Number Lines. Partner 1 will roll the number cube and pick a ***Fraction Card.***

Partner 2 will draw the model described by the number cube. For example, if Partner 1 rolls "Set" and draws "$\frac{2}{3}$", Partner 2 must draw a set model that represents $\frac{2}{3}$.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 11

Equivalence with Equal Shares

Lesson Objectives	• The student will partition various quantities to show equivalence. • The student will organize and verbalize steps in solving to explain mathematical thinking.	
Vocabulary	**equivalent fractions:** Different fractions that represent the same portion of a whole.	
Reviewed Vocabulary	one-eighth, one-fourth, one-half, one-sixth, one-third, equal share	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 137 – 150) • 2 different colored pencils or markers	• Student Booklet (pp. 66 – 72) • Different colored markers or pencils (2 per student)

Preview

Say: **Today we will model equal shares that are the same amount but have different names.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice modeling sharing scenarios.

Ask questions such as:

- How many people are sharing? *(3)*
- How many cupcakes are they sharing? *(2)*
- How should each cupcake be divided? *(into 3 equal parts)*
- What fraction of each cupcake will each friend receive? *(one-third)*
- What is the equal share? *(two-thirds)*

Modeled Practice — Time: 8 min

1. Students will find a non-unit fractional share ($\frac{2}{4}$) as done in previous lessons by dividing and sharing parts of area models representing sandwiches – 4 friends share 2 sandwiches.

   Use ***Modeled Practice Sheet #1***. Following the directions, shade the first rectangle with a colored pencil and partition it into fourths. Have students complete as the lesson progresses.

Say: **Read with me: Lindsay and her 3 friends want to share 2 sandwiches equally. What is the equal share if they eat 2 sandwiches?**

How many friends are sharing the sandwiches? *(4)*

How many sandwiches are being shared? *(2)*

How do we break up the first sandwich to share equally among 4 friends? *(cut it into 4 pieces)* **What is each equal share called?** *(one-fourth)* **How many fourths are in the first sandwich?** *(4)*

Divide and shade one-fourth for each friend. Have students do the same.

Say: **What is the equal share each friend will get from the first sandwich?** *(one-fourth of a sandwich)*

Divide each rectangle representing the friends' equal shares into fourths and then shade the equal share from the first sandwich.

How many fourths are in the second sandwich? *(4)*

Show me how to share this second sandwich.

Shade the second rectangle in a different color and partition it into fourths. Check that students have done the same.

Say: **What is the equal share each friend receives from the second sandwich?** *(one-fourth of a sandwich)*

Shade the rectangle representing each friend's share to show one-fourth from the second sandwich.

How much of the sandwiches does each friend receive altogether? Let's count: one-fourth, two-fourths. How much? *(two-fourths of a sandwich)*

Write "$\frac{2}{4}$ of a sandwich" on the "Equal Share" line. Point to the number sentence.

2. Students will consider a different way to share the sandwiches so that each is cut into fewer parts ($\frac{1}{2}$) – 4 friends share 2 sandwiches.

   Use ***Modeled Practice Sheet #2***. Following the directions, shading as instructed. Have students complete as the lesson progresses.

Say: **We just shared 2 sandwiches among 4 friends. What is the equal share each friend received?** *($\frac{2}{4}$ of a sandwich)*

We are going to share the same sandwiches another way.

How many sandwiches are being shared? *(2)* **Shade each sandwich a different color.**

Check students' work.

Say: **How many friends are sharing the sandwiches?** *(4)*

In the first problem, we split each sandwich into fourths because we were only sharing one sandwich at a time. I am wondering if there is a way to share the sandwiches equally without having to make so many cuts.

How many equal shares do we need to make? *(4)* **How can we cut these 2 sandwiches so that we make 4 equal shares?** *(answers vary: in halves, in fourths)*

Allow students to offer suggestions.

Say: **We need to divide these 2 sandwiches so that 4 friends get an equal share. That means each sandwich would have 2 equal shares, for a total of 4 equal shares. What is the size of an equal share if a sandwich has 2 equal shares?** *(one-half)*

If we cut each sandwich in half, how many one-half size parts do we have? *(4)*

Divide each sandwich in half. Check students' work.

Say: **Will each friend get an equal share?** *(yes)* **How do you know?** *(there are 4 equal shares and 4 friends)*

Say: **How much of a sandwich does each friend receive?** *(one-half of a sandwich)*

Divide and shade each rectangle to show the equal shares. How many friends share the first sandwich? *(2)* **How many friends share the second sandwich?** *(2)*

Write "$\frac{1}{2}$ of a sandwich" on the "Equal Share" line. Check students' work.

Say: **In the first problem, we said each friend got two-fourths of a sandwich as the equal share. In the second problem, we said each friend got one-half of a sandwich as the equal share. Did the amount each friend receive change?** *(no)*

Did these 2 fractions, two-fourths and one-half, represent the same amounts? *(yes)*

How can we tell? *(compare the size of the equal shares)*

Compare the size of the equal shares. Are they the same size? *(yes)*

We shared the 2 sandwiches in different ways, but the equal shares were the same size. Since the fractions one-half and two-fourths represent the same amount, they are *equivalent fractions*. What are fractions that represent the same amount called? *(equivalent fractions)*

***Equivalent fractions* are different fractions that represent the same portion of a whole.**

Two-fourths and one-half are *equivalent fractions* because both represent the same portion or amount. What 2 fractions are *equivalent*? *(two-fourths and one-half)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 69. Students will find the equal share for 2 sharing stories resulting in equivalent fractions.

Say: **Work with a partner to find an equal share when 6 friends share 3 chocolate bars one at a time.**

Give students time to model the equal share.

Say: **How much candy bar does each friend get to eat from each bar?** *(one-sixth)* **What is the total equal share when the friends eat a piece of all 3 candy bars?** *(three-sixths of a chocolate bar)*

How could you share the chocolate bar so there are fewer parts? Work on your own to find the equivalent equal share.

Give students time to work alone to find the equal share, and then discuss the answer as a class.

Monitor students' work and provide corrective feedback when necessary.

Ask questions such as:

- What is the equal share? *(one-half)*
- Do the two equal shares represent the same fraction? How do you know? *(yes, because they have the same shaded area)*
- If you were sharing, would you want to share the candy bars in halves or sixths? Why? *(in halves because there would be fewer pieces to cut)*

Activity 2: Have students turn to the ***Practice Sheet*** on page 70. Students will name each equal share, and then match equivalent equal shares drawn as area models. Students may work on their own or with a partner.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 12

Equivalence with Strip Diagrams

Lesson Objectives	• The student will model equivalent fractions by folding paper strips. • The student will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	equivalent fractions	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 151 – 166) • 5 Paper Strips: 1″ x 8.5″ • 3 different colored pencils or markers	• Student Booklet (pp. 73 – 80) • Paper Strips: 1″ x 8.5″ (5 per student) • Different colored pencils or markers (3 per student)

Preview

Say: **Today we will model more equivalent fractions by folding paper strips.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice matching equivalent fractions.

Ask questions such as:

- How do you know by looking at the models that the fractions are equivalent? *(the same size of area is shaded)*
- If you were sharing sandwiches, which one of these two ways would you choose? *(the one with fewer cuts)*

Modeled Practice — Time: 8 min

1. Model equivalent fractions: $\frac{1}{2}$ and $\frac{2}{4}$. Students fold paper strips to model and compare the fractions for equivalence.

Teacher Note
Use a paper cutter to make the paper strips, cutting the short direction of the page.

Have students turn to ***Modeled Practice Sheet #1***. Distribute 3 paper strips to each student.

Say: **We will use this paper strip to model one-half. The strip represents 1 whole.**

Hold up the first paper strip. Model as prompted. Have students complete as the lesson progresses.

Say: **Fold the strip in half and then draw a line down the crease of the paper.**

How much of the paper strip is one-half? *(1 part of the strip, half of the strip)*

Each part represents one-half.

Draw a line to mark the crease in the paper strip and write "$\frac{1}{2}$" on each part. Have students do the same.

Say: **On the sheet, divide and shade the first area model to represent the paper strip. Write the fraction it represents.**

Shade the first model and write "$\frac{1}{2}$" next to it. Encourage students to use their paper strips to determine where to divide the model if they are having trouble.

Say: **This paper strip represents one-half. We will use the next paper strip to model an amount that is equivalent to one-half. What does equivalent mean?** *(they represent the same amount, or value)*

Equivalent fractions are 2 different fractions that represent the same amount. What is an equivalent fraction? *(fractions that represent the same amount)*

Fold the second paper strip in half. How much does each part represent? *(one-half)*

Now, fold the paper strip in half again. When you open it, what does each part of the paper strip represent? *(one-fourth)*
Each part of the paper strip represents one-fourth.

Draw lines to mark the creases in the paper strip and write "$\frac{1}{4}$" on each part. Have students do the same.

Say: **Compare this paper strip to the one representing one-half.**

Line up the paper strips on the table so the long sides match.

Say: **How many fourths are equivalent to, or the same amount as, one-half?** *(two-fourths)*

Two-fourths are the same amount as one-half.

Have students divide and shade the second rectangle on the sheet, using the paper strip as a guide to make equal divisions.

Say: **When you shade and divide the rectangle, what fraction does the shaded area represent?** *(two-fourths)*

Write "$\frac{2}{4}$" next to the area model. Check students' work.

Say: **How can you tell if the 2 amounts are the same?** *(answers vary: the same amount is shaded; the shaded areas are the same length)*

How does the size of the one-fourth parts compare to the size of the parts representing one-half? *(they are smaller)* **The parts are smaller, but the shaded area is the same.**

Do you think there could be another fraction that is equivalent to one-half and two-fourths? Why or why not? *(answers vary)*

2. Model equivalent fractions: $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$.

Say: **Fold your third paper strip in half, then in half, and in half again.**

Fold the third paper strip into eighths, and assist students in doing the same.

Say: **Unfold the paper strip. How many parts has the strip been folded into now?** *(8)*

What fraction is represented by 1 part of the paper strip? *(one-eighth)* **One-eighth is represented by 1 part of the paper strip.**

Draw lines to mark the creases in the paper strip and write "$\frac{1}{8}$" on each part. Have students do the same.

Say: **Compare this paper strip to the ones representing one-half.**

Line up the paper strips so the long sides are touching.

Say: **How many eighths are equivalent to one-half?** *(four-eighths)*

As you model, have students divide and shade the third rectangle on the sheet, using the paper strip as a guide to make equal divisions. Write "$\frac{4}{8}$" after the area model.

Say: **Compare the paper strip to the one representing two-fourths. How many eighths do you predict will be equivalent to two-fourths?** *(four-eighths)* **Why?**

Have students discuss how many eighths will be equivalent to two-fourths and why. If students struggle to make a prediction, have them think about how both fractions are equivalent to one-half.

Say: **One-half is equivalent to two-fourths and equivalent to four-eighths. This means that two-fourths and four-eights are equivalent.**

These models have different numbers of parts: 2, 4, and 8, but the shaded areas are the same. When the parts are smaller, more of them are shaded to make the same amount. The number and size of the shaded parts changes, but the total amount shaded stays the same.

Refer to the completed sheet with area models to show that all three fractions are equivalent.

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on pages 75. Students will create paper strips to represent thirds and sixths.

After students fold and label the strips for thirds and sixths, they will compare the strips to find equivalent fractions. Students draw, shade, and label equivalent fractions: $\frac{1}{3}$ and $\frac{2}{6}$, $\frac{2}{3}$ and $\frac{4}{6}$.

Ask questions such as:

- How many sixths are equivalent to one-third? How many sixths are equivalent to two-thirds? *(2, 4)*
- How do you know when 2 fractions are equivalent? *(they represent the same shaded area)*

Activity 2: Have students turn to the ***Practice Sheet*** on page 76. Students will work with a partner to solve the first problem using the strips made during the lesson to find equivalent fractions.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 13

Equivalence with Area Models

Lesson Objectives	• The student will model equivalent fractions with area models. • The student will create and use representations to organize, record, and communicate mathematical ideas with peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	denominator, equivalent fractions, numerator	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 167 – 180) • Colored pencil or marker	• Student Booklet (pp. 81 – 87) • Paper Strip models from Lesson 12 • Colored pencil or marker (1 per student) • *Equivalence Matching* Cards (1 set per pair)

Preview

Say: **Today we will represent equivalent fractions using area models.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice using paper strips to find equivalent fractions.

Ask questions such as:

- How many equal parts are in the whole? *(8)*
- What is each equal part called? *(one-eighth)*
- How many eighths are equivalent to one-half? *(4)*
- What fraction represents the shaded area? *(four-eighths)*
- How many pieces of sandwich will Raquel have to share? *(4)*

Modeled Practice — Time: 8 min

1. Model the equivalent fractions $\frac{1}{3}$ and $\frac{2}{6}$ using an area model.

   Have students turn to ***Modeled Practice Sheet #1***. Divide, shade, and label rectangles as directed. Have students complete as the lesson progresses.

Say: **We want to model a fraction that is equivalent to one-third. Divide and shade the first rectangle to show one-third, using the dotted lines as a guide. Label the rectangle below as $\frac{1}{3}$.**

In the next rectangle, we are going to model a fraction that is equivalent to one-third. How will we know if the fraction is equivalent to one-third? *(it will represent the same amount, or area)*

The equivalent fraction will represent the same area as the model of one-third, so we will start by dividing and shading the

second rectangle to also represent one-third. Label this rectangle below as $\frac{1}{3}$.

We found equivalent fractions before by folding paper strips. When we folded the paper strips, did the parts get larger or smaller? *(smaller)*

Did the shaded area stay the same? *(yes)*

Were there more or less parts? *(more parts)* **There were more parts, but the same amount of shaded area.**

If we divide this area model with our pencil, we can find another equivalent fraction without changing the amount of the shaded area.

Divide the second rectangle in half horizontally. Model as directed. Check students' work.

Say: **How many parts are in the whole now?** *(6)* **What fraction does each part represent?** *(one-sixth)* **What fraction of the model is shaded?** *(two-sixths)* **Write $\frac{2}{6}$ below the shaded area.**

Is the shaded area the same in both models? *(yes)* **This means the fractions represented by the area models are equivalent. Name the 2 equivalent fractions these models represent.** *(one-third and two-sixths)*

Write a large "=" between the 2 fractions. Have students do the same.

Say: **We know that one-third and two-sixths are equivalent because they represent the same amount. Let's see if we can find a fraction that is equivalent to two-thirds.**

Divide and shade the bottom area models to show two-thirds.

Give students time to divide, shade, and label the area models. Do the same, and write "$\frac{2}{3}$" below the first model with the students.

Say: **In the second area model we want to find a fraction that is equivalent to two-thirds that represents the same area. How can we find an equivalent fraction?** *(divide the rectangle in half)*

If we divide the rectangle in half, the model will represent a different fraction than the first, but the amount shaded will not change.

Divide the area model in half horizontally. Have students do the same.

Say: **How many equal parts are in the whole?** *(6)* **How many parts are shaded?** *(4)* **What fraction does the area model represent?** *(four-sixths)*

This area model represents four-sixths. Is four-sixths equivalent to two-thirds? *(yes)* **How do you know?** *(the shaded area is the same)*

Two-thirds and four-sixths are equivalent fractions because they represent the same value, or amount. The fractions look different, but they represent the same amount.

How does the size of the parts in the four-sixths model compare to the parts in the two-thirds model? *(they are smaller)* **What can you say about the number of parts?** *(there are more)*

The model representing four-sixths has smaller parts than the model for two-thirds. The whole is made up 6 parts instead of just 3 parts. However, there are more of those smaller parts shaded, 4 instead of 2, so the area or amount shaded did not change.

2. Model the equivalent fractions $\frac{1}{4}$ and $\frac{3}{4}$ using an area model.

Have students turn to ***Modeled Practice Sheet #2***. Divide and shade the top 2 circles and write "$\frac{1}{4}$" below the first following the teacher script. Have students do the same.

Say: **Marla ate one-fourth of a pie on Monday. She ate the same amount of pie on Tuesday. Marla's mother cut the pie into 8**

pieces after baking it. What fractional amount of the pie did she eat on Tuesday?

We will model a fraction that is equivalent to one-fourth. Divide and shade the top circles to show one-fourth, using the dotted lines as a guide.

Right now the second area model also represents one-fourth of the pie. We want to find a fraction that is equivalent to one-fourth using this model. In the previous example, we divided the model in half. When we divide models into more parts, the parts have to be equal in size.

Can we divide the model in half horizontally or across to find an equivalent fraction? *(no)* **Why not?** *(because the parts will not be the same size)*

Teacher Note
Students may need to see that dividing the circle in half horizontally as in the previous example will not result in a model showing an equivalent fraction.

Say: **When we divided the previous area models in half, we were actually dividing each part of the model in half, just like we did with the paper strips.**

Let's try to divide the parts in this model in half. They are a different shape, but we can divide them like we would cut a pie.

Divide the circle with two lines so there are 8 equal pieces. Assist students in doing the same.

Say: **Now, how many equal parts are in the whole?** *(8)* **What fraction does each shaded part represent?** *(one-eighth)* **What fraction does the area model represent?** *(two-eighths)*

Marla ate two-eighths of a pie on Tuesday, which is the same amount as one-fourth of a pie she ate on Monday. She ate 2

smaller pieces instead of 1 larger piece, but the amount is the same.

Write "$\frac{2}{8}$" below the area model. Have students do the same.

Say: **Is the shaded area the same in both models?** *(yes)* **What does this mean?** *(the two fractions are equivalent)* **What fractions are equivalent?** *(one-fourth and two-eighths)*

Write a large "=" between the 2 fractions. Have students do the same.

Say: **Now we will model a fraction that is equivalent to three-fourths. Let's use the same strategy to find a fraction that is equivalent to three-fourths.**

Divide and shade the bottom area models to show three-fourths.

Give students time to divide and shade the area models. Do the same, and write "$\frac{3}{4}$" below the first model. Have students do the same.

Say: **With the second area model, we want to find a fraction that is equivalent to three-fourths, which means it will have the same shaded area. How can we find an equivalent fraction?** *(divide each part of the model in half)*

If we divide each part in half, the model will represent a different fraction than the first, but the size of the shaded area will not change.

Divide the model into 8 equal parts. Have students do the same.

Say: **How many equal parts are in the whole?** *(8)* **How many parts are shaded?** *(6)* **What fraction does the area model represent?** *(six-eighths)*

This area model represents six-eighths. Is six-eighths equivalent to three-fourths? *(yes)* **How do you know?** *(the shaded area is the same)*

Three-fourths and six-eighths are equivalent fractions because they represent the same value, or the same shaded area. Do the fractions look different? *(yes)* **Do they represent the same amount?** *(yes)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 84. Students will practice recognizing equivalent fractions from models and drawing models to represent equivalent fractions.

Have students label the fraction shown by the first model. They will shade the same amount on the second model, and then determine how to divide it to find the equivalent fraction. Finally, students fill in the blank to name the equivalent fraction they have shaded.

Ask questions such as:

- What fraction does each model represent?
- How do you know the two models represent equivalent fractions?
- How many equal parts are in the whole? How can you tell by looking at the fraction?
- How many parts are shaded? How can you tell by looking at the fraction?

Activity 2: Students will practice matching equivalent fractions with models. Each student pair gets 1 set of ***Equivalence Matching*** **Cards**, 12 in all. Each partner receives 6 cards. The goal is to match up cards with equivalent fractions. Once partners have matched up any cards they have, they take turns asking their partner if they have a card equivalent to $\frac{1}{2}$, $\frac{1}{3}$, etc.

If students have difficulty determining if they have a match, the requesting partner (Partner 1) can show the card in question so Partner 2 can compare it to the area models on his or her cards.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 14

Equivalent Fractions on the Number Line

Lesson Objectives	• The student will identify equivalent fractions by labeling and comparing them on the number line. • The student will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	equivalent fractions, number line	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 181 – 190)	• Student Booklet (pp. 88 – 92) • *What Fraction Am I?* Cards (1 set per pair)

Preview

Say: **Today we will find equivalent fractions on the number line.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students turn to the ***Modeled Practice Sheet*** to practice labeling number lines with halves, thirds, fourths, sixths, and eighths. Students will begin using this sheet and will continue using it during Modeled Practice.

Say: **We are going to label the fractions on these number lines. What is the first number on these number lines?** *(0)* **What is the highest number on these number lines?** *(1)* **The range of the number lines is 0 to 1. Run your finger along the whole number line. This number line represents 1 whole.**

Check that students run their finger from 0 to 1.

Say: **Look at the first number line. How many equal parts are in the whole?** *(2)*

What fraction is 1 equal part of the number line? *(one-half)*

When a whole is divided into 2 parts we call those parts halves. How many halves is 0, the first hash mark? *(zero-halves)*

How many halves is 1 whole, the third hash mark? *(two-halves)*

Have students label all three hash marks with $\frac{0}{2}$, $\frac{1}{2}$, and $\frac{2}{2}$.

Say: **Look at the second number line. How many equal parts in the whole?** *(3)* **What fraction is 1 equal part of the number line?** *(one-third)* **What fraction is 2 equal parts of the number line?** *(two-thirds)*

Use similar questions to assist students in labeling the remaining number lines. Ask questions such as:

- How many equal parts are in the whole?
- What fraction is represented by 1 equal part? 2 equal parts?

Modeled Practice — Time: 8 min

1. Find equivalent fractions on the number line: fractions equivalent to $\frac{1}{2}$.

Continue to use the ***Modeled Practice Sheet***. Model the steps and have students complete as the lesson progresses.

Say: **Now that we have labeled the number lines, we want to find fractions that are equivalent to each other. What does it mean for 2 fractions to be equivalent?** *(the fractions represent the same amount)* **On a number line, 2 fractions are equivalent when they represent the same distance.**

Look at the first number line. What does the hash mark in the center represent? *(one-half)* **Draw a circle around one-half and label it.**

Check students' work.

Say: **We are going to find other fractions that are equivalent to one-half. Fractions that are equivalent to one-half will be the same distance from 0 on the number line.**

Fold the right side of your paper over to line up with the one-half hash mark on the number line.

Help students fold their paper over after you have completed this step.

Say: **Look at the number line labeled with thirds. Is there a fraction that is equivalent to one-half?** *(no)* **How do you know?** *(because there is no fraction that is the same length as one-half)*

Look at the number line labeled with fourths. Is there a fraction equivalent to one-half? *(yes)* **Which fraction is the same distance and how do you know?** *(two-fourths; it is the same distance from 0)* **Two-fourths is the same distance from 0 as one-half.**

Draw a circle around the label "$\frac{2}{4}$." Have the students do the same.

Say: **Look at the number line labeled with sixths. Is there a fraction equivalent to one-half?** *(yes)* **Which fraction is the same distance and how do you know?** *(three-sixths; it is the same distance from 0)* **Three-sixths is the same distance from 0 as one-half.**

Draw a circle around the label "$\frac{3}{6}$." Have the students do the same.

Say: **Look at the number line labeled with eighths. Is there a fraction equivalent to one-half?** *(yes)* **Which one and how do you know?** *(four-eighths; it is the same distance from 0)* **Four-eighths is the same distance from 0 as one-half.**

Draw a circle around the label "$\frac{4}{8}$." Have the students do the same.

Say: **Look down the page at the number lines. What fractions did we find that are equivalent to one-half? What other fractions represent the same distance from 0?** *(two-fourths, three-sixths, four-eighths)*

Have students discuss which fractions are equivalent to $\frac{1}{2}$.

Say: **One-half, two-fourths, three-sixths, and four-eighths are all equivalent to one another. They all represent the same length on the number line.**

2. Find equivalent fractions on the number line: fractions equivalent to $\frac{1}{3}$ and $\frac{2}{3}$.

Continue to use the ***Modeled Practice Sheet***.

Say: **On the number line labeled with thirds, we have labeled 2 fractions between 0 and 1: one-third and two-thirds.**

Draw a triangle around the labels "$\frac{1}{3}$" and "$\frac{2}{3}$." Check students' work.

Say: **Fold the right side of your paper over to line up with one-third.**

Look down to the other number lines on the page. The very next number line is fourths. Is there a fraction with a denominator of 4 that is equivalent to one-third? *(no)* **How do you know?** *(the parts are not the same length, one-fourth is too short and two-fourths is too long)*

Look at the next number line that is representing sixths. Is there a fraction that is equivalent to one-third? *(yes)* **What fraction is the same distance from 0 as one-third?** *(two-sixths)* **How do you know?** *(it is the same distance from 0 as one-third)* **Label $\frac{2}{6}$ and draw a triangle around the fraction.**

Are there any other fractions equivalent to one-third shown on these number lines? *(no)* **Why not?** *(the eighths are not the same distance, it is either too short or too long, not exact)*

Move the folded edge of your paper back to line up with two-thirds. Look down at the other number lines.

Demonstrate how to fold the paper to line up with two-thirds.

Say: **What fraction is equivalent to two-thirds?** *(four-sixths)* **How do you know?** *(it is the same distance from 0 as two-thirds)*

Two-thirds is equivalent to four-sixths. Label $\frac{4}{6}$ and draw a triangle around the fraction.

3. Find equivalent fractions on the number line: fractions equivalent to $\frac{1}{4}$ and $\frac{3}{4}$.

Continue to use the ***Modeled Practice Sheet.***

Say: **On the number line labeled with fourths, we have labeled 3 fractions between 0 and 1: one-fourth, two-fourths, and three-fourths. We have already found the fractions that are equivalent to two-fourths. Now let's find equivalent fractions for one-fourth and three-fourths.**

Draw a rectangle around the labels "$\frac{1}{4}$" and "$\frac{3}{4}$." Have students do the same.

Teacher Note
Students may continue to fold their paper over to help line up the equivalent fractions or they may just use their finger to see which fractions are in line.

Say: **Is there a fraction that is equivalent to one-fourth?** *(yes)***What fraction is the same distance from 0 as one-fourth?** *(two-eighths)*

Two-eighths is the same distance from 0 as one-fourth. What fraction is equivalent to two-eighths? *(one-fourth)* **Label $\frac{2}{8}$ and draw a rectangle around the fraction.**

Check that students have correctly labeled the fractions on the number line.

Say: **Look at the halves, thirds, and sixths number lines. Are there any other fractions equivalent to one-fourth shown on these number lines?** *(no)*

What fraction is equivalent to three-fourths? *(six-eighths)* **How do you know?** *(it is the same distance from 0 as three-fourths)*

Three-fourths is equivalent to six-eighths. Label $\frac{6}{8}$ and draw a rectangle around the fraction.

Check that students have correctly labeled the fractions on the number line.

Say: **Name a fraction equivalent to one-half.** *(two-fourths, three-sixths, four-eights)*

Name a fraction equivalent to one-third. *(two-sixths)*

Name a fraction equivalent to three-fourths. *(six-eights)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 89. Students will use the number lines labeled on the ***Modeled Practice Sheet*** to find equivalent fractions. Have students discuss the first answer with a partner. Confirm that each pair has written down *all* possible answers shown by the number lines.

Ask questions such as:

- How do you know the fractions are equivalent? *(it is the same distance from 0)*
- Is there more than 1 equivalent fraction? *(answers vary)*

Activity 2: Students will take turns drawing ***What Fraction Am I? Cards*** with a partner. Partner 1 will read a card (without reading the answer in parentheses), while Partner 2 guesses the fraction. Students may use the number lines on the ***Modeled Practice Sheet*** as needed.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 15

Comparing Unit Fractions

Lesson Objectives	• The student will compare unit fractions. • The student will develop and use representations to model and interpret mathematical phenomena.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	compare, denominator, numerator	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 191 – 206) • Fraction bars: 2 wholes, 1 set of $\frac{1}{3}$ and $\frac{1}{8}$	• Student Booklet (pp. 93 – 100) • Fraction bars: 2 wholes, 1 set of $\frac{1}{3}$ and $\frac{1}{8}$ (1 set per student)

Preview

Say: **Today we will compare different unit fractions to see which is greater.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice modeling unit fractions. Dotted lines have been added to some shapes to assist with equal division of the wholes.

Ask questions such as:

- What is the whole?
- How many equal parts are in the whole? What part of the fraction tells you? *(answers vary; the denominator)*
- How many parts are shaded? What part of the fraction tells you? *(answers vary; the numerator)*

Modeled Practice — Time: 8 min

1. Compare fractions using fraction bars $\frac{1}{8}$ and $\frac{1}{3}$.

   Hold up the 2 whole fraction bars.

Say: **Imagine these whole fraction bars are granola bars. Would you rather have one-eighth of a granola bar, or one-third of a granola bar?** *(allow a variety of answers)* **Why?**

Watch For 	**Students may think the fraction with the largest denominator is the greater number.** **For example, students may state that $\frac{1}{8}$ is larger than $\frac{1}{2}$.** **If this happens, ask students whether they would want $\frac{1}{2}$ of a cupcake or $\frac{1}{8}$, meaning would they rather share their cupcake with 1 other person or**

7 other people. Emphasize that if the denominator is larger, then there are more equal parts in the whole, which means each part is smaller.

Say: **Let's see how big one-eighth of a granola bar is. Trade in 1 whole granola bar for eighths and show me the size of one-eighth.**

Trade in the fraction bar and wait for students to hold up one-eighth.

Say: **How much is one-eighth?** *(1 part out of a whole divided into 8 equal parts)*

Trade in the other whole granola bar for thirds. Show me the size of one-third.

Trade in the fraction bar and wait for students to hold up one-third.

Say: **One-third is 1 part out of a whole divided into 3 equal parts. Let's compare the 2 parts. Is the one-eighth part larger or smaller than the one-third part?** *(smaller)*

Line up the one-eighth and one-third parts one above the other.

Say: **The one-eighth part is smaller, which means one-eighth is less than one-third.**

Teacher Note
If students have difficulty identifying that the one-eighth part is smaller than one-third, use a number line to show that one-eighth is a smaller part.

If a whole granola bar is divided equally to share with 8 people, is each part bigger or smaller than the granola bar pieces divided equally to share with 3 people? *(smaller)*

The more people that share the 1 whole, the smaller the part will be for each person. If the whole were divided into 100 equal parts, would each part be bigger or smaller than the one-eighth pieces? *(smaller)*

Which number in a fraction, the numerator or the denominator, tells us how many equal parts are in the whole? *(the denominator)*

What can you say about the number of parts in the whole and the size of those parts? *(the greater the number of equal parts, the the smaller each part is)*

2. Compare fractions using an area model for $\frac{1}{2}$ and $\frac{1}{4}$.

Have students turn to ***Modeled Practice Sheet #1***. Model and have students complete as the lesson progresses.

Say: **Yessica has 1 cake to share with friends. Will each person get more if she shares the cake with 2 friends or 4 friends?**

Think about this scenario. If you had a cake to share, would you rather share it with 1 friend or 3 friends? When would you get the most cake? *(when sharing with 1 friend)*

Have students discuss.

Say: **If you share 1 cake equally between 2 people, how much cake does each receive?** *(one-half)* **Shade one-half of the circle.**

Write "$\frac{1}{2}$" to the right of the circle and have students do the same.

Say: **How much cake does each person receive if 4 people are sharing equally?** *(one-fourth)* **Shade one-fourth of the circle.**

Write "$\frac{1}{4}$" to the right of the circle and have students do the same.

Say: **Look at the shaded area. Which shaded area is larger?** *(the top circle)*

Is one-half of the cake greater than or less than one-fourth of the cake? *(greater than)*

Fill in the blanks below the models: "$\mathbf{\frac{1}{2}}$ of a cake is **greater** than $\mathbf{\frac{1}{4}}$ of a cake. Have the students do the same.

Say: **Is the fraction one-half greater than or less than the fraction one-fourth?** *(greater than)*

Fill in the blanks "$\frac{1}{2} > \frac{1}{4}$." Have students do the same.

Say: **One-half is greater than one-fourth because one-half of the cake is more than one-fourth of the same cake.**

3. Compare fractions using a number line for $\frac{1}{6}$ and $\frac{1}{3}$.

Have students turn to ***Modeled Practice Sheet #2***.

Say: **Tony ran one-sixth of a mile. Javier ran one-third of a mile. Who ran the farthest?**

To find out who ran the farthest, we have to know which is greater: one-sixth or one-third. Who do you think ran the farthest?

Give students a chance to share their predictions.

Say: **Let's model these fractions on a number line. How far did Tony run?** *(one-sixth of a mile)* **Label the hash marks and model one-sixth on the number line.**

Give students a chance to label the hash marks and shade one-sixth on the number line while you do the same.

Teacher Note

If students have difficulty modeling a fraction on a number line, prompt them to consider into how many equal parts the number line is divided. Assist them in labeling each hash mark from 0 to 1.

Say: **How far did Javier run?** *(one-third of a mile)* **Label the hash marks and model one-third on the next number line.**

Look at the shaded lines. Which line is longer: one-sixth or one-third? *(one-third)* **Why is one-third longer than one-sixth?** *(one-third part is larger because it is divided into less pieces)*

Who ran farther? *(Javier)*

Fill in the blanks so it says "$\frac{1}{6}$ of a mile is **less** than $\frac{1}{3}$ of a mile. **Javier** ran the farthest." Check students' work.

Say: **Is the fraction one-sixth greater than or less than the fraction one-third?** *(less than)*

Fill in the blanks "$\frac{1}{6} < \frac{1}{3}$." Have students do the same.

Say: **Why is one-sixth less than one-third?** *(because it is 1 part of a whole broken into 6 parts, and one-third is 1 part of a whole broken into only 3 parts)*

We are comparing 1 part from each whole. The denominator tells us how many parts are in the whole. When there are 6 equal parts in the whole, each part is smaller than when there are 3 equal parts in the whole.

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 96. Students will work with a partner to determine the inverse relationship between the denominator and the size of the part by shading a unit fraction with denominators of 2, 3, 4, 6, and 8.

Lead students toward the conclusion that the size of an equal part is smaller when the denominator is larger.

Ask questions such as:

- How many equal parts are in the whole?
- What part of the fraction tells you?

Activity 2: Have students turn to the ***Practice Sheet*** on page 97. Students will use the area models from the previous activity to guide their comparison of the unit fractions. Have students work in pairs to determine whether or not Noah is correctly comparing the fractions and address the misconception present in his thinking.

Have students independently compare the remaining pairs of fractions, using the area models from the previous activity for assistance as needed.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 16

Comparing Fractions with Like Denominators

Lesson Objectives	• The student will compare fractions with like denominators using area models and number lines. • The student will communicate mathematical thinking using precise vocabulary to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	area model, compare, denominator, number line, numerator	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 207 – 224) • Fraction bars: 2 wholes, 1 set of $\frac{1}{6}$ • Different colored pencil or marker	• Student Booklet (pp. 101 – 109) • Fraction bars: 2 wholes, 1 set of $\frac{1}{6}$, 1 piece each of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$ (1 set per student) • Colored pencils or markers (2 different colors per student)

Preview

Say: **Today we will compare fractions using like denominators.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice comparing unit fractions.

Give students 5 fraction bar parts, one each: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ in a pile.

Say: **Each of these parts represents a different unit fraction. Arrange them in order, from largest to smallest.**

Give students a chance to arrange the fractional parts.

Say: **Which fraction is the largest?** *(one-half)* **Place the one-half fraction in the first fraction bar on your sheet. Trace the dotted line on the model and write the fraction name.**

Have students use the fraction parts to show each fraction on the bar model outline, and then shade the area and write the fraction to the right of the model.

Say: **Look at the denominators of the fractions. What can you say about the denominators of the unit fractions compared to the size of the fractional part?** *(the larger the denominator, the smaller the part; the smaller the denominator, the larger the part)*

As the denominator gets larger, the size of the fractional parts gets smaller.

Fill in the blanks at the bottom of the page.

Modeled Practice Time: 8 min

1. Compare fractions with like denominators ($\frac{2}{6}$ and $\frac{4}{6}$)by using fraction bars. Students will model each fraction, then compare the shaded areas.

   Have students turn to ***Modeled Practice Sheet #1***. Model both fractions with the fraction bar pieces on top of a 2 whole fraction bar while students do the same.

Say: **Read the problem. Ready? Read: Margaret received two-sixths of a cereal bar, while Sara received four-sixths of a cereal bar. Did Margaret get more or less than Sara?**

These two fractions have the same denominators. What does that mean? *(they have the same number of parts in the whole)* **How many parts in the whole do they have?** *(6)*

We will use the sixth fraction bar parts to model the 2 amounts. How many do we need for the first fraction? *(2)* **How many do we need for the second fraction?** *(4)*

Shade the models to represent each fraction while students do the same.

Say: **Let's talk about the 2 fractions. What can you say about the number of equal parts in both fractions?** *(they have the same number of parts, 6)* **Would you know this if you only had the fractions and not the model?** *(yes, they both have 6 as the denominator)* **The denominator means that both fractions have 6 equal parts in the whole.**

What can you say about the numerators? *(they are different, one has 2 and one has 4)*

We are comparing sixths in both fractions, so we only have to consider how many sixths are being described by each fraction. Do you have more when there are 2 one-sixth parts or when there are 4 one-sixth parts? *(when there are 4)*

Is two-sixths greater than or less than four-sixths? *(less than)*

Write "<" in the circle between the two fractions. Check students' work.

Say: **Each sixth-sized part is the same size, so 2 of them are a smaller amount than 4 of them. Two-sixths is less than four-sixths.**

Does Margaret receive more or less of the cereal bar than Sara? *(less)*

Write "less" in the space to complete the sentence at the bottom of the sheet. Have students do the same.

2. Compare fractions with like denominators ($\frac{5}{8}$ and $\frac{2}{8}$) by comparing area models.

Have students turn to ***Modeled Practice Sheet #2***. Shade above the area models for the fractions, each with a different color. Have students do the same.

Say: **Read the problem. Ready? Read: Ana's shoe is five-eighths of a foot long. Her brother's shoe is two-eighths of a foot long. Who has the bigger shoe?**

How many equal parts are in the whole for the fractions? *(8)* **How many parts do we shade for the first fraction?** *(5)* **How many do we shade for the second fraction?** *(2)* **Shade each fraction on a number line so we can compare the lengths.**

What can you say about the number of equal parts in the whole for the fractions? *(they have the same number of parts, 8)* **How do we know this by looking at the fractions?** *(they both have 8 as the denominator)*

What can you say about the numerators? *(they are different, one has 5 and the other has 2)*

We are comparing eighths in both fractions, so we only have to consider how many eighths are being described by each fraction.

Is five-eighths greater than or less than two-eighths? *(greater than)*

Write ">" in the circle between the two fractions. Check students' work.

Say: **Five-eighths is greater than two-eighths. Who has the bigger shoe?** *(Ana)*

Write "Ana" in the space to complete the sentence at the bottom of the sheet. Have students do the same.

3. Compare fractions with like denominators ($\frac{1}{4}$ and $\frac{3}{4}$) by comparing the shaded areas of 2 area models.

Have students turn to ***Modeled Practice Sheet #3***.

Say: **We want to know if one-fourth of the circle shown is greater than or less than three-fourths of the circle shown.**

As in the previous examples, both fractions have the same denominators. This means we should be able to compare the numerators, or the number of parts shaded, to determine which fraction is greater.

Without looking at the models, is one-fourth greater than or less than three-fourths? *(less than)*

Teacher Note
If students do not answer, remind them that if the parts are the same size, then the fraction with more parts is greater than the fraction with fewer parts.

Say: **How many equal parts are in the whole?** *(4)* **How many parts do we shade for the first fraction?** *(1)* **How many do we shade for the second fraction?** *(3)*

Shade the area models to represent one-fourth and three-fourths, each with a different color. Have students do the same.

Say: **Look at the shaded parts of the area models. Is one-fourth greater than or less than three-fourths?** *(less than)*

How do you know? *(less parts are shaded)*

Write "<" in the circle between the two fractions. Check students' work.

Say: **Read it.** *(one-fourth is less than three-fourths)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 105. Students will shade models to compare fractions with like denominators.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- Do the fractions have the same number of parts in the whole?
- Are the wholes the same size?
- Which fraction describes more parts in the whole?
- Is the first fraction greater than or less than the second fraction?

Activity 2: Have students turn to the ***Practice Sheet*** on page 106. Students will compare fractions with like denominators with a partner. Each partner will shade and write the name of the fraction for several different denominators without looking at each other's papers. Then, they will write and compare each pair of fractions to determine which is greater.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 17

Comparing Fractions with Like Numerators

Lesson Objectives	• The student will compare fractions with like numerators using area models. • The student will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	compare, denominator, numerator	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 225 – 238) • Fraction bars: 1 set of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ • 2 different colored pencils or markers	• Student Booklet (pp. 110 – 116) • Fraction bars: 1 set of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ (1 per student) • Different colored pencils or markers (2 per student) • Like Numerator Comparison Cards (6 cards per pair) • Area Model Mat (1 per pair)

Preview

Say: **Today we will compare fractions with the same numerator.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice reasoning about the size of fractions with like denominators.

Ask questions such as:

- How do the sizes of the parts in the whole compare? How do you know? *(answers vary)*
- How many parts are shaded? How do you know?

Modeled Practice — Time: 8 min

1. Compare fractions with like numerators ($\frac{2}{3}$ and $\frac{2}{6}$).

   Have students turn to the ***Modeled Practice Sheet.*** Have the fraction parts available to use.

Say: **When comparing unit fractions, the larger the denominator, the smaller the part, meaning the entire fraction is less. But what if we want to compare fractions that have numerators other than 1? We are going to compare two-thirds and two-sixths.**

Use the fraction bar parts to model two-thirds and two-sixths on your sheet.

Put the fraction bar parts on top of the rectangles.

Say: **How many parts did you shade to model each fraction?** *(2)* *(there are the same number)* **There is the same number of shaded parts in both models.**

Which number in a fraction tells you the number of shaded parts? *(the numerator)* **What are the numerators for these fractions?** *(2)*

Describe the size of each part. If this was food, when would you get a bigger part of the item? *(the parts are larger in the model of two-thirds)* **The parts are larger for the fraction two-thirds. Why?** *(there are fewer parts in the whole)*

What number in a fraction tells you how many parts are in the whole? *(the denominator)* **What are the denominators for these two fractions?** *(3 and 6)*

By looking at the denominators, how many total parts in two-thirds? *(3)* **How many total parts in two-sixths?** *(6)* **Two-thirds has less parts or divisions, so that means each part is larger in size than dividing the whole into six total parts.**

What can we say about the number and size of the shaded parts for two-thirds and two-sixths? *(they have the same number of shaded parts but the parts are bigger for two-thirds)*

Is two-thirds greater or less than two-sixths? *(greater than)* **How do you know?** *(one-third part is larger with one-sixth part)*

Remove the fraction parts and shade the areas underneath. Have students do the same. Write ">" in the circle between the fractions.

Say: **What does this mean about the size of the fraction?** *(two-thirds is greater than two-sixths)*

2. Compare fractions with like numerators ($\frac{3}{8}$ and $\frac{3}{4}$).

Say: **Divide and shade the area models to show three-eighths and three-fourths.**

Divide and shade the area models.

Say: **What can you say about the number of shaded parts?** *(they have the same number, 3)* **Which number of the fraction tells you the number of shaded parts?** *(the numerator)*

Compare the size of each part. What can you say about the size of the parts? *(the parts in three-eighths are smaller than the parts in three-fourths)* **Why?** *(because there are more parts in the whole)*

Which number of the fraction tells you the number of equal parts in the whole? *(the denominator)*

Is three-eighths greater than or less than three-fourths? *(less than)* **How do you know?** *(the shaded area is smaller)*

Three-eighths is less than three-fourths.

Draw "<" in the circle between the fractions. Check students' work.

Say: **Why is three-eighths less than three-fourths?** *(they have the same number of parts, but the parts in three-eighths are smaller because there are more parts in the whole)*

When fractions have the same numerator, the denominator will tell us which fraction is greater. The larger the denominator, the smaller the parts are because the whole has been divided more times.

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 112 and 113. Have students discuss the first problem with a partner.

Say: **Solve the first problem with a partner. After we discuss the answer, you will work on your own to solve the rest of the problems.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- What is the number of shaded parts? Which part of the fraction tells you?

- What is the size of the parts? What part of the fraction determines this?
- Which fraction is larger/smaller? Why?

Activity 2: Using ***Like Numerator Comparison Cards*** (6 per pair) and the ***Area Model Mat***, students will play a game comparing fractions with like numerators. Each partner will be assigned a number: 1 or 2. Partners will take turns drawing a card. On each card, a player will be assigned a fraction. The players will model each fraction on the Area Model mat, and then compare the fractions. Whoever has the largest (or smallest, as the teacher decides) fraction gets a point.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 18

Comparing Fractions with Like Numerators on the Number Line

Lesson Objectives	• The student will compare fractions with like numerators by placing them on a number line. • The student will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	compare, denominator, number line, numerator	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 239 – 250) • Fraction bars: 1 set of $\frac{1}{4}$ and $\frac{1}{8}$ • 2 different colored pencils or markers	• Student Booklet (pp. 117 – 122) • Fraction bars: 1 set of $\frac{1}{4}$ and $\frac{1}{8}$ (1 set per student) • Different colored pencils or markers (2 per student) • Number Line Mat (1 per pair)

Preview

Say: **Today we will compare fractions with like numerators by placing them on a number line.**

Engage Prior/Informal Knowledge Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice comparing fractions with area models.

Say: **Before we compare fractions on the number line, let's review numerators and denominators of fractions to see if we can make predictions about the size of the fractions.**

Read the problem. Ready, read: Ethan bought $\frac{2}{4}$ of a pound of raisins and $\frac{2}{8}$ of a pound of walnuts. Did he buy more or less raisins than walnuts?

What part of the fraction tells us how many equal parts are in the whole? *(the denominator)* **Which number in a fraction is the denominator?** *(the bottom number)*

How many equal parts are in the whole for two-fourths? *(4 equal parts)* **How many for two-eighths?** *(8)* **How does one-fourth of a pound compare to one-eighth of a pound?** *(one-fourth is larger than one-eighth)* **Why?** *(because there are fewer parts that the whole is divided into)*

What part of the fraction tells us how many parts are being described, or shaded? *(the numerator)* **What do the numerators of these two fractions tell us?** *(that 2 parts are described, or shaded)* **What can you predict about the size of these fractions, based on this information?** *(two-fourths is larger because both fractions have 2 shaded parts, but fourths are bigger than eighths)*

Modeled Practice — Time: 8 min

1. Compare fractions $\frac{2}{4}$ and $\frac{2}{8}$ by placing them on number lines, using fraction bars as a scaffold.

   Have students turn to the ***Modeled Practice Sheet***. While completing the directions, shade the two sections of the number line while students do the same.

Say: **Read the problem. Ready, read: At the grocery store, Javier bought two-fourths of a pound of blackberries, while Marco bought two-eighths of a pound of blackberries. Who bought more blackberries?**

To find out who bought more blackberries, we are going to compare the fractional amounts on a number line. How many total equal parts are in the whole for two-fourths? *(4 equal parts)* **The top number line is divided into fourths.**

Because the number line is divided into fourths, we can shade to show the length of two-fourths.

How many parts of the whole should be shaded to show the length of two-fourths? *(2 parts)* **Shade two parts on the number line.**

Label the fraction that represents the shaded part of the number line. Let's count the length: one-fourth, two-fourths. Write $\frac{2}{4}$ at the hash mark above the number line.

Model the second fraction on the bottom number line. Divide, shade, and label $\frac{2}{8}$ as directions are completed, having students complete following teacher modeling.

Say: **On the bottom number line, we will model the second fraction, two-eighths. How many total equal parts are in the whole?** *(8 equal parts)* **The bottom number line is divided into eighths.**

How many parts of the whole should be shaded to show two-eighths? *(2 parts)* **Shade two parts on the number line.**

Label the fraction that represents the shaded part of the number line. Let's count the length: one-eighth, two-eighths. Write $\frac{2}{8}$ at the hash mark above the number line.

Compare the two lengths on the number lines to compare the two fractions.

Say: **Compare the lengths on the number lines. Is two-fourths greater than or less than two-eighths?** *(greater than)*

Write ">" in the circle between the fractions while students do the same.

Say: **What can you say about the length of the parts in each fraction?** *(fourths are longer than eighths)* **How many parts were shaded for each fraction?** *(2)*

2 parts were shaded for each fraction, but fourths are longer than eighths. We used a number line to compare, which is why we compare the lengths. This story problem involved weight, so thinking about weight and length are similar in this scenario. Therefore, two-fourths is longer than two-eighths and two-fourths weighs more than two-eighths. Who bought more blackberries? *(Javier)*

2. Compare the fractions $\frac{3}{8}$ and $\frac{3}{6}$ by placing them on number lines. Continue to use the ***Modeled Practice Sheet***.

Say: **Read the problem. Ready, read: Javier needs to buy nails that are three-eighths of an inch long. The ones he bought are three-sixths of an inch long. Do the nails need to be shorter or longer than the ones he bought?**

How many total equal parts are in the whole for three-eighths? *(8 equal parts)* **The top number line is divided into eighths.**

On the top number line, model three-eighths. How many parts of the whole should be shaded to show the length of three-eighths? *(3 parts)* **Shade three parts above the number line.**

Label the fraction that represents the shaded part of the number line. Count the length: one-eighth, two-eighths, three-eighths. Write $\frac{3}{8}$ at the hash mark above the number line.

On the bottom number line, model the second fraction we are comparing, three-sixths. Are there more or fewer parts in the whole? *(fewer)*

Is the length of each part going to be larger or smaller? *(larger)* **Since we will shade the same number of parts, would you predict that three-sixths will be greater or less than three-eighths?** *(greater than)*

How many total equal parts are in the whole? *(6 equal parts)* **How many parts of the whole do we shade?** *(3 parts)* **Shade three parts on the number line.**

Label the fraction that represents the shaded part of the number line. Let's count the length: one-sixth, two-sixths, three-sixths. Write $\frac{3}{6}$ at the hash mark above the number line.

Compare the lengths of the two fractions to determine which is greater.

Say: **Compare the lengths on each number line. Is three-eighths greater than or less than three-sixths?** *(less than)*

Write "<" in the circle between the fractions while students do the same.

Say: **What can you say about the length of parts for each fraction?** *(eighths are smaller than sixths)* **How many parts were shaded for each fraction?** *(3)*

3 parts were shaded for each fraction, but eighths are smaller than sixths. Therefore, three-eighths is smaller than three-

sixths. Is the nail he bought longer or shorter than the one he needed? *(longer)*

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 119. Students will compare fractions by shading them on the number lines. The first problem can be discussed with a partner.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- Which fraction has larger parts?
- How many parts are shaded in the whole?
- Is the first fraction greater than or less than the second fraction?

Activity 2: Have students turn to the ***Practice Sheet*** on page 120. With a partner, students will compare fractions to find the answer to the riddle. Students circle the letter of the greater fraction in each pair. The circled letters will then be written in the blanks with the corresponding number at the bottom of the page. Encourage students to use the ***Number Line Mat*** when comparing fractions.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 19

Comparing Fractions with Like Numerators or Denominators

Lesson Objectives	• The student will compare fractions with like numerators or denominators using area models and number lines. • The student will apply and explain a variety of appropriate strategies to solve problems.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	area model, compare, denominator, number line, numerator	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 251 – 266) • 2 different colored pencils or markers	• Student Booklet (pp. 123 – 130) • Different colored pencils or markers (2 per student) • Fraction Comparison Cards (3 cards per pair) • Area Model or Number Line Mat (1 per pair)

Preview

Say: **Today we will compare fractions using different models.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice comparing fractions with like numerators on he number line.

Ask questions such as:

- How many parts are in the whole? *(answers vary)*
- How does the size of the parts in the whole compare for each fraction?
- How many parts are shaded?
- Is the first fraction greater than or less than the second fraction?

Modeled Practice — Time: 8 min

1. Compare the unit fractions $\frac{1}{2}$ and $\frac{1}{4}$. Students will discuss what they know about the size and number of the parts in each fraction, and then draw a model to confirm the comparison.

   Have students turn to ***Modeled Practice Sheet #1***. Students will shade to model the fractions following teacher directions.

Say: **Read the problem. Ready, read: Cristi ordered one-half of a pound of turkey and one-fourth of a pound of cheese. Did she order more turkey or cheese?**

Compare the fractions. How many parts will be in the whole for one-half? *(2)* **How many parts will be shaded?** *(1)*

How many parts will be in the whole for one-fourth? *(4)* **How many parts will be shaded?** *(1)*

Are halves larger or smaller than fourths? *(larger)* **How do you know?** *(there are fewer parts in the whole, so each part is larger)*

Divide and shade the models for each fraction. Label the models turkey and cheese.

Is one-half greater than or less than one-fourth? *(greater than)*

Write ">" in the circle between the fractions. Check students' work.

Say: **Did Cristi order more turkey or cheese?** *(turkey)*

Write "turkey" on the line to complete the sentence. Have students do the same.

Say: **In both examples the number of shaded parts is the same. What is different?** *(the size of each part)* **What can you say about halves compared to fourths?** *(halves are larger, so one-half is greater than one-fourth)*

2. Compare the fractions $\frac{3}{6}$ and $\frac{3}{4}$. Students will discuss what they know about the size and number of the parts in each fraction, and then draw a model to confirm the comparison.

Have students turn to ***Modeled Practice Sheet #2***. Students will shade to model the fractions following teacher directions.

Say: **Read the problem. Ready, read: Alan ate three-sixths of a cake. Raul ate three-fourths of a cake. Who ate the most cake?**

Compare the fractions. How many parts are in the whole for three-sixths? *(6)* **How many parts were eaten?** *(3)*

How many parts are in the whole for three-fourths? *(4)* **How many parts were eaten?** *(3)*

Both boys ate 3 parts of the cake. Did they eat the same amount of a cake? *(no)* **Why not?** *(the parts were different sizes)*

Alan ate sixths while Raul ate fourths. Are sixths larger or smaller than fourths? *(smaller)* **How do you know?** *(there are more parts in the whole, so each part is smaller)*

Divide and shade the models for each fraction. Label the models Alan and Raul.

Is three-sixths greater than or less than three-fourths? *(less than)*

Write "<" in the circle between the fractions. Check students' work.

Say: **Who ate the most cake?** *(Raul)*

Write "Raul" on the line to complete the sentence. Have students do the same.

Say: **What is the same in both fractions?** *(the number of shaded parts)* **How can we describe the difference in these parts?** *(the sizes of the parts are different)* **Sixths are smaller than fourths, so three-sixths is less than three-fourths.**

3. Compare the fractions $\frac{7}{8}$ and $\frac{3}{8}$. Students will discuss what they know about the size and number of the parts in each fraction, and then draw a model to confirm the comparison.

Have students turn to ***Modeled Practice Sheet #3***. Students will divide and shade the models following teacher directions.

Say: **Read the problem. Ready, read: The beetle is seven-eighths of an inch long, while the ant is three-eighths of an inch long. Which insect is smaller?**

Compare the fractions. How many parts are in the whole for seven-eighths? *(8)* **How many parts will be shaded?** *(7)*

How many parts are in the whole for three-eighths? *(8)* **How many parts will be shaded?** *(3)*

Divide and shade the models for each fraction. Label the models beetle and ant.

Because there are 8 parts in the whole inch for each measurement, the parts are the same size. We will compare the

number of parts. What part of the fraction tells the number of shaded parts? *(the numerator)*

Is seven-eighths greater than or less than three-eighths? *(greater than)*

Write ">" in the circle between the fractions.

Say: **Which insect is smaller?** *(the ant)*

Write "ant" on the line to complete the sentence. Have students do the same.

Say: **The same size of parts are being compared, eighths, but the number of parts is different. Seven-eighths is more than three-eighths.**

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheet*** on page 127. Students will compare pairs of fractions that have like numerators or denominators. The student will identify the type of comparison, and then reason about the size and number of the parts. Students will draw a model to support their conclusion.

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- Do the two fractions have the same number of parts in the whole?
- If no, is the size of the parts larger or smaller?
- If yes, how many parts are described by each fraction?
- Is the first fraction larger or smaller than the second?

Activity 2: Using the ***Fraction Comparison Card***s and ***Area Model or Number Line Mat***, Students will compare fractions in pairs. Partner 1 will choose a card and tell Partner 2 the first fraction on the card. Partner 1 will give clues that allow Partner 2 to guess the second fraction, using clues such as:

- The second fraction has the same numerator/denominator as the first.
- The first fraction has a numerator/denominator that is 2 more/less than the second fraction.

Once both fractions are identified, the partners will draw a model for comparison on the ***Area Model or Number Line Mat***. Partners then switch roles.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

25

Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Fraction Models
Lesson 20

Measurement with Fractional Quantities

Lesson Objectives	• The student will measure fractional lengths in inches. • The student will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	denominator, numerator, units	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 267 – 290) • 1 colored pencil or marker • Ruler	• Student Booklet (pp. 131 – 141) • Colored pencil or marker (1 per student) • Ruler (1 per student) • Small objects for students to measure (4 per pair)

Preview

Say: **Today we will use a ruler to measure and compare fractional lengths.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students complete the ***Engaged Practice Sheet*** to practice measuring objects with whole numbers.

Ask questions such as:

- Where does the length of the ruler start? *(at 0)*
- What is the length of the object? *(answers vary)*
- What do you think the marks in between the whole numbers represent? *(answers vary: fractions, etc.)*

Modeled Practice — Time: 8 min

1. Students will use a ruler to measure the length of the pencil in inches.

   Have students turn to ***Modeled Practice Sheet #1***. Model how to use and mark the length of objects on the ruler, and then line it up with the picture. Also model how to transfer the length from the actual ruler to the picture. Have students shade the length of the objects to the picture of the ruler.

Say: **We are going to measure the pencil in inches. Put your ruler along the pencil, with the left end at 0.**

Hold the ruler where the pencil eraser ends, and then move it next to the ruler picture. On the picture of the ruler, shade the length of the pencil.

Does the ruler look like another tool we have been using to model fractions? *(yes, a number line)*

A ruler has numbers just like a number line. Unlike the number lines we have used previously, this one doesn't just

show from 0 to 1. It shows several whole numbers. What are the whole numbers on this ruler? *(1, 2, 3, 4, 5, 6, 7)*

The length from 0 to 1 is a whole, but so is the length from 1 to 2 and from 2 to 3. How many whole numbers are shown on this ruler? *(7)*

The length between any 2 whole numbers can be broken up into fractional parts. Look at the end of the shaded area on the ruler. What 2 whole numbers is it between? *(3 and 4)*

Fill in the blanks under the ruler as students do the same. Continue to fill in the blanks after each of the following questions.

Say: **Looking between 3 and 4, how many equal parts are there on the ruler?** *(8)* **How many equal parts are between each set of whole numbers?** *(8)*

What fraction does an equal part between 3 and 4 represent? *(one-eighth)* **The ruler is divided into eighths. There are 8 equal parts between each pair of whole numbers.**

How many equal parts past 3 is the end of the shaded area? *(4)* **What fraction do these 4 equal parts represent?** *(four-eighths)*

What is the length of the pencil? Let's count the parts together: one, two, three, three and one-eighth, three and two-eighths, three and three-eighths, three and four-eighths. How long is the pencil? *(3 and four-eighths)* **What is the unit?** *(inches)*

We have learned about equivalent fractions in previous lessons. We want to see if there are any other fraction names for the length, four-eighths.

Draw arches from the hash mark at 3 to $3\frac{1}{4}$, from $3\frac{1}{4}$ to $3\frac{1}{2}$, from $3\frac{1}{2}$ to $3\frac{3}{4}$, and from $3\frac{3}{4}$ to 4.

Say: **Look at the sizes of the marks. If we ignore the smallest marks, how many equal parts are between 3 and 4?** *(4)* **How many equal parts are shaded?** *(2)* **What is the fraction?** *(two-fourths)*

Draw larger arches from the hash mark at 3 to $3\frac{1}{2}$ and from $3\frac{1}{2}$ to 4.

Say: **Now look at only the largest mark between 3 and 4. How many equal parts?** *(2)* **How many equal parts are shaded?** *(1)* **What is the fraction?** *(one-half)*

All 3 fractions are correct, but we usually read measurements using the fraction with the smallest denominator.

We would say the pencil is 3 and one-half inches long.

2. Students will use a ruler to measure the length of the scissors in inches.

Have students turn to ***Modeled Practice Sheet #2***. Model the positioning of the ruler. Shade the length of the scissors on the picture of the ruler as students do the same.

Say: **Now we are going to measure the length of the scissors in inches. Put your ruler along the scissors, with the left end at 0.**

On the picture of the ruler, shade the length of the scissors, using your finger to mark the length on the actual ruler.

Look at the end of the shaded area on the ruler. What 2 whole numbers is it between? *(5 and 6)*

Fill in the blanks under the ruler as students do the same. Continue to fill in the blanks after each of the following questions.

Say: **One equal part between 5 and 6 represents what fraction?** *(one-eighth)* **The ruler is divided into eighths.**

How many equal parts past 5 is the end of the shaded area? *(2)* **What fraction do these two equal parts represent?** *(two-eighths)*

What is the length of the scissors? Let's count the parts together: one, two, … five and one-eighth, five and two-

eighths. How long are the scissors? *(5 and two-eighths)* **What is the unit?** *(inches)*

We want to see if there are any other fraction names for the length, two-eighths.

Draw arches from the hash mark at 5 to $5\frac{1}{4}$, from $5\frac{1}{4}$ to $5\frac{1}{2}$, from $5\frac{1}{2}$ to $5\frac{3}{4}$, and from $5\frac{3}{4}$ to 6.

Say: **Look at the sizes of the marks. If we ignore the smallest marks, how many equal parts are between 5 and 6?** *(4)* **How many equal parts are shaded?** *(1)* **What is the fraction?** *(one-fourth)*

The scissors are 5 and one-fourth inches long.

3. Students will critique a fictional student's measurement for accuracy, and then provide the correct measurement.

Have students turn to ***Modeled Practice Sheet #3***.

Say: **Andrea measured this pen and says it is 5 and three-fourths inches long. Is she correct?** *(no)* **How do you know?** *(because the ruler starts at 1 instead of 0)*

When we use a ruler, the end of the object must start at 0. Use your ruler to measure the pen. Let's measure the pen.

Give students a chance to measure the pen.

Say: **What is the length?** *(4 and three-fourths)*

Since Andrea held the ruler incorrectly, her measurement was an inch longer than it should have been. The actual measurement is 5 and three-fourths inches.

Practice Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 135 and 136. Students will practice measuring the length or width of various objects. The measurements will be in fourths.

Ask questions such as:

- How do you line up the ruler with your object? Where does the beginning of the object start?
- What is the length/width of the object?
- How does the length of this object compare to the length of the other objects you measured?

Activity 2: Have students turn to the ***Practice Sheet*** on page 137. Students will measure objects in the classroom to the nearest eighth of an inch. Have students work with a partner to record the name of the object and its length or width.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.