ESTAR
INTERVENTION

Tier 2 Mathematics Intervention

Module: *Multiplication & Division Fact Strategies (MDFS)*

Teacher Lesson Booklet

The Meadows Center for Preventing Educational Risk—Mathematics Institute
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The Meadows Center

FOR PREVENTING EDUCATIONAL RISK

THE UNIVERSITY OF TEXAS AT AUSTIN

COLLEGE OF EDUCATION

Mathematics Institute for Learning Disabilities and Difficulties

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Fact Strategies
Lesson 1

Multiplication as Equal Grouping

Lesson Objectives	• The student will write multiplication equations for equal-groups models, relating repeated addition to multiplication. • The student will apply mathematical vocabulary to solve multiplication problems using equal-groups models.	
Vocabulary	**equal-groups model:** a diagram that shows a number or set of objects separated into equal parts **repeated addition:** an addition situation in which all groups have the same number of objects **multiplication:** an operation that gives the total number when you put equal groups together **factor:** the numbers multiplied together to find a product [for example: 3 (factor) × 6 (factor) = 18] **product**: the answer to a multiplication problem [for example: 3 × 6 = 18 (product)]	
Reviewed Vocabulary	addition, equation	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 1-8) • Whiteboard with marker • Number line • Counters (15)	• Student Booklet (pp. 1-4) • Whiteboard with marker (1 per student) • Counters (15 per student)

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Preview

Say: **Today we learn about *multiplication* and its relationship to addition.**

Engage Prior/Informal Knowledge — Time: 3 min

Practice skip counting (repeated addition). Write the problems below on the whiteboard. Have a **number line** available for a visual reference.

Say: **I have written some addition problems. The problems can be solved quickly by skip counting. What number is repeated in each problem?** *(5, 2, 10)* **This tells us how to skip count.**

5 + 5 + 5 2 + 2 + 2 + 2 10 + 10 + 10 + 10 + 10 + 10

Have students skip count together as a group to find the total for each addition problem.

Modeled Practice — Time: 8 min

1. Show the relationship of repeated addition to multiplication through equal-groups models.

   Distribute 12 counters and a whiteboard to each student. Have students write out the addition equation, equal-groups equation, and multiplication equation on the whiteboard as each is discussed in the lesson. Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

   Say: **Put your counters into 3 equal groups of 4 on top of the whiteboard. Equal means the same. What does equal mean?** *(the same)* **How many counters are in each equal group?** *(4)* **How many total groups?** *(3)* **Circle each group.**

   This model is called an *equal-groups model*. An *equal-groups model* is a diagram that separates objects into equal parts or groups. What is the name of the model we just made? *(equal-groups model)*

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What does this model show? *(3 groups of 4 counters)* **How many counters are on the board in all?** *(12)* **How do you know?** *(acceptable answers include counting the counters or adding)*

Write the equal-groups equation "3 groups of 4" on your whiteboard.

How many equal groups? *(3)* **How many in each group?** *(4)* **We can write an addition equation for this *equal-groups model*: 4 + 4 + 4 = 12. This addition equation is an example of *repeated addition* because we add the same number, 4, 3 times.**

Write the *repeated addition* equation on your whiteboard.

4 + 4 + 4 is the same as 3 groups of 4.

Instead of writing a long addition equation, 4 + 4 + 4 = 12, we can write 3 groups of 4 equals 12 as a *multiplication* equation, 3 × 4 = 12. Write the *multiplication* equation on your whiteboard.

***Multiplication* is the combining of equal groups. Are the groups equal?** *(yes)* **How many are in each group?** *(4)*

What does 3 × 4 equal? *(12)*

Explain why 4 + 4 + 4 is the same as 3 × 4. *(accept reasonable answers that include discussion about equal groups)*

The numbers 3 and 4 are called the *factors*. *Factors* are the numbers multiplied together to find the answer.

What are *factors*? *(numbers multiplied together)*

A *product* is the whole amount, or the answer, to a *multiplication* problem. What is a *product*? (*the answer to a multiplication problem)* **What is the *product* of 3 × 4?** *(12)*

Using the same 12 counters, make 4 equal groups of 3 on your whiteboard. Circle the fourth group.

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How many are in each group? *(3)* **How many total equal groups?** *(4)* **How many in all?** *(12)*

Write the equal-groups equation "4 groups of 3."

We did not change the total amount of counters, but we did change the groupings. What is the *repeated addition* equation for this new *equal-groups model*? *(3 + 3 + 3 + 3 = 12)* **Write it.**

What is *multiplication*? *(the combining of equal groups)* **When we made 3 groups of 4, the *multiplication* equation was 3 × 4 = 12. What do you think is the corresponding *multiplication* equation for this *equal-groups model*?** *(4 × 3 = 12)* **Write it.**

In *multiplication* you can change the order of the *factors* without changing the total amount. What were the 2 *multiplication* equations using the *factors* 3 and 4? *(3 × 4 = 12 and 4 × 3 = 12)*

What is the *product* in both *multiplication* equations? *(12)*

2. Show a non-example of multiplication using an unequal group.

Have students clear their whiteboards. Give each student 1 additional counter. Have students complete the steps as the lesson progresses.

Say: **Put the counters into 2 equal groups of 4 and 1 group of 5.**

What does equal mean? *(the same as)* **How many are in each group now?** *(4 and 5)*

What is the addition equation for 2 groups of 4 and 1 group of 5? *(4 + 4 + 5 = 13)*

Is this a *repeated addition* equation? *(no)* **Why?** *(there is not the same amount in each group)*

Is 4 + 4 + 5 an *equal-groups model*? *(no)* **Why not?** *(one group has more; they are not all equal)*

***Multiplication* is the combining of equal groups. If the groups are not all equal, then *multiplication* cannot be used.**

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Can you write 4 + 4 + 5 as a *multiplication* equation? *(no)* **Why not?** *(all the groups are not equal or the same)*

3. Students will draw an equal-groups model and write an equal-groups equation and multiplication equation.

   Have students clear their whiteboards. On the teacher's whiteboard write "3 groups of 4." Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

Say: **We want to model 3 equal groups of 4 using a drawing. First, draw 3 circles on your whiteboard to show the 3 groups.**

Now draw 4 dots in the first 2 circles.

We have 3 total groups with 4 dots in 2 groups and 0 dots in 1 group.

How would you describe this model? Is it a picture of 3 equal groups of 4? *(no, it is 2 groups of 4)* **Why not?** *(it only has 2 groups of 4, not 3 groups of 4)*

Draw 4 dots in the third circle.

Looking at the 3 groups now, is this a correct drawing of an *equal-groups model*? *(yes)* **How do you know?** *(all the groups have the same or equal amount)*

Now draw 1 more circle with 4 dots in it.

What is the equal-groups equation for the model we just drew? *(4 groups of 4)*

How many groups of 4 are there? *(4)* **How many in all?** *(16)*

What is the *repeated addition* equation for this *equal-groups model*? *(4 + 4 + 4 + 4 = 16)* **Write it.**

What *multiplication* equation can we write for this *equal-groups model*? *(4 × 4 = 16)* **Write it.**

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The answer to the *multiplication* equation is called what? *(the product)* **What number is the *product?*** *(16)*

4. Students will write an equal-groups equation and multiplication equation using a drawing of an equal-groups model using numerals.

   Clear the whiteboard. Write the number "4," 5 times, circling each. Write out the addition equation, equal-groups equation, and multiplication equation on the whiteboard as each are discussed in the lesson.

   Say: **Look at my drawing. What is the equal-groups equation for my drawing?** *(5 groups of 4)*

   How many groups of 4 does this model show? *(5)*

   How is this model different from the other 2 models we just saw? *(there is a number in each group, not dots or counters)*

   What is the *repeated addition* equation for this model? *(4 + 4 + 4 + 4 + 4 = 20)* **How do you know?** *(there are 5 groups of 4, so to find the total we would add 4 five times)*

   What is the *multiplication* equation for this model? *(5 × 4 = 20)*

   Which numbers are the *factors* in the equation? *(5 and 4)*

   What is the corresponding *multiplication* equation using the *factors* 5 and 4? *(4 × 5 = 20)*

   What is the *product*? *(20)*

Practice Time: 8 min

Activity 1: Using counters and drawings, students will make equal-groups models and write the addition and multiplication equations. Distribute 15 counters to each student. Work along with the students at first, then fade teacher assistance.

Have students use the counters to make groups of 3 on top of their whiteboards, circling each group. Check each student's work. Have a

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student volunteer write the addition and multiplication equations on the whiteboard and discuss the equations with the group.

Ask similar questions from the lesson such as:

- How many equal groups of 3 did you make? *(5)*
- How could we find the total number of counters? *(accept reasonable answers such as skip count or repeated addition; use several strategies suggested by students to find the total)*
- What is the *repeated addition* equation for the *equal-groups model*? *(3 + 3 + 3 + 3 + 3 = 15)*
- What is another way to make equal groups using the same 15 counters? *(3 equal groups of 5)*
- What is the *multiplication* equation for the *equal-groups model*? *(3 × 5 = 15)*
- What is the corresponding *multiplication* equation using the *factors* 3, 5, and the *product* 15? *(5 × 3 = 15)*

Have students clear their whiteboards and write the number "2," 5 times on their board, drawing a circle around each number. Use skip counting to illustrate what is happening in the model.

Ask similar questions from the lesson such as:

- How would we find the total for this *equal-groups model*? *(skip count or add the numbers together)*
- How many groups are there? *(5)*
- How many are in each group? *(2)*
- What is the *multiplication* equation for this *equal-groups model*? *(5 × 2 = 10)*
- Is there another way to write this *multiplication* equation? *(yes)* What is the corresponding *multiplication* equation? *(2 x 5 = 10)*

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Activity 2: Have students turn to the ***Practice Sheets*** on pages 1 and 2. Students will continue to practice with equal-groups models on their activity sheets with a math partner. Discuss answers as a group. Address any errors at this time.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module: Multiplication & Division Fact Strategies
Lesson 2

Multiplication Through Arrays

Lesson Objectives	• The student will use arrays of objects to model multiplication. • The student will model factors and products using arrays. • The student will apply mathematical vocabulary to solve multiplication problems using array models.	
Vocabulary	**array:** a rectangle divided into the same number of items in each row and the same number of items in each column **Commutative Property of Multiplication:** a property of multiplication (but not of division) that states changing the order of the factors will not change the product (for example, $5 \times 8 = 40$ and $8 \times 5 = 40$) **row:** items arranged in a horizontal line **column:** items arranged in a vertical line	
Reviewed Vocabulary	equal-groups model, equation, factor, multiplication, product, repeated addition, sum	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 9-16) • Counters (18) • Whiteboard with marker	• Student Booklet (pp. 5-8) • Counters (24 per student) • Whiteboard with marker (1 per student)

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Preview

Say: **Today we will use *arrays* as a model for multiplication. We will apply the *Commutative Property of Multiplication*.**

Engage Prior/Informal Knowledge — Time: 3 min

Review and discuss the Commutative Property of Addition. Students will make predictions whether the Commutative Property applies to multiplication. Students will write answers on their whiteboards and share their answers with the group.

Write "5 + 8 = ?" and "8 + 5 = ?" on the whiteboard.

Ask questions and give directions such as:

- What is 8 + 5? *(13)* Then what is 5 + 8? *(13)* Are the sums the same? *(yes)*
- How do both facts equal the same amount? *(accept answers that relate to number families and examples of the Commutative Property)*

Teacher Note
Students do not have to name the Commutative Property, but apply it correctly and understand it.

- Think about 12 – 5 and 5 – 12. Can we switch the order of the numbers in a subtraction problem and get the same answer? *(no)*
- Why not? *(accept reasonable answers that include: in subtraction, the first number is the total, or the whole, and the second number, or part, is subtracting from the total; 5 – 12 does not equal 7; the numbers are not the same in this number family; 12 is more than 5 so you would be subtracting more than what you started with)*
- If we switch or change the order of the numbers in an addition problem, will the sum be the same? *(yes)* Write an example on your whiteboard. *(answers will vary)*

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- If we switch or change the order of the factors in a multiplication problem, will the product be the same? *(yes)* Write an example on your whiteboard. *(answers will vary)*

- How is the ability to change the order of the parts in addition similar to factors in multiplication? *(you can change the order of the factors, or switch the total groups with the amount in each group)*

Modeled Practice — Time: 8 min

1. Model an array with counters.

   Distribute 18 counters to each student or student pair. Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

Say: **Arrange the counters into 3 equal groups on top of the whiteboard.**

What is the name of the model we just made? *(equal-groups model)*

How many are in each group? *(6)*

Today we will create another model of multiplication. Use the counters from your equal-groups model and arrange the 3 groups into straight lines with 6 counters in each line.

Model a 3-row by 6-column array on top of the whiteboard while students work.

Say: **This is an *array*. An *array* is another way to model multiplication. What is this model of multiplication called?** *(an array)*

An *array* has an equal number of items in each *row* and an equal number of items in each *column*.

A *row* runs across the page. Run your finger across the *row*.

Count the number of *rows* in this *array*. Ready, count: 1, 2, 3.

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A *column* runs up and down the page. Run your finger up and down the *column*.

Count the number of *columns* in this *array*. Ready, count: 1, 2, 3, … 6.

This is a 3-by-6 *array*.

2. Write a repeated addition equation for the array to show the relationship between addition and multiplication. Allow students time to complete the steps.

Say: **How many *rows* in this *array*?** *(3)* **Circle each *row*.**

Instead of groups, we now have *rows*.

How many counters are in 1 *row*? *(6)*

There are 3 *rows* of 6 counters.

How do we find the total number of counters? *(accept reasonable answers that use addition or multiplication)*

How many counters in all? *(18)*

Write 3 *rows* of 6 on your whiteboard.

What is the repeated addition equation for this *array*? *(6 + 6 + 6 = 18)*

How do you know? *(the 3 rows of 6 are like having 3 equal groups of 6)*

Write the repeated addition equation on your whiteboard.

Repeated addition is 1 way to combine equal groups.

An *array* is like an equal-groups model, but now the objects are organized into *rows* and *columns*.

What is the multiplication equation for 3 *rows* of 6 equals 18? *(3 × 6 = 18)* **Write the multiplication equation on your whiteboard.**

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Leave the 3 by 6 *array* on your whiteboard and erase the circles. Then turn your whiteboard.

This is another *array*. Are there still 18 counters? *(yes)*

How many *rows,* going across, are there? *(6)*

How many *columns,* going up and down, are there? *(3)*

What is the repeated addition equation for this *array*? *(3 + 3 + 3 + 3 + 3 + 3 = 18)* **Write it on your whiteboard.**

What is the multiplication equation for 6 *rows* of 3 equals 18? *(6 × 3 = 18)* **Write it.**

What numbers are the factors? *(6 and 3)* **What is 18 called?** *(the product)*

If 3 × 6 = 18 and 6 × 3 = 18, does 3 × 6 = 6 × 3? *(yes)*

Write out "3 × 6 = 6 × 3" on the whiteboard.

Say: **Think back to the beginning of the lesson. What did we notice about 5 + 8 and 8 + 5?** *(they both equal 13)*

3 × 6 and 6 × 3 are the same because both equal 18.

In mathematics, the *Commutative Property of Multiplication* says that factors can switch and it does not change the product. Just like in addition, changing the order of the numbers does not change the sum.

3. Students will draw an array to show the Commutative Property of Multiplication.

Have students clear their whiteboards.

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Teacher Note
Students should start the array by drawing 3 dots in a column, then draw another dot in each column until 12 dots are drawn. If needed, use the counters as a visual to help students draw and organize their array. Place 3 counters in a column, then add a counter to each column until all counters are used.

Say: **On your whiteboard draw an *array* that has a total of 12 dots in 3 *rows*. How many *rows*?** *(3)*

Check students' work.

Say: **How many dots are in each *row*?** *(4)*

How many total dots are there? *(12)*

What is the multiplication equation for the *array*? *(3 × 4 = 12)* **Write it.**

Now turn your whiteboard. How many *rows* are there? *(4)*

How many dots in each *row*? *(3)*

How many total dots? *(12)*

What is the multiplication equation for the *array*? *(4 × 3 = 12)* **Write it.**

What is the product for both *arrays*? *(12)*

The product is the same when we switched or changed the order of the factors.

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Note to Teacher

While this lesson teaches rows and columns, the order of the factors can be row × column or column × row. It is more important for the students to understand that the order of the factors can be switched and still result in the same product.

Practice — Time: 8 min

Activity 1: Students will arrange counters in an array and then draw the array. Have students turn to the ***Practice Sheet*** on page 5.

Say: **Read the problem. Ready, read: "There are 24 desks in a classroom. Your teacher wants to arrange them in equal *rows*. What are some ways to arrange the desks so that the 24 desks are in equal *rows* and *columns*?"**

Ask questions such as:

- What is the problem asking? *(different ways to arrange 24 desks in equal rows and columns)*
- How many total desks are there? *(24)*
- What are different ways to arrange the desks? *(4 rows of 6; 6 rows of 4; 3 rows of 8; 8 rows of 3; 12 rows of 2; 2 rows of 12; 1 row of 24 24 rows of 1)*
- What are the multiplication equations to match the different *arrays* of desks? *(4 × 6; 6 × 4; 3 × 8; 8 × 3; 12 × 2; 2 × 12; 1 × 24; 24 × 1)*

Activity 2: Students will work in pairs to complete multiplication problems using arrays. Have students turn to the ***Practice Sheet*** on page 6. Work along with the students at first, then fade teacher assistance.

Ask questions or provide instructions for problem 1, such as:

- Complete both arrays so that there are 3 *rows* of 5 dots.

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- With your pencil circle each *row*. How many *rows* are there in the *array*? *(3)* What is another way to describe the *rows*? *(rows are like the number of groups)*
- How many dots are in each *row*? *(5)*
- Circle each *column*. How many *columns* are there in the *array*? *(5)*
- What sentence will you write below the first *array*? *(3 rows of 5)*
- What sentence will you write below the other *array*? *(5 columns of 3)*
- Without counting every dot, how can you find the total number of dots in this *array*? *(count by 5s because there are 5 in each row; accept other reasonable answers that have to do with skip counting, addition, or multiplication)*
- What is the multiplication equation for 3 *rows* of 5? *(3 × 5 = 15)*
- What is the corresponding multiplication equation using the factors 3 and 5? *(5 × 3 = 15)*
- If you added a fourth *row* of 5 dots, what is the multiplication equation? *(4 × 5 = 20 or 5 × 4 = 20)*

Work with students to identify rows and columns. Have students complete problems 2 and 3 with a partner.

Connect to the Commutative Property of Multiplication with questions and observation of students' work. Allow students to elaborate on the connection of $4 \times 10 = 10 \times 4$. Discuss students' answers for problems 2 and 3.

Say: **What did you write in the last blank for problem 2?** *(4 × 10 = 10 × 4)*

How do you know that these two multiplication equations are equal? *(the products, or answers, are the same)*

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Note to Teacher
If students struggle to explain, provide a prompt: "the Commutative Property states that factors can change, but what stays the same?" Have students who may need additional scaffolds build or draw an array.

Explain your strategy for solving problem 3. *(reasonable answers may include drawing the array row by row, and then counting the number of columns)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 3

Multiplication Through Area Models

Lesson Objectives	• The student will use area models to represent and solve multiplication problems. • The student will apply mathematical vocabulary to solve multiplication problems using area models.	
Vocabulary	**area model:** a model for multiplication using square units	
Reviewed Vocabulary	array, column, Commutative Property of Multiplication, equal-groups model, equation, factor, multiplication, product, repeated addition, row, square units	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 17-24) • Graph paper • Counting cubes (optional)	• Student Booklet (pp. 9-12) • Whiteboard with marker (1 per student) • Graph paper (2 sheets per student)

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Preview

Say: **Today we will be using *area models* to represent and solve multiplication problems another way.**

Engage Prior/Informal Knowledge — Time: 3 min

Review equal-groups models and arrays. Have students draw 4 groups of 2 on their whiteboards.

Ask questions and provide instructions such as:

- How did you choose to represent 4 groups of 2? *(accept reasonable answers that include drawing 4 groups of 2 objects, writing the number 2 four times, or drawing an array)*
- What does 4 groups of 2 equal? *(8)*
- How did you get your answer? *(accept reasonable answers that include skip counting, repeated addition, or multiplication)*
- For the array model ask, "Does your array have an equal number of objects in each row?" *(yes)*
- For the equal-groups model ask, "How many objects did you draw in each circle?" *(2)*
- Explain how the array model is similar to the equal-groups model. *(accept reasonable answers such as equal number of objects or objects are in equal groups)*
- Write the multiplication equation for the equal-groups model or array. *($4 \times 2 = 8$)*
- Write the corresponding multiplication equation using the factors 4 and 2 and the product 8. *($2 \times 4 = 8$)*
- What numbers are the factors? *(2 and 4)* What is the product? *(8)*

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Modeled Practice — Time: 8 min

1. Show multiplication with an area model using graph paper. Demonstrate on graph paper while students observe.

   On graph paper, draw a rectangle with 4 rows of 5 square units. Label the dimensions of the rectangle.

   Say: ***Area models*** **are similar to arrays because you show the group of items in rows and columns. The difference between an array and an** ***area model*** **is that the rows and columns are pushed tightly together, forming a square or a rectangle made up of square units.**

   Count the square units in the column to find the width of the rectangle. Ready, count: 1, 2, 3, 4. How many? *(4)*

   Count the square units in the row to find the length of the rectangle. Ready, count: 1, 2, 3, 4, 5. How many? *(5)*

   The width and the length tell us how many rows and columns are in the rectangle.

   Are there an equal number of square units in each row? *(yes)*

   Are there an equal number of square units in each column? *(yes)*

   In an area model, there are an equal number of square units in each row and an equal number of square units in each column.

   Circle each row to identify 4 groups of 5.

   Say: **Because there are equal-size groups we say this** ***area model*** **represents 4 groups of 5.**

   With the ***area model*****, how can you find the total number of square units in 4 groups of 5?** *(skip count, repeated addition, or multiplication)*

   What is the total number of square units? *(20)*

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We have seen equal groups before with equal-groups models and arrays. How do you write 4 groups of 5 as a repeated addition equation? *(5 + 5 + 5 + 5 = 20)*

Write the repeated addition equation below the area model.

Say: **How do you write 4 groups of 5 as a multiplication equation?** *(4 × 5 = 20)*

Write "20 = 4 × 5."

Say: **What are the 2 factors in the multiplication equation?** *(4 and 5).*

In the multiplication equation 20 = 4 × 5, which number is the product? *(20)*

On the graph paper draw a rectangle with 5 rows of 4 square units. Label the dimensions of the rectangle.

Say: **Compare this *area model* to the first. What is the same, and what is different?** *(each model has the same number of square units inside, 20; one has 4 square units across and 5 square units down, while the other has 5 square units across and 4 square units down)*

Counting the rows, what does this *area model* represent? *(5 groups of 4)*

What is the multiplication equation for 5 groups of 4? *(5 × 4 = 20)*

The Commutative Property of Multiplication says we can switch or change the order of the factors but the product stays the same.

How do these 2 *area models* prove the Commutative Property of Multiplication is true? *(accept explanations that involve the factors of 4 and 5 switching positions but the product or area of the rectangle stays the same)*

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2. Students will draw a rectangle with 6 rows of 5 square units on the graph paper. Review the connection between repeated addition and multiplication. Distribute graph paper to students. Allow students time to complete the steps.

Say: **Draw a rectangle with 6 rows and 5 square units in each row on your graph paper.**

Check students' work.

Say: **Instead of counting all the square units to find the area, can we skip count?** *(yes)* **What number should we skip count by and why?** *(5s, because there are 5 in each row.)*

If students suggested counting by 6, ask if there is another way because counting by 6 can be difficult.

Say: **Circle each row of 5 to see the equal groups.**

Let's skip count together to find the total square units in this *area model*. Ready, count: 5, 10, 15 … 30.

Skip counting is similar to repeated addition. What is the repeated addition equation for this *area model*? *(5 + 5 + 5 + 5 + 5 + 5 = 30)* **Write it under your model.**

That is a lot to write. Multiplication is a short way to solve the same problem. What is the multiplication equation for this *area model*? *(6 × 5 = 30)* **Write it under the addition equation.**

The Commutative Property of Multiplication says we can change the order of the factors but the product will stay the same.

We can see this property when we rotate or turn the paper. Is the rectangle still made up of 30 square units? *(yes)*

What is the multiplication equation for this rectangle now? *(5 × 6 = 30)* **Write it under the first multiplication equation.**

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By turning this paper, what do we know about multiplication equations? *(the order of the factors does not change the product)*

What are the 2 multiplication equations shown in this *area model*? *(6 × 5 = 30 and 5 × 6 = 30)*

We can write 5 × 6 = 30 and 6 × 5 = 30. I can also write 6 × 5 = 5 × 6. Are they equal? *(yes)* **Why?** *(they have the same product)*

Can I write 5 × 6 = 5 × 4? *(no)* **Why?** *(the products are different or do not equal the same)*

3. Provide a non-example using 4 rows instead of 5 first. Then have students explore and draw other area models with a total of 30 square units on their own using the graph paper from the previous problem.

Say: **Let's draw another *area model* with 30 as the product.**

Our last model had 5 rows. Let's try drawing one with 4 rows.

Demonstrate on graph paper while students observe. Draw an area model with 30 square units in 4 rows of 7, giving the last row 2 extra square units as a non-example.

Say: **Do all the rows have an equal number of square units?** *(no)*

Is this an *area model*? *(no)* **Why not?** *(all the rows are not equal)*

Cross out this area model by putting an "X" over it.

Say: **What other factor should we try to put into equal rows?** *(looking for 3, 2, or 1 as a suggested factor)*

Draw the *area model* using the factor you would like to try.

Does your *area model* have a total of 30 square units? *(yes)*

What are the factors in the new *area model*? *(answers may vary; 3 and 10; 2 and 15; 1 and 30)*

Write the multiplication equation. *(answers may vary; 3 × 10 = 30; 2 × 15 = 30; 1 × 30 = 30)*

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Write the corresponding multiplication equation below it.
(answers may vary; 10 × 3 = 30; 15 × 2 = 30; 30 × 1 = 30)

Practice Time: 8 min

Activity 1: Guide students to draw area models on the graph paper. Have students use the same graph paper or distribute a clean sheet to each student.

Have students draw a rectangle with 3 groups of 6. Next, have students circle each row, writing the repeated addition equation for the area model.

Say: **Find the total number of square units in the rectangle. What are ways to find the total number of square units of the rectangle?** *(multiply 3 × 6; skip count by 3s or 6s; repeated addition)*

What is the total number of square units of this rectangle? *(18)*

Write the multiplication equation for this *area model.* *(3 × 6 = 18)*

Thinking about the Commutative Property of Multiplication, what is the corresponding multiplication equation for the *area model*? *(6 × 3 = 18)*

Watch For

Students may struggle using graph paper as a mathematics manipulative.
For example, students may have trouble drawing a rectangle on graph paper.
Direct students to use unit cubes to create an array before drawing an area model on the graph paper.

Activity 2: Assign each partner group a number (16, 24, 15, 20). Have students work together to draw an area model on graph paper for their number. Have students identify the number of rows and number of square units in each row. Have students write the addition and multiplication

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equations for the model before sharing their area models. Lead the discussion to aid connections between repeated addition and multiplication.

Teacher Note

Optional alternatives to this assignment could include:

- Use cubes instead of graph paper.
- Supply students with one dimension of the rectangle and the total. Allow students to draw to find the other dimension.
- Work on the same number together as a whole group.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 4

Models of Multiplication Review

Lesson Objectives	• The student will review the use of equal-groups models, arrays, and area models to represent and solve multiplication problems. • The student will apply mathematical vocabulary to solve multiplication problems in various models.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	area model, array, column, Commutative Property of Multiplication, equal-groups model, equation, factor, multiplication, product, repeated addition, row, square units	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 25-38) • Whiteboard with marker • Graph paper	• Student Booklet (pp. 13-19) • Whiteboard with marker (1 per student) • Graph paper (1 sheet per student)

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Preview

Say: **Today we will review the different ways to solve multiplication problems using 3 different representations and models.**

Engage Prior/Informal Knowledge — Time: 3 min

Review vocabulary terms related to multiplication. List the words "multiplication," "factor," and "product" on the whiteboard with space for examples after each word. Allow 1 to 2 students to share their explanation for each word. Write their explanations on the whiteboard.

Ask questions such as:

- What is multiplication? *(multiplication is the operation of putting equal groups together)*
- What does factor mean? *(factors are the numbers multiplied together to find the product)*
- What does product mean? *(product is the answer to a multiplication problem)*
- What does the word equal mean? *(the same amount or the same as)*

Watch For

Students sometimes struggle with the meaning of equal.
Refer to equal as a balance. Guide students away from describing equal as the end result.
For example, say, "If I had 4 slices of cheese and you had 4 slices of cheese, we would have an equal amount of cheese. We would have the same amount."

Remind students that we have used equal-groups models, arrays, and area models to represent multiplication problems. Have students write the answers on their whiteboards.

- Draw an example of one of the multiplication models we have used. *(answers will vary)*

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- Write the multiplication equation for the equal-groups model, array, or area model you drew. *(answers will vary)*

Modeled Practice — Time: 8 min

1. Students will review drawing an equal-groups model and writing an equal-groups sentence, a repeated addition equation, and a multiplication equation.

   Distribute whiteboards and marker to each student. Allow students time to complete the steps.

   Say: **We are going to model 7 equal groups of 5 using a drawing.**

   How many circles do we draw? *(7)* **How do you know?** *(7 equal groups)*

   Draw 7 circles on your whiteboard to show the 7 groups.

   How many dots do we draw in each group? *(5)*

   Draw 5 dots in the first group and 5 dots in the second group.

   Instead of drawing 5 dots in each group, what number can we write in each group? *(5)*

   Write the number 5 in the remaining groups.

   What is the equal-groups sentence for this model? *(7 groups of 5)* **Write it.**

   What can we skip count by to find how many in all? *(skip count by 5s)* **Why?** *(each group has 5)*

   Count by 5s. Ready, count: 5, 10, 15 … 35. How many in all? *(35)*

   What is the repeated addition equation for this equal-groups model? *(5 + 5 + 5 + 5 + 5 + 5 + 5 = 35)* **Write it.**

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What multiplication equation can we write for this equal-groups model? *(7 × 5 = 35)* **Write it.**

What are the factors in this multiplication equation? *(7 and 5)*

What is the corresponding multiplication equation that can be written using 7 and 5 as factors? *(5 × 7 = 35)* **Write it.**

What is the answer to a multiplication problem called? *(the product)*

Which number is the product? *(35)*

2. Students will review drawing an array and writing a repeated addition equation to show the relationship between addition and multiplication.

   Have students clear their whiteboards. Allow students time to complete the steps.

Say: **Next we are going to draw an array. An array is like an equal-groups model, but now the objects are organized into rows and columns.**

Does a row run across the page or up and down? *(across)*

What about a column? *(up and down)*

How is an array similar to an equal-groups model? *(same number of objects in each row and the same number of objects in each column)*

On your whiteboard, draw an array that has a total of 20 dots in 2 rows.

How many dots are in each row? *(10)* **How did you figure that out?** *(accept reasonable answers such as: divided 20 by 2; knew that half of 20 is 10)*

What is the repeated addition equation for this array? *(10 + 10 = 20)* **Write it.**

How do you know? *(the 2 rows of 10 are like having 2 equal groups of 10)*

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What is the multiplication equation for this array? *(2 × 10 = 20)* **Write it.**

Leave the 2-by-10 array on your whiteboard and turn your whiteboard sideways.

This is another array. Are there still 20 dots? *(yes)*

How many rows are there now? *(10)*

How many columns? *(2)*

What is the repeated addition equation for this array? *(2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20)* **Write it on your whiteboard.**

What is the multiplication equation for 10 rows of 2 equals 20? *(10 × 2 = 20)* **Write it.**

While students are working, write out "2 × 10 = 10 × 2" on the whiteboard.

Say: **If 2 × 10 = 20 and 10 × 2 = 20, does 2 × 10 = 10 × 2?** *(yes)*

2 × 10 and 10 × 2 are the same because both equal 20.

In mathematics, the Commutative Property of Multiplication says the factors can switch and it does not change the product.

3. Students will review area models using graph paper. Review the connection between repeated addition and multiplication.

Put aside the whiteboards. Distribute graph paper to students. Allow students time to complete the steps.

Say: **Area models are similar to arrays because you show the group of items in rows and columns.**

What is the difference between an array and an area model? *(in an area model, the rows and columns are tightly together; an area model has squares or is a rectangle made up of square units)*

Draw a rectangle that has 8 rows, with 5 square units in each row, on the graph paper.

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Check students' work.

Say: **How many equal-size groups of 5 are there?** *(8)*

Skip count by 5s together to find the total square units in this area model. Circle the rows of 5 as you count. Ready, count: 5, 10, 15 … 40.

How do you write 8 groups of 5 as a repeated addition equation? *(5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40)* **Write it below the area model.**

How do you write 8 groups of 5 as a multiplication equation? *(8 × 5 = 40)* **Write it below the repeated addition equation.**

What are the factors in this multiplication equation? *(8 and 5)*

Rotate your paper. Is the rectangle still made up of 40 square units? *(yes)*

What is the multiplication equation for this rectangle now? *(5 × 8 = 40)* **Write it under the first multiplication equation.**

What are the two multiplication equations for this area model? *(8 × 5 = 40 and 5 × 8 = 40)*

Practice — Time: 8 min

Activity 1: Have students turn to the ***Practice Sheets*** on pages 13, 14, and 15. The students will work with a partner to complete this activity. Assign each pair a different model to represent the multiplication problem 6 × 3.

Note to Teacher

For the area model: Instruct students how to draw 6 rows in the box. First, draw a horizontal line cutting the box in half, and draw 2 lines in the top half to create 3 rows. Then, draw 2 lines in the bottom half to create 3 rows. After the rows are drawn, students should draw 2 lines going down to create 3 columns.

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As the students work, ask individual groups some of the following questions:

- How many equal-size groups will you have to make for 6 × 3? *(6)*
- How many will be in each group? *(3)*
- What are your factors in the model? *(6 and 3)*
- Without counting each 1, how will you find the product? *(skip count by 3s; repeated addition)*
- What does 6 × 3 equal? *(18)* What is 18 called? *(the product)*
- With 3 equal-size groups, how many would be in each group? *(6)*

Math partners will share their model of multiplication with the group.

Say: **Explain your model. Include which factor represents the number of groups and which factor represents the number in each group.**

Does everyone agree with this representation of 6 × 3?

If a student disagrees, ask the student to explain why.

Compare representations to each another. Display all 3 representations: the equal-groups model, array, and area model. Remind students of the Commutative Property of Multiplication.

Say: **What is the corresponding multiplication equation using the factors 6 and 3 with a product of 18?** *(3 × 6 = 18)*

Cover 1 group of 3 on the equal-groups model with your hand.

Say: **If I cover 1 group of 3 on the equal-groups model, does it still represent 6 × 3 = 18?** *(no)* **Explain why.** *(there are no longer 6 groups of 3)* **What would be the new multiplication equation?** *(2 × 6 = 12)*

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Cover 1 dot in the array with your finger.

Say: **If I cover 1 of the dots in the array, does it still represent 6 × 3 = 18?** *(no)* **Explain why.** *(the groups do not have an equal number of dots anymore)*

Activity 2: Have students turn to the ***Practice Sheets*** on pages 16 and 17. Work along with students at first, then fade teacher assistance.

Say: **You will work the first problem, 8 × 8, using a model of your choice to solve: equals-group model, array, or area model. When you are finished working, describe your model to a partner.**

Ask questions such as:

- What is 8 × 8? *(64)*
- Explain which of the 3 representations is easiest for you to use when solving multiplication problems. *(accept reasonable answers)*

Students will continue to solve problems with a representation of their choice. Provide corrective feedback as needed while students are working. Allow time for students to share their models and their reasons for choosing the model.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 5

The Multiplication and Division Relationship

Lesson Objectives	• The student will use the relationship of multiplication and division to solve mathematical problems. • The student will write a set of multiplication and division equations in a number family. • The student will use mathematical vocabulary to explain the relationship and the answer to multiplication and division facts.	
Vocabulary	**division:** an operation that tells how many equal groups there are or how many are in each group (for example, 12 ÷ 3 = 4) **quotient:** the factor solved for in a division problem (for example, in 15 ÷ 3 = 5, 5 is the quotient, or equal share) **divisor:** in division, the number that divides another number, the dividend (for example, in 15 ÷ 3 = 5, 3 is the divisor) **dividend:** in division, the number that is being divided (for example, in 15 ÷ 3 = 5, 15 is the dividend) **number family:** a set of multiplication and division equations made from the same numbers (for 5, 7, and 35, the multiplication/division family consists of 5 x 7 = 35, 7 x 5 = 35, 35 ÷ 7 = 5, 35 ÷ 5 = 7)	
Reviewed Vocabulary	array, column, Commutative Property of Multiplication, equation, equations, factor, multiplication, product, row	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 39-52) • Whiteboard with marker	• Student Booklet (pp. 20-26)

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	• Counters (18)	

Preview

Say: **Today we will use *number families* to see how multiplication and *division* are related.**

Teacher Note
Students might refer to number families as "fact families."

Engage Prior/Informal Knowledge — Time: 3 min

Review the models of multiplication. Use the ***Engaged Practice Sheet***. Discuss how each model represents a multiplication problem.

Ask questions such as:

- What is the multiplication equation for the equal-groups model? *(4 × 5 = 20)*
- What is the product, or total, in this multiplication equation? *(20)*
- What is the multiplication equation for the array? *(2 × 6 = 12)*
- What are the factors in this multiplication equation? *(2 and 6)* What is the product? *(12)*
- What is the multiplication equation for the area model? *(1 × 4 = 4)*
- If I had 8 total squares with 4 in each row, how many rows would be in this area model? *(2)* Draw it on your sheet.

Modeled Practice — Time: 8 min

1. Relate division and multiplication facts by modeling the division of 18 counters into an array with 3 equal rows.

   Place 18 counters on the table. Ask a student to assist with the counters.

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Say: **We have 18 counters in all. Arrange the counters into 3 equal rows.**

Are there an equal number of counters in each row? *(yes)*

How many counters are in each row? *(6)*

We started with 18 counters and then divided them into equal-size groups.

Division **is the operation that is used to find how many are in each group or how many equal groups can be made. What operation is used to find out how many are in each group or how many equal groups?** *(division)*

Write the division equation "18 ÷ 3 = 6" on the whiteboard.

Say: **This is the *division* equation for 18 divided into 3 rows equals 6 in each row.**

Some of the wording is similar to multiplication because *division* is related to multiplication.

Look at the array of counters. How many rows? *(3)* **How many columns?** *(6)*

What is the multiplication equation that represents the divided counters? *(3 × 6 = 18)*

Write the multiplication equation "3 × 6 = 18" under the division equation on the whiteboard.

Say: **If we switch the factors, will the product be the same?** *(yes)*

What is the corresponding multiplication equation using the same 2 factors? *(6 × 3 = 18)*

Write "6 × 3 = 18" on the whiteboard to the right of the first multiplication equation.

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Say: **Thinking of the array for 3 × 6, these counters represents another *division* equation.**

How many counters total? *(18)* **How many in each row?** *(6)*

How many rows? *(3)*

18 divided by 6 equals 3 is another way we can arrange the counters to represent *division*.

Write "18 ÷ 6 = 3" to the right of the first division equation.

Say: **These 4 equations make up the *number family* for 3, 6, and 18.**

Write "3," 6," and "18" on the whiteboard below the four equations.

Say: **A *number family* is a set of multiplication and *division* equations made from the same numbers.**

Do not erase the 4 equations from the 3, 6, and 18 number family. Keep the counters together in the array.

2. Discuss a non-example for the number family 3, 6, and 18.

Write "2 × 9 =" on the whiteboard.

Say: **What is the product for 2 × 9?** *(18)* **How did you solve it?** *(accept reasonable answers such as counted by 2s, doubled 9)*

Write "18" as the product on the whiteboard.

Say: **18 is in the 3, 6, 18 *number family*. Are 2 and 9 in the same *number family?*** *(no)* **Why not?** *(2 and 9 are not numbers in this set)*

3. Discuss vocabulary terms related to multiplication and division. Use the number family previously written on the whiteboard and the array of counters. Allow students time to complete the steps.

Say: **In the multiplication equation 6 × 3 = 18, what are 6 and 3 called?** *(factors)*

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What is 18 called? *(the product)* **The product is the total.**

In the *division* equation 18 ÷ 3 = 6, 18 is the called the *dividend*. The *dividend* shows how many in all or the total. What is 18 called in *division*? *(dividend)*

On the whiteboard label "18" as the "dividend."

Say : **3 is the *divisor*. The *divisor* is the number that tells us how many groups. What is 3 called?** *(divisor)* **What does the *divisor* tell us?** *(how many groups)*

On the whiteboard label "3" as the "divisor."

Say: **6 is the *quotient*. The *quotient* represents the number in each group or the number of groups the total was divided into. What is 6 called?** *(quotient)* **What does the *quotient* tell us?** *(how many in each group)*

On the whiteboard, label "6" as the "quotient."

Say: **In the *number family* for 3, 6, and 18, 3 can also be the *quotient* when 6 is the *divisor*.**

However, in this *number family* 18 will always be the *dividend* because the *dividend* is the total or number being divided.

Use the array of counters to further explain the meaning of the vocabulary terms.

Say: **In the array of counters, what number is the *dividend* or the total number of counters?** *(18)*

What number is the *divisor*, or amount, of groups or rows? *(3)*

What number is the *quotient*, or the number of counters in each group or row? *(6)*

4. Use a number family triangle and a graphic organizer to practice number families and vocabulary terms. Allow students time to complete the steps.

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Use the ***Modeled Practice Sheet***. Students will write the 4 equations for the number family.

Say: **Look at the *number family* triangle. The set of numbers for this *number family* is 3, 5, and 15.**

We are going to write 4 equations.

When we write the multiplication equations, what number is the product? *(15)* **How do you know?** *(it is the total)*

What is a multiplication equation for this number family? *(3 × 5 = 15 or 5 × 3 = 15)* **Write it.**

What is the corresponding multiplication equation for this number family? *(3 × 5 = 15 or 5 × 3 = 15)* **Write it.**

How many *division* equations belong to this *number family*? *(2)*

Remember in a *division* equation the product from the multiplication equation is not the *quotient*.

15 is the *dividend,* or the total, that is being divided in *division*.

What is 15 called? *(dividend)*

When we write the *division* equation, we start with the total, or the *dividend*.

What is a *division* equation for this *number family*? *(15 ÷ 3 = 5 or 15 ÷ 5 = 3)* **Write it.**

What is the corresponding *division* equation? *(15 ÷ 3 = 5 or 15 ÷ 5 = 3)* **Write it.**

What 3 numbers are in each equation? *(3, 5, and 15)*

Give an example of an equation that does not belong in this *number family*. *(answers will vary)*

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Use the graphic organizer to review vocabulary terms for division. The teacher and students will complete together.

Say: **What is the *division* equation in the middle?** *(15 ÷ 5 = 3)*

Each number has a name. Which number is the *dividend*? *(15)* **Write "15" in the box.**

What number is the *divisor*? *(5)* **Write "5" in the box.**

What number is the *quotient*? *(3)* **Write "3" in the box.**

What is the corresponding *division* equation with these 3 numbers? *(15 ÷ 3 = 5)* **Write it in the box.**

In the *division* equations, what number do they both start with? *(15)* **What is 15 called?** *(dividend)*

Practice — Time: 8 min

Activity 1: Students will work in a Round Robin activity to write 4 equations for each number family. Have students turn to the ***Practice Sheet*** on page 22. Give each student 3 numbers in a number family to write in the triangle for Round 1. Be careful not to use any doubles facts as these number families only have 2 equations (1 multiplication equation and 1 division equation) instead of 4.

Say: **For your first practice set you will be given a *number family*. Everyone will have a different set of numbers.**

How many numbers are in a *number family*? *(3)*

First, everyone will write 1 equation, either multiplication or *division,* using the 3 numbers in that *number family*.

Then, you will all pass your papers to the person on your left. You will receive a new set of numbers for a different *number family*. Write another equation for that *number family*.

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Be careful to not write the same equation that someone else already wrote. We will continue to switch until all 4 equations for each *number family* have been written.

To review, how many multiplication equations will be in each *number family*? *(2)* **How many *division* equations?** *(2)*

Provide teacher assistance and corrective feedback as students work. Do not let students pass booklets until you tell everyone to pass.

Teacher Note
If students need additional practice, provide every student with another set of numbers and have the group do a second round of Round Robin.

Activity 2: Students will practice number families. Have students turn to the ***Practice Sheets*** on pages 23 and 24. Students will work with a math partner to answer questions.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 6

Breaking Apart Numbers Through 12

Lesson Objectives	• The student will break apart numbers through 12. • The student will organize and verbalize steps/strategies in solving to explain mathematical thinking through communication to peers and teachers.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	addition, Commutative Property of Addition, equation, number family, part, subtraction, whole number	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 53-68) • Whiteboard with marker	• Student Booklet (pp. 27-34) • Whiteboard with marker (1 per student) • 2 number cubes (per pair)

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Preview

Say: **Today we will break apart numbers through 12. Knowing how to break apart numbers easily can help us solve multiplication problems.**

Engage Prior/Informal Knowledge Time: 3 min

Review finding pairs of numbers that make 3, 4, and 5. Write a number on the whiteboard. Ask students to write different ways to make each number on their whiteboards. Have students name the different parts added together to make the whole; for example, 2 and 1 are parts to the whole number 3.

Ask questions such as:

- What 2 parts make the whole number 3? *(0 + 3; 3 + 0; 2 + 1; 1 + 2)*
- What 2 parts make the whole number 4? *(0 + 4; 4 + 0; 1 + 3; 3 + 1; 2 + 2)*
- What 2 parts make the whole number 5? *(0 + 5; 5 + 0; 1 + 4; 4 + 1; 2 + 3; 3 + 2)*

Modeled Practice Time: 8 min

1. Break apart 6.

   Have students turn to ***Modeled Practice Sheet #1***. Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

Say: **We are going to break apart the whole number 6 into 2 parts. We will use our knowledge of addition and subtraction to break apart numbers.**

1 part is given and 1 part is missing. What part is given? *(1)*

What part is missing? *(5)*

1 + 5 = 6. Write "5" on the line.

Can we subtract to find the missing part? *(yes)*

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What do we subtract from the whole 6? *(1)*

What is 6 – 1? *(5)*

What 2 parts make 6? *(5 and 1)*

1, 5, and 6 are numbers in a number family.

Write the 2 addition equations and the 2 subtraction equations in this number family.

What are the addition equations in this number family? *(1 + 5 = 6 and 5 + 1 = 6)*

Does 5 – 1 belong to this number family? *(no)* **Why not?** *(5 – 1 = 4, and 4 is not a number in this number family)*

What are the subtraction equations? *(6 – 1 = 5 and 6 – 5 = 1)*

Let's break apart 6 again. What part is given? *(2)*

Is 8 the missing part? *(no)* **Why not?** *(8 + 2 = 10, not 6)*

What part is missing? *(4)* **Write "4" on the line.**

If we subtracted to find the missing part, what do we subtract from the whole 6? *(2)*

What is 6 – 2? *(4)*

Break apart 6 again on your own.

What part is given? *(3)*

What is the missing part? *(3)*

2. Break apart 7.

Continue using *Modeled Practice Sheet #1*.

Say: **What whole number are we breaking apart?** *(7)* **What part is given?** *(1)*

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What part is missing? *(6)*

What 2 parts make 7? *(1 and 6)*

Write "7 = __ – 1" on the whiteboard.

Say: **What number will make this equation true?** *(8)*

How do you know? *(8 – 1 = 7, or 7 is the same as 8 – 1)*

7 equals 8 – 1 because 8 – 1 equals 7.

Write "7 = 7" under 7 = 8 – 1.

Say: **Go back to your sheet. Break apart 7 in 2 different ways on your own.**

Turn to your math partner and share how you broke apart 7 in 2 ways. *(answers will vary)*

What other 2 parts make 7? *(2 and 5; 3 and 4)*

Does it matter which number you write first on the line? *(no)*

Why not? *(accept answers related to the Commutative Property of Addition)*

You can write the parts in any order on the lines.

3. Break apart 8.

Use ***Modeled Practice Sheet #2***. Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

Say: **There are 4 ways to break apart 8. Some of the parts are given. What parts are given?** *(1, 2, 3, and 4)*

Find the missing parts on your own.

Let's check our answers together. What number and 1 make 8? *(7)*

What number and 2 make 8? *(6)*

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What number and 3 make 8? *(5)*

What number and 4 make 8? *(4)*

Write the addition and subtraction equations in the number family using 6, 2, and 8.

Does 6 – 2 belong to this number family? *(no)* **Why not?** *(6 – 2 = 4, and 4 is not a number in this number family)*

What are the subtraction equations in the number family? *(8 – 6 = 2 and 8 – 2 = 6)*

What are the addition equations? *(2 + 6 = 8 and 6 + 2 = 8)*

Write an equation that does not belong to this number family. *(answers will vary)*

4. Break apart 10.

Continue to use ***Modeled Practice Sheet #2***.

Say: **Break apart 10 in different ways with your math partner.**

Wait 1 minute for students to work.

Say: **What are some ways to break apart 10?** *(1 and 9; 2 and 8; 3 and 7; 4 and 6; 5 and 5)*

Read the problem below. *(9 = __ – 1)*

What number will make this equation true? *(10)*

How do you know? *(10 – 1 = 9, or 9 is the same as 10 – 1)*

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Practice Time: 8 min

Activity 1: Students will break apart 9 and 11. Have students turn to the ***Practice Sheets*** on pages 29 and 30. Students will work with a math partner to complete the activity.

Say: **Work on the first problem on your own. Then we will discuss the answer. After we discuss, you and your math partner will work together to solve the rest of the problems. Record your work on your sheet.**

Monitor students' work and provide corrective feedback when necessary. Allow students to explain how they solved the problem.

Ask questions such as:

- How many ways can you break apart 9? *(4)*
- What number and 1 make 9? *(8)* How did you solve it? *(answers will vary)*
- What number and 2 make 9? *(7)* How did you solve it? *(answers will vary)*
- What number and 3 make 9? *(6)* How did you solve it? *(answers will vary)*
- What number and 4 make 9? *(5)* How did you solve it? *(answers will vary)*
- Does it matter which number you write first on the line? *(no)*
- Why not? *(accept answers related to the Commutative Property of Addition)*
- What are a pair of numbers that make up 11? *(1 and 10; 2 and 9; 3 and 8; 4 and 7; 5 and 6)*
- What are the addition and subtraction equations in the number family? *(6 + 5 = 11; 5 + 6 = 11; 11 – 6 = 5, 11 - 5 = 6)*

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Activity 2: Have students turn to the ***Practice Sheet*** on page 31. Students will review breaking apart numbers through 12 using 2 number cubes. Students will work in pairs and take turns rolling the number cubes. The first student rolls both number cubes at the same time and writes the 2 parts on the number cubes in the column of the correct sum. The next player rolls and writes the parts on the number cubes on their sheet. The players continue to take turns rolling until one player has filled all empty spaces. If a player rolls the same parts that are already written on their sheet, then that player must pass.

Ask questions such as:

- Are there any other ways to make 7, 8, 9, 10, 11, or 12 that were not found by rolling the number cubes? *(yes)*
- Why were these parts not found when rolling the number cubes? *(because the highest number of the cube is 6, so 1 and 7 to make 8 does not show up on the number cubes)*
- Which sums were most likely to be rolled? *(6, 7, and 8)* Why? *(because there are more ways to make these numbers)*

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module E
Lesson 7

Multiply by 9s

Lesson Objectives	• The student will identify multiples of 9 and 10 to solve unknown multiplication facts in which 1 of the factors is 9. • The student will identify the multiplication and division equations in a number family. • The student will apply and explain a variety of appropriate strategies to solve problems.	
Vocabulary	**multiple:** repeated groups of the same amount; the product of any 2 whole numbers (for example, 8 is a multiple of 4 because it is the product of 4 and the whole number 2) **Make 10 Subtract the Factor:** a strategy in which the student uses the Distributive Property to find the product of facts with a factor of 9 (for example, $9 \times 6 = 10 \times 6 = 60 - 6 = 54$) **doubles fact:** a multiplication fact in which the factors are the same (for example, 6×6)	
Reviewed Vocabulary	dividend, division, equation, factor, number family, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 69-82) • Hundreds chart • Whiteboard with marker • Multiplications Strategies Poster	• Student Booklets (pp. 35-41) • Whiteboard with marker (1 per student) • Highlighter (1 per student) • Multiplication Strategies Bookmark (1 per student)

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Preview

Say: **Today we will solve multiplication problems with 9 as a factor. We will also write the corresponding division equations in the number family.**

Engage Prior/Informal Knowledge — Time: 3 min

Students will need to use mental subtraction in order to be successful with 9s facts. Review breaking apart the whole number 10 into 2 parts. Use the ***Engaged Practice Sheet***. Have a ***hundreds chart*** available if needed.

Say: **Today we will learn a strategy where we will use our knowledge of breaking apart 10 and practice subtracting larger numbers.**

Ask questions such as:

- What is 10 – 8? *(2)* If 10 – 8 = 2, what is the corresponding addition fact to make 10? *(2 + 8 = 10 or 8 + 2 = 10)*
- What is 10 – 4? *(6)* What 2 parts make 10? *(4 and 6)*
- What is 10 – 5? *(5)* 10 – 1? *(9)* 10 – 6? *(4)* 10 – 7? *(3)* 10 – 3? *(7)* 10 – 2? *(8)*
- How many groups of 10 are in 30? *(3 tens)* When 3 ones are taken away, how many groups of 10 are left? *(2 tens)* How many ones are left? *(7)* What is 30 – 3? *(27)*
- What is 50 – 5? *(45)* If 50 – 5 = 45, what number plus 5 equals 50? *(45)* What is 20 – 2? *(18)* 90 – 9? *(81)* 40 – 4? *(36)*
- How can you use your knowledge of numbers that break apart 10 to help with larger subtraction facts? *(the number left in the ones place will be the number pair that makes 10)*
- What are some things you notice when subtracting from a group of 10? *(the tens place will have one less group of 10; the ones place will be the number pair to 10)*

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Modeled Practice — Time: 8 min

1. Using the ***hundreds chart***, have students compare the multiples of 10 with the multiples of 9.

   Use *Modeled Practice Sheet #1.* Distribute a highlighter to each student and have them highlight the multiples of 9 when appropriate in the lesson.

Say: **When we skip count by 10, starting at 10, what do these numbers have in common?** *(all end in a zero)*

10, 20, 30, 40, and so on, end in zero. They are called *multiples* of 10.

***Multiples* are repeated groups of the same amount.**

What is the first *multiple* of 10? *(10)* **The second *multiple* of 10?** *(20)* **Third *multiple*?** *(30)*

We can find *multiples* of other numbers in the same way. If we had 1 group of 9, what is the first *multiple*? *(9)*

Highlight "9" on your hundreds chart. How many jumps backwards is it from 10 to 9? *(1)*

Find 20 on your hundreds chart. How many jump backwards is it from 20 to 18? *(2)*

20 – 2 is the same as 2 × 9. What is 2 × 9? *(18)* **Highlight "18."**

18 is the second multiple of 9. If we were to make a picture we would draw 2 equal groups of 9.

What is the third *multiple* of 10? *(30)*

We are going to use 30 to help us solve 9 × 3. What factor are we multiplying 9 by? *(3)*

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The other factor tells us how many times to jump backwards. What is the other factor, or how many times are we going to jump backwards? *(3 or the number of times of the factor)* **Do it. What number are you on?** *(27)*

How many jumps backwards is it from 30 to 27? *(3)* **What is 3 × 9?** *(27)* **Highlight "27."**

27 is the third *multiple* of 9. Look at the numbers we have highlighted. We have highlighted 3 *multiples* of 9. Count by 9s. *(9, 18, 27)*

Looking at the hundreds chart, what do you think will be the next *multiple* of 9? *(36)* **How do you know?** *(accept reasonable answers such as added 9 more to 27, counted on 9 more from 27)*

Highlight "36." How many jumps is it from 40 to 36? *(4)*

What factor times 9 equals 36? *(4)*

Look at the number 36. What number is in the ones place? *(6)*

6 + what number makes 10? *(4)*

36 is 4 away from 40, just like 6 is 4 away from 10.

Look at 27. 7 plus what number equals 10? *(3)*

How many jumps is it from 30 to 27? *(3)*

What do you see happening with each *multiple* of 9? *(accept reasonable answers such as: the product moves back from the multiple of 10 the same number of spaces as the other factor; the factor times 9 is the same number of jumps from the factor times 10; or, the multiples of 9 are in a diagonal on the hundreds chart)*

2. Introduce the Make 10 Subtract the Factor strategy.

   Use ***Modeled Practice Sheet #2***. Students will work along with the teacher.

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Teacher Note
This lesson will demonstrate how the Distributive Property of Multiplication works. The students do not need to memorize the term or the definition of this property.

Say: **Here are the steps in the *Make 10 Subtract the Factor* strategy to solve unknown 9s facts.**

Step 1 is "think of 9 as 10." Why? *(because 9 = 10 – 1)*

What is step 1? *(think of 9 as 10)*

Step 2 is "multiply 10 times the other factor." What is step 2? *(multiply 10 times the other factor)*

Step 3 is "subtract the other factor." What is step 3? *(subtract the other factor)*

We subtract the other factor because 9 = 10 – 1.

Read the fact. *(9 × 6)* **Can we use the strategy?** *(Yes)* **Why?** *(1 factor is 9)*

What is the first step of this strategy? *(think of 9 as 10)*

Write "10" on the first line.

What is the second step? *(multiply 10 times 6)*

Write the factor "6" on the next line. What is 10 × 6? *(60)* **Write it below on the line.**

What is the third step? *(subtract the other factor 6, or 60 – 6)* **Write the other factor "6" on the next line.**

What is 60 – 6? *(54)* **Write it on the line below.**

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What is 9 × 6? *(54)*

3. Relate multiplication to division through number families.

Say: **What numbers are in this number family?** *(9,6, and 54)*

We know 9 × 6 = 54. What is the corresponding multiplication equation in the number family? *(6 × 9 = 54)*

Write the multiplication equations on the whiteboard.

Say: **When we learn multiplication facts, we also learn the corresponding division facts. In division, we write the dividend, the whole or total, first. What is the dividend here?** *(54)* **What is 54 ÷ 9?** *(6)* **What is 54 ÷ 6?** *(9)*

Write the division equations on the whiteboard.

4. Provide a non-example to help illustrate when this strategy can be used.

Write "6 × 3" on the whiteboard.

Say: **Read the fact.** *(6 × 3)*

Can you use the strategy to solve this fact? *(no)*

Why not? *(it does not have 9 as a factor)*

Can you use the strategy for 4 × 8? *(no)*

The strategy will only work for multiplication facts in which 9 is 1 of the factors.

Continue using ***Modeled Practice Sheet #2.***

Say: **Read the second fact.** *(9 × 9)* **What do you notice about the factors?** *(they are the same)*

9 × 9 is an example of a *doubles fact*. A *doubles fact* is a multiplication fact in which the factors are the same.

Can you use the strategy to solve 9 × 9? *(yes)*

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Why? *(one of the factors is 9)*

Use the strategy and solve.

Wait 30 seconds for students to work.

Say: **What is 9 × 9?** *(81)*

Give an example of a multiplication problem where we can use the strategy. *(accept multiplication problems with a factor of 9)*

Practice Time: 8 min

Activity 1: Students will practice the Make 10 Subtract the Factor strategy to solve 9s facts.

Have students turn to the ***Practice Sheet*** on page 38. Keep the ***Multiplication Strategies Poster*** visible while students work. Distribute the ***Multiplication Strategies Bookmark*** to each student.

Say: **Work with your partner using the *Make 10 Subtract the Factor* strategy and your knowledge of number families to complete the next few problems. Use the Multiplication Strategies Bookmark to help you remember the steps of the *Make 10 Subtract the Factory* strategy.**

Ask questions such as:

- What is step 1? *(think of 9 as 10)* Why? *(because 9 = 10 – 1)*
- What is step 2? *(multiply 10 times the other factor)*
- What is step 3? *(subtract the other factor)*

Check students' answers before continuing.

Say: **Look at problem #3. This fact is a *doubles fact. Doubles facts* only have 1 multiplication equation and 1 division equation.**

What is another *doubles fact* that we already know? *(1 × 1 = 1; 2 × 2 = 4; 5 × 5 = 25; 10 × 10 = 100; accept any doubles multiplication fact the students know automatically)*

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Explain in your own words why you think a *doubles fact* number family only has 1 multiplication equation and 1 division equation. *(accept answers that address that there is only 1 factor that is repeated and if the factors switch order, the problem stays the same)*

Activity 2: Have students turn to the ***Practice Sheet*** on page 39. Students will work with a math partner to complete the problems. Check that students are able to use the Make 10 Subtract the Factor strategy

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 8

Distributive Property of Multiplication

Lesson Objectives	• The student will use the Distributive Property when solving unknown multiplication problems. • The student will select, apply, and demonstrate knowledge of mathematical representations to solve problems using language related to multiplication.	
Vocabulary	**Distributive Property of Multiplication over Addition:** a property of multiplication over addition that states if a, b, and c are whole numbers, then $a \times (b + c) = (a \times b) + (a \times c)$; this property is essential to understanding multiplication of multi-digit numbers such as 4×27, in which the problem becomes $4 \times (20 + 7)$, which equals $(4 \times 20) + (4 \times 7)$; it is also important for later work with fractions, equations, and algebra	
Reviewed Vocabulary	expression, factor, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 83-104) • Whiteboard with marker • Scissors • 12 Cubes (optional)	• Student Booklet (pp. 42-49) • Whiteboard with marker (1 per student) • Graph paper • Scissors (1 per student) • Glue stick (1 per student)

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Preview

Say: **Today we will break apart a factor in a multiplication problem to make facts we can solve quickly. We will then use those facts to solve a challenging multiplication problem.**

Engage Prior/Informal Knowledge — Time: 3 min

Practice breaking apart 12. Write "12" on the whiteboard. Have students write all the different ways to break apart 12 on their whiteboards. Ask students to identify the 2 parts the make the whole number.

Ask questions such as:

- How many ways can you break apart 12? *(6)*
- How did you break apart 12? *(1 and 11, 2 and 10, 3 and 9, 4 and 8, 5 and 7, 6 and 6)*

Have students complete the part-part-whole number puzzle on the ***Engaged Practice Sheet***. Each circle will be the sum of the 2 circles below it. Students must use the numbers that are present to determine the missing numbers. Everyone should have the same numbers in each circle when complete.

Watch For

Students may struggle in identifying parts of the whole number.
For example, when breaking apart 12, students may identify 7 as the whole instead of a part.
Use cubes to break apart 2 parts from the whole. Have students break up the cubes into 2 parts: 7 and 5. Put the parts together to identify the whole 12.

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Modeled Practice — Time: 8 min

1. Introduce the Distributive Property by finding the area of 2 parts of the original rectangle.

Teacher Note
This lesson will discuss the Distributive Property of Multiplication. The students do not need to memorize the term or the definition.

Cut out the 3 × 7 area model from ***Modeled Practice Display #1,*** but do not cut the model into two pieces yet. Place the model on ***Modeled Practice Display #2***. Review how to find the area by skip counting.

Say: **Look at the area model. Count the rows. How many?** *(3)* **How many columns?** *(7)*

This area model is 3 square units by 7 square units. The dimensions represent the multiplication fact 3 × 7.

In previous lessons how did we find the area or total number of square units in this rectangle? *(multiply 3 × 7; repeated addition)*

To solve this problem another way, I am going to break apart 1 of the factors.

Cut the rectangle along the line between the grey and white sections.

Say: **I have cut the large rectangle to make 2 smaller rectangles or area models.**

Hold up the 2 new rectangles.

Say: **Let's find the area, the number of square units in the rectangle, for 1 of the rectangles.**

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Hand the grey rectangle to a student to allow them an opportunity to figure out the dimensions. Run your finger along the length and width when using the word dimensions to help illustrate the meaning of the word.

Say: **What are the dimensions, or the length of the side and the width of the side, for the rectangle shaded grey?** *(3 by 2, or 2 by 3)*

By cutting the rectangle into 2 parts, I have broken apart 1 of the factors to create 2 facts that I know.

To find the area of the first model, we multiply 3 × 2. What is 3 × 2? *(6)*

On ***Modeled Practice Display #2***, place the grey rectangle in the blank box and fill in the multiplication expression below.

Say: **We have found the area for 1 part of the original rectangle. Now we need to find the area for the other part.**

Hand the white rectangle to a student to allow them an opportunity to figure out the dimensions.

Say: **What are the dimensions for this rectangle?** *(3 by 5, or 5 by 3)*

To find the area, what do we multiply? *(3 times 5)*

What is 3 × 5? *(15)*

On ***Modeled Practice Display #2***, place the white rectangle in the blank box and fill in the multiplication expression below.

Touch the rectangles as you discuss the area of each. Slide the rectangles together to the lower box on ***Modeled Practice Display #2*** to demonstrate how adding the two smaller rectangles will equal the area of the original rectangle.

Say: **What is the area for the smaller rectangle?** *(6)* **What is the area for the larger rectangle?** *(15)*

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To find the area of the whole rectangle we add the 2 parts or the 2 smaller rectangles. What is 6 + 15? *(21)*

Write "6 + 15" in the blanks.

Say: **What is the multiplication fact for the original rectangle?** *(7 × 3 or 3 × 7)* **Then what is 3 × 7?** *(21)*

Watch For 	**Students may struggle with 2-digit addition. For example, addition of 2-digit numbers such as 15 + 6 or 12 + 30 may be a prerequisite for this skill.** **Provide corrective feedback to students during the lesson if their sum is incorrect. Have students solve the addition problems with a partner to check for accuracy or use a hundreds chart.**

2. Solve 4 × 8 using the Distributive Property of Multiplication.

   Have students turn to ***Modeled Practice Sheet #1*** and work along with the teacher.

Say: **Read the problem.** *(4 × 8)*

4 × 8 is a challenging multiplication fact, so we are going to break apart 1 of the factors to help us solve it.

What are the 2 factors in the problem? *(4 and 8)*

Instead of using scissors, we are going to cut the area model by drawing a dark line. Count 2 rows then draw a dark horizontal line.

Check that students have counted the rows correctly and drawn the horizontal line correctly.

Say: **How many rectangles are there now?** *(2)*

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The area model now has 2 smaller rectangles. What is the multiplication expression for the first small rectangle? *(2 × 8)* **Write the multiplication expression "2 × 8" beside it.**

What is the multiplication expression for the bottom rectangle? *(2 × 8)* **Write "2 × 8" beside it.**

Fill in the blanks in the problem to match our area model.

Check that students have written (2 × 8) + (2 × 8).

Say; **I have 2 multiplication problems that I can solve easily using skip counting.**

What is 2 × 8? *(16)* **Write it.**

What is 2 × 8? *(16)* **Write it.**

Just like in the last example, we have to put the 2 parts together to find the whole, or the total, square units. What do we do last with the 2 products? *(add the 2 products together to get the answer to the original problem, 4 × 8)*

What is 16 + 16? *(32)* **Write it.**

What does 4 × 8 equal? *(32)*

How do you know that the 2 smaller rectangles and the larger model represent the same product? *(the area of the smaller rectangles is the same as the larger 4 × 8 rectangle)*

What 3 numbers are in this number family? *(4, 8, and 32)*

What is the corresponding multiplication equation for this number family? *(8 × 4 = 32)*

What are the division equations for this number family? *(32 ÷ 8 = 4 and 32 ÷ 4 = 8)*

3. Review the steps to breaking apart a factor to solve.

   Point to each step on the sheet as it is discussed.

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Say: **By breaking the large rectangle apart into 2 smaller rectangles, I can see how the factor 4 is broken apart.**

Let's review how we solved 4 × 8.

First, we chose to break 4 apart. What did we break 4 into? *(2 and 2)* **Why?** *(because we know our 2s facts with skip counting)*

Second, we multiplied each 2 by the other factor. What was the other factor? *(8)*

Then we solved the 2 facts by skip counting.

What are the 2 facts? *(2 × 8 and 2 × 8)*

What is 2 × 8? *(16)* **What is 2 × 8?** *(16)*

Once we found the 2 products for the smaller area models, or facts, what did we do with the 2 products? *(added them together)*

What is 16 + 16? *(32)* **What is 4 × 8?** *(32)* **How do you know?** *(the area of the smaller rectangles is the same as the larger 4 × 8 rectangle)*

To solve a problem like 2 × 9, would you break apart 9? *(no)*

Why not? *(it's not a challenging fact; we know our 2s facts; we can solve by skip counting by 2s)*

When we come across a multiplication problem we don't know, we can think about breaking apart 1 of the factors to make 2 facts that we can solve and then add the products together.

What are some multiplication facts you might need to break apart 1 of the factors to help you solve? Why? *(accept reasonable answers such as 8 × 6 or 7 × 8; a multiplication problem with a factor of 2, 5, or 10 would not be a reasonable answer because students should use skip counting to solve)*

4. Explore other ways to solve 4 × 8 using the Distributive Property of Multiplication. Have students turn to ***Modeled Practice Sheet #2***. Allow students time to complete the steps.

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Say: **We are going to solve 4×8 another way by breaking apart the rectangle into 2 smaller rectangles. Draw a line through the model to make 2 smaller rectangles.**

Wait 5-7 seconds for students to work. Check students' work.

Say: **Next to each rectangle write the fact. Turn to your math partner and share how you broke apart the model.**

Solve the facts.

What do we do with the 2 products? *(add them together)* **Do it.**

Is the product of 4×8 still 32? *(yes)*

When we broke apart the area model differently, the product was still 32.

Practice

Time: 8 min

Activity 1: Students will use graph paper to illustrate the Distributive Property.

Have students turn to the ***Practice Sheets*** on pages 45 and 46. Distribute scissors and glue stick to every student. Work along with the students at first, and then fade teacher assistance.

Have students cut out the area models on page 45. Instruct students to cut along the edges of the model, but not to cut the model into 2 pieces yet.

Have students place each of the 2 models next to its corresponding multiplication problem on page 46. Check that students have correctly matched the model with the problem.

Teacher Note

If students struggle with the area model's orientation, remind them that the number of units going down will be the first factor being multiplied. The units going across will be the second number being multiplied.

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Next have students cut along the line of the 4 × 6 rectangle, between the grey shaded part and the white part, to create 2 rectangles.

Students will glue the grey part of the original rectangle into the first blank box.

Say: **What is the original problem?** *(4 × 6)* **Which factor was broken apart?** *(6)* **Why?** *(answers may vary)*

What 2 numbers was it broken into? *(1 and 5)* **Why?** *(1s and 5s are facts we already know rather than 6s)*

What is the new multiplication problem for that area model? *(4 × 1)*

Students will glue the white part of the original rectangle into the second blank box.

Say: **What is the new multiplication problem for that area model?** *(4 × 5)*

Guide students to complete the missing blanks below the boxes.

Say: **Instead of writing "6," what will you write instead?** *(1 + 5)*

Under each model, write the multiplication expression for the rectangles.

Check that students write "4 × 1" and "4 × 5" in the blanks and check their answers. Ask follow up question such as:

- What is the final step? *(add the 2 products together)*
- Can you answer the original multiplication problem, 4 × 6? What is the product? *(yes, 24)*
- What are the 2 multiplication problems you cut 4 × 6 into? *(4 × 1 and 4 × 5)*

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Allow students to complete #2 and discuss with their math partners how the answer was found before moving to #3.

Activity 2: If students need additional practice, have them solve the problems on page 47. If students are ready to move on, then proceed to the *Independent Practice Sheets*.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 9

Multiplying by 6s

Lesson Objectives	• The student will learn 6s facts using the Distributive Property to break apart the factor 6. • The student will apply their understanding of the term Distributive Property by following the steps of the Break Apart Strategy.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Distributive Property, division, equation, expression, factor, product, Make 10 Subtract the Factor, number family	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 105-128) • Multiplication Strategies Poster	• Student Booklet (pp. 52-63) • Whiteboard with marker (1 per student) • Multiplication Strategies Bookmark (1 per student)

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Preview

Say: **In this lesson we will solve unknown multiplication facts with 6 as a factor. We will use our knowledge of breaking apart numbers and the Distributive Property.**

We know some facts automatically, which means you know the answer instantly without using your fingers or counting in your head. Some facts are more challenging, so this strategy will be very helpful.

Engage Prior/Informal Knowledge Time: 3 min

Review breaking apart 6 into 2 different parts. Use the ***Engaged Practice Sheet***.

Say: **What number is the whole?** *(6)*

What are some different parts that make 6? *(1 and 5; 2 and 4; 3 and 3)*

When we multiply by the factor 6, we can break it apart into 2 parts to make facts that we already know.

Which 2 parts, or numbers, can we multiply automatically and quickly? *(1 and 5)* **Why 1 and 5?** *(we know any number multiplied by 1 equals itself and we know 5s facts by skip counting by 5)*

What is 1 × 9? *(9)* **What is 6 × 1?** *(6)* **What is 5 × 6?** *(30)* **3 × 5?** *(15)* **5 × 5?** *(25)* **5 × 8?** *(40)* **5 × 7?** *(35)*

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Modeled Practice — Time: 8 min

1. Have students solve a multiplication problem with a factor of 6 using the Distributive Property.

   Use ***Modeled Practice Sheet #1.*** Allow students time to complete the steps.

Teacher Note
This lesson will discuss the Distributive Property of Multiplication. The students do not need to memorize the term or the definition.

Say: **6 × 4 might be a fact you do not know automatically.**

Instead, we can use the Break Apart Strategy for 6s. Break apart the factor 6 to make 2 facts that we know. Think of 6 as the whole number. If 1 is a part, what is the other part that makes 6? *(5)*

In the Break Apart Strategy, step 1 is to break apart 6 into 1 and 5. What is step 1 when multiplying by 6? *(break apart 6 into 1 and 5)* **Write "1" and "5" in the boxes and between the parentheses.**

Look at the area model on the left of your sheet. Count 1 row and draw a dark horizontal line.

Check students' work.

Say: **Breaking apart 6 into 1 and 5 is like cutting the area model into 2 smaller rectangles like we have done previously.**

What is the multiplication expression for the small rectangle? *(1 × 4 or 4 × 1)* **Write the multiplication expression beside it.**

What is the multiplication expression for the other rectangle? *(5 x 4 or 4 x 5)* **Write the multiplication expression beside it.**

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Write the multiplication expressions on the lines in step 2.

Check students' work.

Say: **Step 2 is to multiply 1 and 5 by the other factor. What is step 2?** *(multiply the other factor by 1 and 5)* **What is the other factor?** *(4)*

We multiply 4 by both of the parts that we broke 6 into.

What is the multiplication expression for the first area model? *(1 × 4)* **What is the multiplication expression for the second area model?** *(5 × 4)*

Why do I have to multiply 4 by both 1 and 5? *(the problem is 4 × 6, if we break apart 6, 4 then has to be multiplied by both of the parts that equal 6)*

What is 1 × 4? *(4)* **Write it on the line.**

What is 5 × 4? *(20)* **Write it on the line.**

Step 3 is to add the products together to find the answer. What is step 3? *(add the products together)*

What products do I add together? *(4 and 20)* **Write it on the lines in step 3.**

4 square units make up the first rectangle. How many square units are in the other rectangle? *(20)* **What is 4 + 20?** *(24)*

What is 6 × 4? *(24)* **Write it.**

What 3 numbers are in this number family? *(6, 4, and 24)*

If we switch the order of the factors will the product stay the same? *(yes)*

What is the corresponding multiplication equation in this number family? *(4 × 6 = 24)*

Does 24 ÷ 3 belong in this number family? *(no)* **Why not?** *(3 and 8 are not numbers in this number family)*

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What are the division equations in this number family? *(24 ÷ 4 = 6 and 24 ÷ 6 = 4)*

Let's review the steps in the Break Apart Strategy. What is step 1? *(break apart 6 to 1 and 5)*

What is step 2? *(multiply 1 and 5 by the other factor)*

What is step 3? *(add the products together to find the answer)*

2. Provide a non-example for the Break Apart Strategy. Compare solving using the Make 10 Subtract the Factor against the Break Apart Strategy.

Say: **Would we use the 6s strategy for 7 × 8?** *(no)* **Why not?** *(neither factor is 6)* **How could you solve this fact?** *(break part 1 of the factors, then multiply by the other factor and add the products together)*

What about 10 × 6? Would we use the Break Apart Strategy and break apart 6 to 5 and 1? *(no)* **Why not?** *(we already know the 10s facts; we can skip count by 10s; we know multiples of 10)* **What is 10 × 6?** *(60)*

Think about 6 × 9. Do we already know a strategy to solve this fact? *(yes)* **What is the name of the strategy?** *(Make 10 Subtract the Factor)* **Could you solve this fact using either the 9s or the 6s strategy?** *(yes)*

Assign some students to solve 6 × 9 using the Break Apart Strategy and other students to solve using the Make 10 Subtract the Factor strategy. Tell students to work on the margins of their sheet. Wait 30 seconds for students to work.

Say: **Is the product 54 if we solve it either way?** *(yes)*

Let's quickly review how to use the Make 10 Subtract the Factor strategy to solve 6 × 9. Step 1 is what? *(think of 9 as 10)*

Step 2 is what? *(multiply 10 times the other factor)*

What is the other factor? *(6)* **What is 10 × 6?** *(60)*

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What is step 3? *(subtract the other factor)*

What is 60 – 6? *(54)* **What is 6 × 9?** *(54)*

3. Have students solve a multiplication problem with a factor of 6 using the Break Apart Strategy. Use ***Modeled Practice Sheet #2.*** Allow students time to complete the steps.

Say: **Read the problem. Ready, read: "Coach Martinez is setting up cones for a soccer obstacle course. She wants to set up 6 rows of 8 cones for her team to dribble around. How many cones does Coach Martinez need?"**

What is the problem asking? *(how many total cones)*

What is the multiplication expression for this word problem? *(6 × 8)* **Write it.**

What is step 1? *(break apart 6 to 1 and 5)* **Write "1" and "5" on the lines in step 1.**

What is the other factor? *(8)* **Write the 8 on the next line in step 1.**

What is step 2? *(multiply 1 and 5 by the other factor 8)*

Complete the rest of the problem on your own.

What products did you add together in step 3? *(8 and 40)*

How many cones does Coach Martinez need? *(48 cones)*

Practice **Time: 8 min**

Activity 1: Students will solve 6s facts with a partner. Have students turn to the ***Practice Sheets*** on pages 55 and 56. Gradually fade assistance as students become more comfortable with the strategy. Keep the ***Multiplication Strategies Poster*** visible while students work. Remind students to use the ***Multiplication Strategies Bookmark*** if needed.

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Say: **For the first few problems on your sheet I want you and your partner to practice solving multiplication problems with a factor of 6.**

Allow students time to complete their work. Provide corrective feedback.

Watch For

For #3: Watch for students who want to break apart both 6s. Although it is not incorrect to break apart both 6s, it does add extra steps that the student does not need to do. Remind the students of the 3 steps. Break one factor (6) apart into 1 x 5 and multiply by the other factor. The second 6 is considered the other factor in this problem.

Activity 2: Have students turn to the ***Practice Sheets*** on pages 57, 58, and 59. Students will continue to work with a partner practicing the 6s strategy and reviewing previously taught strategies such as Make 10 Subtract the Factor in a game of *Facts Tic-Tac-Toe.* Students are to work out any problem that is not automatic on their whiteboard. They must prove their answer to their partner before placing an X or O in the square. Students will only need one person's practice sheet. The other sheet can be saved for another day.

Say: **With your partner, you will play Tic-Tac-Toe. Before you place your "X" or "O" in the square, you must first answer the multiplication problem.**

If the fact is one that you know automatically, you may tell your partner the answer and place your "X" or "O."

If you do not know the answer automatically, you need to work out the problem on your whiteboard and allow your partner to check your work before you can place your "X" or "O" in the square. If the answer is incorrect, do not mark the square.

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There are 3 games on each page. To win a game you need to have 3 squares in a row.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 10

Multiplying by 7s

Lesson Objectives	• The student will learn the 7s facts using the Distributive Property to break apart the factor 7. • The student will apply their understanding of the term Distributive Property by following the steps of the Break Apart Strategy.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Distributive Property, division, equation, expression, factor, number family, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 129-152) • Multiplication Strategies Poster	• Student Booklet (pp. 64-75) • Whiteboard with marker (1 for each student) • Multiplication Strategies Bookmark

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Preview

Say: **In this lesson we will solve unknown multiplication facts with 7 as a factor. We will use our knowledge of breaking apart numbers and the Distributive Property to help us solve.**

Engage Prior/Informal Knowledge — Time: 3 min

Review breaking apart 7 into different parts. Use the ***Engaged Practice Sheet***.

Say: **What number is the whole?** *(7)*

What are some different parts that make 7? *(1 and 6; 2 and 5; 3 and 4)*

When we multiply by the factor 7 we can use 2 of its parts to make 2 facts that we already know.

Which 2 parts or numbers can we multiply automatically and quickly? *(2 and 5)* **Why 2 and 5?** *(we know 2s facts by skip counting by 2 and we know 5s facts by skip counting by 5)*

What is 2 × 6? *(12)* **What is 8 × 2?** *(16)* **What is 5 × 8?** *(40)* **5 × 4?** *(20)* **5 × 9?** *(45)* **7 × 5?** *(35)*

Modeled Practice — Time: 8 min

1. Have students solve a multiplication problem with a factor of 7 using the Distributive Property and Break Apart Strategy. Have students complete the steps as the lesson progresses.

   Use ***Modeled Practice Sheet #1.*** Allow students time to complete the steps.

Teacher Note
This lesson will discuss the Distributive Property of Multiplication. The students do not need to memorize the term or the definition.

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Say: **Read the problem.** *(7 × 4)*

7 × 4 might be a fact you do not know automatically, so we are going to break apart the factor 7 to help us solve it.

In the Break Apart Strategy, step 1 is to break apart 7 to 2 and 5. What numbers? *(2 and 5)* **Why 2 and 5?** *(because we know these facts easily by skip counting by 2s or 5s)* **Write "2" and "5" in the boxes and between the parentheses.**

Look at the area models on the left of your sheet. Count down 2 rows and draw a dark horizontal line.

Check students' work.

Say: **Breaking apart 7 into 2 and 5 is like cutting the area model into 2 smaller rectangles like we have done previously.**

What are the multiplication expressions for the 2 area models? *(2 × 4 and 5 × 4)* **Write the multiplication expressions beside the models.**

What is step 2? *(multiply 2 and 5 by the other factor)*

What is the other factor in the problem? *(4)*

What are the 2 facts we will use to solve? *(2 × 4 and 5 × 4)* **Write "2" and "5" on the lines in step 2.**

Why do I have to multiply 4 to both 2 and 5? *(the problem is 7 × 4, so if we break apart 7, 4 then has to be multiplied to both of the parts that equal 7)*

What is 2 × 4? *(8)* **Write it on the line in step 3.**

Check students' work.

Say: **What is 5 × 4?** *(20)* **Write it on the line in step 3.**

Check students' work.

Say: **Step 3 is to add the products together to find the answer. What is step 3?** *(add the products together)*

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What products do I add together? *(8 and 20)*

What is 8 + 20? *(28)* **Write it on the line.**

Check students' work.

Say: **8 square units make up the first rectangle. How many square units are in the other rectangle?** *(20)*

Then what is 7 × 4? *(28)* **Write it.**

What 3 numbers belong in this number family? *(4, 7, and 28)*

If we switch the order of the factors what happens to the product? *(stays the same)* **How do you know?** *(accept reasonable answers, such as 7 × 4 and 4 × 7 both equal 28, the Commutative Property)*

What is the corresponding multiplication equation in this number family? *(4 × 7 = 28)*

What are the division equations in this number family? *(28 ÷ 7 = 4 and 28 ÷ 4 = 7)*

Let's review the Break Apart Strategy to solve 7s. What is step 1? *(break apart 7 into 2 and 5)*

What is step 2? *(multiply 2 and 5 by the other factor)*

What is step 3? *(add the products together to find the answer)*

Think about the factors in the problem 7 × 5. What is the best strategy to solve 7 × 5? *(skip count by 5s)* **Why?** *(because we know 5s facts by skip counting by 5)*

2. Students will use the Break Apart Strategy to solve a multiplication word problem.

Use ***Modeled Practice Sheet #2.*** Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

Say: **Read the problem. Ready, read: "8 friends went to a birthday party. On the way out they were told to grab a bag. Each bag**

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had 7 prizes. How many total prizes were used to make the 8 bags?"

What is the problem asking? *(how many total prizes were in the 8 bags)*

What is the multiplication expression for this word problem? *(8 × 7 or 8 × 7)* **Write it.**

What factor are we going to break apart? *(7)*

What is step 1? *(break apart 7 to 2 and 5)* **Write "2" and "5" on the line.**

What is step 2? *(multiply 2 and 5 by the factor 8)*

Complete the rest of the problem on your own.

What is 2 × 8? *(16)*

What is 5 × 8? *(40)*

What products did you add together in step 3? *(16 and 40)*

What is 8 × 7? *(56)*

How many total prizes were used to make the 8 bags? *(56 prizes)*

What 3 numbers belong in this number family? *(7, 8, and 56)* **Write it on the line.**

What is the corresponding multiplication equation in this number family? *(7 × 8 = 56)* **Write it.**

What are the division equations in this number family? *(56 ÷ 7 = 8 and 56 ÷ 8 = 7)* **Write it.**

Read the multiplication and division equations in this number family. *(7 × 8 = 56; 8 × 7 = 56; 56 ÷ 7 = 8; 56 ÷ 8 = 7)*

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3. Students will use the Break Apart Strategy and choose to break apart the factor 7 or 6 to solve the multiplication word problem.

   Use ***Modeled Practice Sheet #3.*** Have students complete the steps as the lesson progresses. Allow students time to complete the steps.

   Say: **Read the problem. Ready, read: "There are 7 tables in the restaurant. Each table has 6 chairs. How many chairs are there in all?"**

   What is the problem asking? *(how many chairs are there in all)*

   What is the multiplication expression for this word problem? *(7 × 6 or 7 × 6)* **Write it.**

   What factor can we break apart? *(6 or 7)*

   We can break apart 6 into 1 and 5 or 7 into 2 and 5.

   Choose the factor you would like to break apart and solve the problem.

   Turn to your math partner and share how you solved the problem by breaking apart 6 or 7.

   How many chairs are there in all? *(42)*

   The teacher will model solving the word problem by breaking apart the factor 7. Use ***Modeled Practice Sheet #3*** to demonstrate to students.

   Say: **Let's solve by breaking apart 7.**

   What is step 1? *(break apart 7 to 2 and 5)*

   What is step 2? *(multiply 2 and 5 by the factor 4)*

   What is 2 × 6? *(12)*

   What is 5 × 6? *(30)*

   What is step 3? *(add the products)*

   What products do we add together? *(12 and 30)*

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What is 12 + 30? *(42)* **What is 7 × 6?** *(42)*

There are 42 chairs in all.

Practice Time: 8 min

Activity 1: Students will solve 7s facts with a partner. Have students turn to the ***Practice Sheets*** on pages 69 and 70. Gradually fade assistance as students become more comfortable with the strategy. Provide corrective feedback. Keep the ***Multiplication Strategies Poster*** visible while students work. Remind students to use the ***Multiplication Strategies Bookmark*** if needed.

Say: **For the first few problems on your sheet I want you and your partner to solve the multiplication problems with a factor of 7.**

Ask questions such as:

- How is this strategy similar to solving 6s facts? *(breaking apart a factor into facts we know)*
- What are the steps in the Break Apart Strategy to solve 7s facts? *(step 1 is break apart 7 into 2 and 5, step 2 is multiply 2 and 5 by the other factor, step 3 is add the products together)*
- How can your knowledge of number families help you solve multiplication and division problems? *(accept reasonable answers)*

Activity 2: Have students turn to the ***Practice Sheets*** on pages 70 and 71. Students will continue to work with a partner practicing the Break Apart Strategy and reviewing previously taught strategies such as Make 10 Subtract the Factor and the 6s strategy in a game of *Four in a Row*. Students are to work out any problem that is not known automatically on their whiteboard. They must prove their answer to their partner before placing an "X" or "O" in the square. Students will only need 1person's practice sheet. The other sheet can be saved for another day.

Say: **With your partner you will play Four in a Row. However, before you place your "X" or "O" in the square, you must first answer the multiplication problem.**

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If the fact is one you know automatically, you may tell your partner the answer and place your "X" or "O."

If you do not know the answer automatically, you need to work out the problem on your whiteboard and allow your partner to check your work before you can place your "X" or "O" in the square. If the answer is incorrect, do not mark the square.

There is 1 game per page. To win a game you need to have 4 squares in a row.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many questions as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 11

Selecting a Strategy to Solve Multiplication Facts

Lesson Objectives	• The student will choose the appropriate strategy needed to solve unknown multiplication facts. • The student will apply their knowledge of the Associative, Commutative, and Distributive Properties to solve unknown multiplication facts.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Commutative Property of Multiplication, Distributive Property, number family	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 153-160) • Whiteboard with marker • Multiplication Strategy Poster	• Student Booklet (pp. 76-79) • Multiplication Strategy Bookmark (1 per student)

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Preview

Say: **Today we will review the Distributive Property to solve facts with the factors of 6s, 7s, and 9s. Your goal for learning these strategies is to be able to complete the strategy in your head and then no longer even need the strategy.**

Engage Prior/Informal Knowledge Time: 3 min

Practice mental math for addition and subtraction. Have students think aloud about the process for solving the problem. Encourage students to decompose numbers into tens and ones.

Suggested practice problems:

12 + 30 16 + 40 20 + 4 15 + 3 7 + 14 6 + 15

40 – 4 20 – 2 70 – 7 80 – 8 50 – 5 30 – 3

Ideas for practicing mental math:

- *Wastebasket Basketball* – a student answers a problem correctly and then is allowed to throw paper into the wastebasket for a point.
- *Beat the Teacher* – the problems are written on flashcards, and the teacher shows a card to 1 student at a time; if answered correctly, it goes into a stack for the students, if answered incorrectly it goes into a stack for the teacher. At the end of the round, whichever stack has more cards wins, either the students or the teacher.
- *Tic-Tac-Toe Math* – Create a tic-tac-toe sheet with problems written in each square. The students play with a partner. In order to mark a square, the problem must first be answered correctly.

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Modeled Practice — Time: 8 min

1. Review the Break Apart strategies for 6s and 7s taught in the previous lessons for facts with a factor of 6 or 7. Write on the whiteboard "3 x 7" and "9 x 6."

   Have students discuss the different ways to solve each problem. Allow students to share any personal variations on a strategy. For example, some students may describe solving for 3s by doubling the factor and adding 1 more set. This is the Distributive Property Strategy just explained in different terms.

   Provide an opportunity for each student in the group to explain his or her way of solving for at least 1 of the problems.

2. Have students turn to ***Modeled Practice Sheet #1*** to solve 3 × 7 using the Break Apart Strategy for 7s.

   Display the ***Multiplication Strategy Poster***. Have students refer to their ***Multiplication Strategy Bookmark***.

Say: **What is the first step in the Break Apart Strategy?** *(break the factor apart into 2 known factors)*

To make unknown facts into known facts, we break apart 1 of the factors.

To solve 3 × 7, we will break apart 7. What 2 parts do we break 7 into? *(2 and 5)*

Why do we choose 2 and 5 for 7? *(because they will result in facts that we already know)*

What is the second step in the strategy? *(multiply the 2 new parts by the other factor)*

Step 2 will be to multiply the 2 new parts by the other factor. What is the other factor? *(3)* **Write the 2 new multiplication expressions below 3 × 7.** *(students should write "3 × 2" and "3 × 5")*

What is 3 × 2? *(6)*

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What is 3 × 5? *(15)*

What is the third step in the strategy? *(add the products together to find the answer)*

How do you find the final answer? *(add 6 plus 15)* **Write it.**

What is 6 + 15? *(21)*

What are the 3 numbers in this number family? *(3, 7, and 21)* **Write them.**

Write the 3 corresponding facts on the lines below. *(7 × 3 = 21; 21 ÷ 7 = 3; 21 ÷ 3 = 7)*

3. Review the 9s. Point out similarities and differences between the strategies. Emphasize that the Break Apart Strategies for 6s and 7s employ the Distributive Property over Addition, while the 9s Strategy employs the Distributive Property over Subtraction.

Say: **The strategy we use for 9s facts is very similar to the Break Apart Strategy we use for 6s and 7s.**

When we come across an unknown multiplication fact that has a factor of 9, we can think of 9 as 10 because we know 9 = 10 – 1. How is this similar to the Break Apart Strategy we just reviewed? *(we are using facts we know, the 10s and the 1s)*

In this strategy we still have to multiply the other factor by both parts, but what is very different about the last step? *(we subtract the 2 products instead of adding them)*

When using the 9s strategy you must remember to subtract the 2 products. Why? *(because you multiply by 10, which is 1 greater than 9, instead of breaking the factor into 2 smaller parts)*

4. Have students turn to ***Modeled Practice Sheet #2*** to solve 9 × 6 using the Make 10 Subtract the Factor strategy.

Say: **For 9 × 6 we will use the Make 10 Subtract the Factor strategy to solve.**

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What is step 1 of the 9s strategy? *(think of 9 as 10)* **Write "10" on the first line.**

What is step 2? *(step 2 is to multiply 10 times the other factor)* **What is the other factor?** *(6)* **Write it.**

What is 10 × 6? *(60)* **Write it.**

What is step 3? *(step 3 is to subtract the other factor)* **We subtract the other factor because 9 = 10 – 1.**

What is the subtraction expression? *(60 – 6)* **Write it.**

What is 60 – 6? *(54)* **What is 9 × 6?** *(54)*

What numbers are in this number family? *(9,6, and 54)*

We know 9 × 6 = 54. What is the corresponding multiplication fact in the number family? *(6 × 9 = 54)* **Write it.**

What are the 2 corresponding division facts in the number family? *(54 ÷ 6 = 9 and 54 ÷ 9 = 6)* **Write them both.**

Practice — Time: 8 min

Activity 1: Students will work on multiplication problems in a round robin setting. Each student will have a problem to start. The students will only work the first step in the problem. Then all papers will be passed to the left. The student will then look to see what strategy is being used and complete the second step. The papers will be passed to the left again so that another student can complete the third step. Finally, the papers are passed left 1 more time so that the last student can fill in the answer. Every student should be working on the same step at the same time, just on a different problem.

Have students turn to the ***Practice Sheet*** on page 78.

Say: **I will provide each of you with a different multiplication problem. Write the problem I give you in the center circle on the top Round Robin Practice chart.**

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Provide each student with a problem such as: 9 x 4, 7 x 6, 4 x 6, 9 x 7, 3 x 6, or 6 x 8. The problems must have a 6, 7, or 9 as a factor and cannot be factors students know automatically, i.e., do not include a 2, 5, or 10 as a factor.

Say: **To start the round robin practice, everyone will complete the first step in a strategy and only the first step. If your problem has 2 factors with strategies we have learned, then choose the strategy you prefer.**

Instruct the students to begin and check to make sure each student only completes the first step and does not go any further.

Say: **For step 2 of the strategy, everyone will pass their papers to the person to the left. When you receive a new problem, look to see what strategy the person before you used and then use the same strategy to complete step 2. Stop after you have completed step 2.**

Teacher Note
If a student notices an error during round robin practice, ask the student to continue to work on the next step as if the error was not there. The student may choose to write the previous step in their own box or just correct the error in their work of the problem. The final student may go back and discuss the error with the group.

Check that each student only completed step 2 and then have the students pass the papers to the person on the left.

Say: **With the new problem you have in front of you, look at the first 2 steps. Decide what the third step should be and complete it in the Step 3 box. Stop when you are finished with that step.**

Check that each student only completes step 3 and then have the students pass the papers to the person to the left 1 last time.

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Say: **The last paper you have should have all the steps complete. It is your job to review the steps and write the final answer in the last box.**

Check that students complete the final step by writing in the answer. Ask students about their peers' work – were things done accurately, would they have chosen a different strategy, and if so, why?

Activity 2: Students will complete another round robin practice in the second chart. Provide each student with a different problem that has not been done yet. Have students complete step 1 and then pass papers. At the end of the round robin, ask students questions about the steps, their peers' work, and the strategies that were chosen.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **For the next 5 minutes you will work independently on multiplication problems. To solve these problems, use the strategies we have practiced. At the end of 5 minutes we will share our solutions.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 12

Doubling Numbers Mentally

Lesson Objectives	• The student will double numbers mentally. • The student will verbalize the mathematical steps taken to double a number mentally.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	doubling, even, factor, multiple, odd, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 161-170) • Whiteboard with marker • Index cards (optional)	• Student Booklet (pp. 80-84) • 6-sided number cubes (2 per pair of students) • Whiteboard with marker (1 per student) • Calculator (1 per student, optional)

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Preview

Say: **Today we will use mental math to practice doubling a number.**

Engage Prior/Informal Knowledge — Time: 3 min

Students will practice doubles addition facts and skip counting by 2.

Have students turn to the ***Engaged Practice Sheet***.

Say: **Practice skip counting by 2s. Fill in the missing numbers in the number sequences for problems 1-3.**

Allow time for students to complete the first 3 problems.

Say: **What do you notice about the numbers in these sequences?** *(accept answers that include that the numbers are all even or the sequence is counting by 2)*

Each of the numbers in these sequences are even. What does "even" mean? *(a number that has been doubled, a number that has been added to itself, or a number that can be divided into 2 equal groups)*

Solve the addition problems below. Will the answer to these problems be even or odd? *(even)* **Why?** *(because each problem is adding the same number twice)*

Modeled Practice — Time: 8 min

1. Connect doubling addition facts with multiplying by 2.

On a whiteboard, write the information the students provide as the lesson progresses.

Say: **If we know the doubles addition facts, then we also know the 2s multiplication facts.**

What does 2×8 mean? *(2 groups of 8)*

How do you write 2 groups of 8 as an addition expression? *(8 + 8)*

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Write "2 × 8 = 8 + 8" on the whiteboard.

Say: **Is this a true statement? Is 2 times 8 equal to 8 plus 8?** *(yes)*
How do you know? *(because 2 × 8 means 2 groups of 8 and 8 + 8 is 2 groups of 8)*

Being able to double a number in our head without having to use paper and pencil can help us when we have to add products and use strategies to solve multiplication problems.

2. Mentally practice single-digit doubling.

Use a whiteboard or index cards with 2s multiplication facts.

Say: **Let's practice doubling single digit numbers first, then we will move on to doubling 2-digit numbers.**

We will answer the following multiplication problems together. I will show the problem and hold up 3 fingers. Look at the problem and answer it in your head while I count down on my fingers. When I put my last finger down we will all say the answer together.

Hold up one problem at a time, either written on the whiteboard or on an index card. Hold up 3 fingers. Slowly count down with your fingers, allowing students some time to think. Encourage students to all answer together and to not shout the answer out before you are done counting down.

Suggested order of facts:

- 2 × 5 = 10
- 3 × 2 = 6
- 9 × 2 = 18
- 2 × 6 = 12
- 7 × 2 = 14

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- $4 \times 2 = 8$
- $2 \times 8 = 16$

Watch For

If students struggle with rapid recall of doubles facts try number-picture associations.
For example:
2 x 5 = 10 fingers on a hand
2 x 6 = 12 eggs in a carton
7 x 2 = 14 days in a week
2 x 8 = 16 crayons in a box
These kinds of visual associations sometimes help visual-spatial learners that have difficulty with memorization.

Teacher Note

If the majority of the group misses a problem, go back to that problem at the end to reinforce the correct answer.

Say: **Are the multiples, the products of a whole number multiplied by 2, even or odd?** *(even)* **Why?** *(because even means there are 2 equal groups in that number)*

If we multiply any number by 2, the answer will always be even. What digits will be in the ones place for an even number? *(0, 2, 4, 6, or 8)*

3. Multiply 2-digit numbers by 2.

Work with students to practice doubling 2-digit numbers mentally. Have students decompose numbers in their head, breaking apart the tens from the ones. Then students should double the tens, double the ones, and add the 2 products together.

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Teacher Note

For students who struggle to keep numbers in their heads, suggest they write down the doubled products on a whiteboard before adding.

Say: **We know when we multiply a number by 2 we are doubling the number, or adding it to itself.**

Starting with 12 × 2, what is the addition fact with the same answer as 12 × 2? *(12 + 12)* **What is the answer?** *(24)*

Break the number into tens and ones. How many tens in 12? *(1 ten)* **How many ones?** *(2 ones)*

Write the steps on a whiteboard to help students visualize the process.

Say: **First, double the tens. What is 10 x 2?** *(20)* **Next, double the ones. What is 2 x 2?** *(4)* **Then putting the tens and ones back together, what is 20 + 4?** *(24)*

Did we get the same answer? *(yes)*

How about 14 × 2? What is the repeated addition expression? *(14 + 14)*

To solve, first break 14 into tens and ones. How many tens? *(1 ten)* **How many ones?** *(4 ones)*

Double the tens and the ones. What is 10 doubled? *(20)* **What is 4 doubled?** *(8)*

Putting it back together, what is 20 + 8? *(28)* **What is 14 x 2 or 14 doubled?** *(28)*

How would you double 13? *(break it into tens and ones)* **What is the multiplication expression for doubling 13?** *(13 × 2)* **What is the repeated addition expression for doubling 13?** *(13 + 13)*

Doubling the tens, what do you get? *(20)*

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Doubling the ones, what do you get? *(6)*

Putting it back together, what do you get? *(26)*

What is 13 doubled? *(26)*

4. Double 2-digit numbers that will require regrouping.

Use a whiteboard to write out the steps.

Say: **Let's double a number that will have an answer greater than 28. Double 15. What is the multiplication expression?** *(15 × 2)* **What is the addition expression?** *(15 + 15)*

How many tens in 15? *(1 ten)* **How many ones?** *(5 ones)*

We know 10 doubled equals 20. What does 5 doubled equal? *(10)*

Now putting it together, what is 20 + 10? *(30)*

What is 15 doubled? *(30)*

Double 28. What is the multiplication expression? *(28 × 2)* **What is the addition expression?** *(28 + 28)* **How many tens in 28?** *(2 tens)* **How many ones?** *(8 ones)*

What is 20 doubled? *(40)* **What is 8 doubled?** *(16)*

Now put it back together. What is 40 + 16? *(56)*

What is 28 doubled? *(56)* **What is 28 × 2?** *(56)* **What is a division equation for this fact?** *(56 ÷ 2 = 28 or 56 ÷ 28 = 2)*

Practice Time: 8 min

Activity 1: Students will practice doubling 2-digit numbers. Have the group work the first problem together. Then have the students work with a math partner to complete the rest on the page.

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Have students turn to the ***Practice Sheet*** on page 81. Read the problem together.

Say: **Asheem was working on a project and needed rope. We will read the problem together. Ready, read: "Asheem was doubling a length of rope for his project. The rope was 16 inches long. He said the doubled length would be 23 inches. Is he correct?"**

What is the question asking you to find? *(whether 16 doubled is 23)*

How should we answer this question? *(accept reasonable answers that suggest doubling 16 to check whether 23 is correct)*

Is 23 a multiple of 2? *(no)* **How do you know?** *(it is odd, it is not even)* **So could 16 doubled be 23?** *(no)*

Double 16. First, doubling the tens, what do you get? *(20)* **Next, doubling the ones, what do you get?** *(12)*

What is 20 + 12? *(32)*

What is 16 doubled? *(32)*

When Asheem doubled his rope, what was the new length? *(32 inches)*

Complete the next 2 problems on your own, then discuss your answers with your math partner.

Activity 2: Students will practice doubling by playing the game *Doubles Doom*.

Have students turn to the ***Practice Sheet*** on page 82. Distribute 2 6-sided number cubes to each pair of students. 1 student will roll the number cubes, count up the dots on both number cubes and multiply that number by 2, doubling it. Students will then record the multiplication problem on their sheets.

If the student rolls doubles on the dice, their score is 0 for that round. Students should play 5 rounds (1 round being that both students roll

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once). When 5 rounds are over the students will add up their answers to determine a winner. Calculators may be used to find the total score.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 13

Associative Property of Multiplication

Lesson Objectives	• The student will use the Associative Property of Multiplication to solve multiplication problems involving more than 2 factors. • The student will apply their understanding of the Associative Property of Multiplication to multiply factors in any order.	
Vocabulary	**Associative Property of Multiplication:** a property of multiplication (but not division) that says changing the grouping of the numbers being multiplied will not change the product [for example: (2 x 5) x 3 = 2 x (5 x 3)]	
Reviewed Vocabulary	Associative Property of Addition, Commutative Property of Addition, factor, multiplication, product, sum	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 171-178) • Sticky notes • Whiteboard with marker	• Student Booklet (pp. 85-88) • Whiteboard with marker (1 per student) • Multiplication Table (1 per student, optional)

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Preview

Say: **Today we will work with a mathematical property called the *Associative Property of Multiplication* to solve multiplication problems with more than 2 factors.**

When I say, "Mary associates with the girls on her soccer team," what does the word "associate" mean? *(to hang out with someone or a group of people who know each other; emphasize with students that association is a group with a common purpose)*

Engage Prior/Informal Knowledge Time: 3 min

Review the Commutative Property and Associative Property of Addition. Present the following problems and have students solve on their whiteboards. Then have students explain their method for solving. If any student grouped the numbers differently when adding, discuss why it is possible to do so. If students did not group the numbers differently when adding, ask if it is possible and check with students to confirm that the same answer is found.

5 + 4 + 5 1 + 2 + 9 5 + 2 + 18 13 + 20 + 10

Ask questions such as:

- What 2 numbers did you group together and add first? *(answers will vary)*
- Why did you group those numbers first? *(accept possible answers such as it is easier to add those 2 numbers first, I made 10 first and added the last number, I made 20 first and added the last number, I made 30 first and added the last number)*
- Group the numbers in a different way. Is the sum the same or different? *(same)*
- In the first problem, what 2 numbers make 10? *(5 and 5)* What is 10 + 4? *(14)* In the second problem, what 2 numbers make 10? *(1 and 9)* What is 10 + 2? *(12)* Why is it easier to make 10 and add the third number in the problem? *(answers will vary)*
- In the third problem, what 2 numbers make 20? *(18 and 2)*

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- In the fourth problem, what 2 numbers make 30? *(10 and 20)*

Modeled Practice — Time: 8 min

1. Demonstrate the Associative Property of Multiplication by solving a 3-factor problem.

   Write "___ × ____ × ____" on the whiteboard. Write "3," "2," and "5" on 3 separate sticky notes. Have a blank sticky note ready. Put the 3, 2, and 5 sticky notes in this order on the whiteboard. Draw "(" and ")" (parentheses) on 2 separate sticky notes. Put the parentheses aside until needed.

   Say: **We are going to multiply the factors 3, 2, and 5.**

   We are going to start by multiplying 3 and 2 first.

   What is 3 × 2? *(6)*

   Write "6" on a blank sticky note and place it below 3 × 2. Draw an arrow to show that 3 × 2 equals 6.

Teacher Note
E.g., 3 × 2 × 5 V 6 × 5 =?

   Say: **What is the new multiplication problem? 6 times what number?** *(5)*

   Write "5" on a blank sticky note and place it below, after the 6. Then write "×" between 6 and 5.

   Say: **What is 6 × 5?** *(30)*

   What is the product of 3 × 2 × 5? *(30)*

2. Demonstrate the use of parentheses in multiplication problems with more than 2 factors.

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Use the parentheses sticky notes.

Say: **The *Associative Property of Multiplication* says that changing the grouping of the numbers being multiplied will not change the product.**

These are called parentheses. What are these called? *(parentheses)*

When you use the *Associative Property of Multiplication*, use parentheses to group what is multiplied first.

Put the parentheses around 2 and 5 in the problem.

Say: **To solve this problem, multiply the 2 factors in the parentheses first. What 2 numbers do we multiply first?** *(2 and 5)*

What is 2 × 5? *(10)*

Write "10" on a blank sticky note and place it below 2 × 5.

Say: **We have solved part of the problem: 2 × 5 equals 10.**

What number do we multiply 10 by? *(3)* **Why?** *(it's the last factor)*

Write "3" on a blank sticky note and place it below, after the 10. Then write "×" between 3 and 10.

Say: **What is 3 ×10?** *(30)*

Is the product the same as when we multiplied 3 and 2 first then 5? *(yes)*

We can group the factors in different ways using the parentheses and the product will be the same.

Just like with addition, we can group the numbers in a different way to find the sum. To find the product, we can group and multiply the factors in different ways. This is called the *Associative Property of Multiplication*.

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Rearrange the sticky notes in the order of 2, 3, and 5. Keep the parentheses around the last two factors, now 3 and 5, in the problem. The students will work along with the teacher.

Say: **What are the factors?** *(2, 3, and 5)* **Have we changed the factors?** *(no)*

Write the problem on your whiteboard. Make sure the parentheses are written around the 3 and the 5.

What do the parentheses tell us to do first? *(multiply 3 and 5)*

What is 3 × 5? *(15)* **Write "15" under (3 × 5) on your whiteboard.**

Do the same on your whiteboard. Draw an arrow to show that 3 × 5 equals 15. Instruct students to do the same.

Say: **What factor is left?** *(2)* **Write "2 ×" next to 15. What is the next multiplication problem we have to solve?** *(2 × 15)*

What is 2 × 15? *(30)*

Teacher Note
If students do not have 2 x 15 memorized, then remind them of the previous doubling lesson. Ask the students to break 15 into tens and ones. Double the tens and double the ones. Then put it back together.

Say: **What does 2 × (3 × 5) equal?** *(30)*

We changed the order of the factors in this problem. Was the product the same? *(yes)* **This is how the Commutative Property of Multiplication works.**

Think of the 2 different ways we have solved this problem so far. Which way you do you like best? Why? *(there is no right or wrong answer, allow students to explain their reasoning; some*

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possible reasons may be, "I like multiplying 3 × 10 better than 2 × 15.")

3. Demonstrate the use of parentheses in multiplication problems with more than 2 factors.

   Write "(2 × 4) × 3" on the whiteboard.

Teacher Note
Students may have a multiplication table out during the lesson to quickly find the answer to any unknown fact.

Say: **Read the problem.** *(2 × 4 × 3)* **Write the problem on your whiteboard.**

What do we multiply first? *(2 and 4)* **Why?** *(2 and 4 are in the parentheses; we multiply the numbers in the parentheses first)*

What is 2 × 4? *(8)* **Write it below the expression.**

Check students' work. "8" should be written under (2 × 4) on the students' whiteboards. Use an arrow to show that 2 × 4 equals 8.

Teacher Note
E.g., (2 × 4) × 3 V 8 × 3 = ?

Say: **We have solved part of the problem: 2 × 4 equals 8.**

What do we multiply by 8? *(3)* **What is 8 × 3?** *(24)* **Write it.**

Have students use their multiplication tables if 8 × 3 is not automatically known. Allow time for students to answer the problem and write it on their whiteboards.

Say: **What does 2 × 4 × 3 equal?** *(24)*

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This is 1 way to solve 2 × 4 × 3, but I think 8 × 3 is a challenging multiplication fact that I sometimes forget. Can we maybe solve it another way to get around having to solve 8 × 3? *(yes)*

Write "2 × (4 × 3)" on the whiteboard next to (2 × 4) × 3.

Say: **In the first problem, 2 and 4 were grouped together with the parentheses. We move the parentheses to change the grouping. Write "2 × (4 × 3)" on your whiteboard.**

What is grouped now? *(4 and 3)* **What is 4 × 3?** *(12)* **Write it under 4 × 3.**

Write "12" on the whiteboard under 2 × (4 × 3). Use an arrow to show that 4 × 3 equals 12.

Say: **The problem is not complete. Which factor have we not multiplied by yet?** *(2)*

What is 2 × 12? *(24)*

Teacher Note
If students do not have 2 x 12 memorized, then remind them of the previous doubling lesson. Ask the students to break 12 into tens and ones. Double the tens and double the ones. Then put it back together.

Say: **Do the 2 ways of solving this problem result in the same product?** *(yes)*

Write "(2 × 4) × 3 = 2 × (4 × 3)" on the whiteboard.

Say: **If we know the *Associative Property of Multiplication,* we know that we can change the grouping of the factors and the product will be the same.**

Explain how you think the *Associative Property of Multiplication* could be useful in mathematics. *(accept*

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reasonable answers; some student answers may include multiply easier numbers first, or if 2 factors create an unknown problem, multiply the other factors first)

4. If time allows, have students complete an example on their own.

   Have students group 2, 3, and 4 differently using parentheses on their whiteboards. Some examples are $(2 \times 3) \times 4$ or $(4 \times 2) \times 3$. Ask the following questions with each new multiplication problem created from the 3 factors. Answers will vary depending on how students choose to place the parantheses.

   - Which 2 factors will be multiplied together first?
   - What is the product of the 2 factors?
   - What is the third factor that is left to multiply?
   - What is the new multiplication problem?
   - What is the product of all 3 factors?

   Have students discuss and compare the products each time the factors were arranged in a different order.

Practice — Time: 8 min

Activity 1: Students will work with a math partner on the ***Practice Sheet*** on page 85. In each pair, 1 student will be A and the other B. The A students will work the problem in the A box while the B students will work the problem in the B box. The partners will then share their work with each other, checking that both products found are the same.

Have students complete the second problem on their own. Work along with students at first, and then fade teacher assistance.

Activity 2: Students will be placed in 2 teams. Each team will receive the same 3 factors. The team must group the factors in the way they best see fit. The teams will then present their grouping of the factors, their solution, and the reason why that particular grouping was chosen.

3 factor multiplication problems to use:

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- 5 x 6 x 2
- 2 x 2 x 4
- 5 x 1 x 3
- 10 x 2 x 4

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 14

Doubling Strategy to Solve Facts With 4 as a Factor

Lesson Objectives	• The student will learn the 4s facts by using the doubling facts strategy. • The student will use correct vocabulary to describe multiplication and related facts, as well as the properties of multiplication to solve.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Associative Property of Multiplication, automatic, factor, number family, product	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 179-190) • Doubling Strategy Poster • Whiteboard with markers • Deck of cards (optional) • 10-sided number cubes (optional) • 6 Popsicle sticks or counters in 8 bags	• Student Booklet (pp. 89-94) • 6-sided number cubes (2 per pair of students) • Whiteboard with marker (1 per student) • Calculator (1 per student, optional)

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Preview

Say: **Today we will use our knowledge of doubling and the properties of multiplication to solve multiplication facts with 4 or 8 as a factor.**

Engage Prior/Informal Knowledge — Time: 3 min

Review doubling facts and the properties of multiplication with students. Doubling is a needed skill for solving 4s facts before they are fluent. Have students turn to the ***Engaged Practice Sheet***.

For the first section of the ***Engaged Practice Sheet***, work with students to practice doubling 2-digit numbers mentally. Have students decompose numbers in their head, breaking apart the tens from the ones. Then students should double the tens, double the ones, and add the 2 products together.

For the second section:

Write on the whiteboard, "2 × 7 = 7 × 2" and "(2 × 5) × 6 = 2 × (5 × 6)."

Say: **We have learned about the properties of multiplication. The Commutative Property says changing the order of the factors will not change the product.**

What is 2 × 7? *(14)* **Does the other side represent the same amount?** *(yes, 7 × 2 equals 14)*

The Associative Property says changing how the factors are grouped will not change the product.

What is 2 × 5? *(10)* **What is 10 × 6?** *(60)* **Does the other side represent the same amount?** *(yes, 5 × 6 equals 30 and 2 × 30 equals 60)*

Use both of these properties to change the order or grouping of the factors to solve the next set of problems.

Review the answers with the group. Have students share how they arranged the factors to solve.

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Modeled Practice — Time: 8 min

1. Demonstrate the steps of the doubling strategy using the ***Doubling Strategy Poster.***

Say: **Read the steps together. Ready, read: "Step 1 – Think of 4 as 2 × 2. Step 2 – Double the other factor. Step 3 – Double the product."**

Have students turn to ***Modeled Practice Sheet #1***. The teacher and students should complete the steps as the lesson progresses.

Say: **How many columns are there?** *(4)* **How many dots are in each column?** *(6 dots in each column)*

Circle the first group of 2 columns and then circle the second group of 2 columns.

We can think of this array as 2 groups of 2 columns with each column having 6 dots.

Write "2 groups of 2 columns of 6" below the picture.

Say: **What multiplication problem can we write for 2 groups of 2 columns of 6?** *(2 × 2 × 6)* **Write the multiplication problem on the line for Step 1.**

We separated the array to break apart the factor 4. This is a picture of what happens in step 1: 4 is 2 × 2, 2 groups of 2 columns.

What is step 1? *(think of 4 as 2 × 2)*

Point to Step 2, "Double the other factor." What is step 2? *(double the other factor)*

What is the other factor? *(6)* **Write "2 × 6" on the line for Step 2. What is 6 doubled, or 2 × 6?** *(12)*

Step 3 is to double the product. What is step 3? *(double the product)* **Write "2 × 12" on the line for Step 3.**

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Look at the original expression, 2 × 2 × 6. We multiplied 2 × 6. Now what do we have to multiply? *(2 × 12)* **Why?** *(step 3 is double the product from step 2; there are 2 circles with the same amount of dots)*

What is 2 × 12? *(24)* **What is 4 × 6?** *(24)*

What are the 3 numbers in this number family? *(4, 6, and 24)*

Complete the corresponding multiplication fact. What number can you multiply by 6 to equal 24? (4)

2. Solve the multiplication fact 3 × 4. Have students turn to ***Modeled Practice Sheet #2***. The teacher and students should complete the steps as the lesson progresses.

Say: **How many columns here?** *(4)* **How many dots in each column?** *(3)* **What fact does the model represent?** *(3 × 4 or 4 × 3)*

Circle the first group of 2 columns and then circle the second group of 2 columns.

We can think of this array as 2 groups of 2 columns. Each column has how many dots? *(3)*

Write "2 groups of 2 columns of 3" below the picture.

Say: **What multiplication expression can we write for 2 groups of 2 columns of 3?** *(2 × 2 × 3)* **Write the multiplication problem on the line for Step 1.**

What is step 1 of the 4s strategy? *(think of 4 as 2 × 2)*

What is the second step? *(double the other factor)* **What number is the other factor?** *(3)*

Write "2 × 3" on the line for Step 2.

What is 3 doubled, or 2 × 3? *(6)*

What is step 3? *(double the product from step 2)* **Write "2 × 6" on the line for Step 3.**

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What is 2 × 6? *(12)*

Explain how the picture shows step 3. *(step 3 is double the product from step 2; there are 2 circles with the same amount of dots)*

What is 3 × 4? *(12)*

What are the 3 numbers in this number family? *(3, 4, and 12)*

Complete the corresponding division fact. What number can you divide by 12 to equal 4? (3)

3. Demonstrate how the doubling strategy can be used to solve 8s facts. The only fact with 8 as a factor that students will not have an alternative strategy for is 8 × 8. Have students work along with the teacher on whiteboards.

Display the ***Doubling Strategy Poster***.

Say: **Count by 2s, starting at 2 and stopping at 10. Both 4 and 8 are multiples of 2, or we can break them into equal groups of 2. 8s facts are another set of facts we can use the doubling strategy to solve because they are a multiple of 2.**

Write "8 × 8" on your whiteboard.

To solve for 8s we can use doubling, just like with the 4s facts. What 2 factors can we multiply that equal 8? *(2 × 4)* **How do we break apart 4?** *(2 × 2)* **Write "2 × 2 × 2 × 8" on your whiteboard.**

What is the next step*? (double the other factor)* **What is the other factor in this problem?** (8)

Double 8. What is 8 + 8 or 8 × 2? *(16)* **Now we have 2 × 2 × 16.**

What is step 2? *(double the product)* **What is 16 doubled, or 16 × 2?** *(32)*

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When multiplying by 8s, you have to double the product again. What is the next step? *(double the product again)* **Write "2 × 32" on the whiteboard.**

To find the final product we have to double 32. What do we break 32 into to mentally double it? *(tens and ones)*

Break 32 apart into 30 and 2. What is 30 doubled? *(60)* **What is 2 doubled?** *(4)* **What is 60 + 4?** *(64)*

2 × 2 × 2 × 8 equals 64.

What does 8 × 8 equal? *(64)*

What 3 numbers are in this number family? *(8, 8, 64)*

Teacher Note

It may be beneficial for your students to walk through the steps again. Refer to each step in the problem and have students repeat the steps as you go.

Students may also benefit from just memorizing 8 × 8 = 64 because of the multiple steps of having to double 8 three times.

Practice — Time: 8 min

Activity 1: Have students act out the strategy for 8s with 6 counters or Popsicle sticks as "candy" in 8 sacks, or "goodie bags."

Say: **I am having a birthday party and you all are invited. At the end of the party I will hand out goodie bags with 6 pieces of candy in each bag.**

Lay out the "goodie bags" with 6 Popsicle sticks or counters in each bag.

Say: **We have 8 equal groups of 6. The first person to leave the party is (insert student name here).**

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Have the student move 1 bag of "candy," Popsicle sticks or counters, to the side.

Say: **Then (insert student name here) leaves the party.**

Have the student move a bag to the side.

Say: **2 people have each taken 6 pieces of candy so far. What multiplication expression is 2 groups of 6?** *(2 × 6)*

What is 2 × 6? *(12)*

I have given out 12 pieces of candy so far.

Then 2 more friends leave the party (insert names here).

Have 2 students move 2 bags to the side.

Say: **We know 2 groups of 6 is 12. I just gave away another group of 12. What is 12 doubled?** *(24)*

I have given away 24 pieces of "candy."

The last 4 people all leave together.

Have a student move the last 4 bags of candy to the side.

Say: **I know that 4 bags of candy is 24 because I already gave away 4 bags of candy. So now I can double 24 to find out how many pieces of candy I gave away altogether.**

What is 24 doubled? *(48)*

6 equal groups of 8 is 48. We doubled 6, doubled 12, and doubled 24 to find the answer.

Activity 2: With a partner, students will practice the steps of the doubling strategy for 4s.

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Have students turn to the ***Practice Sheet*** on page 89. Students will complete the problems with their partner. Check students' work and provide corrective feedback as students work.

Watch For

For 4 × 4, students may want to break both 4s into 2 × 2. This adds an extra step the student does not need to do. Remind the student of the 3 steps: think of 4 as 2 × 2, and then double the other factor. They do not need to break both 4s apart into 2 × 2 × 2 × 2.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes practicing the doubling strategy. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answer with the group. Provide corrective feedback using mathematical language from the lesson. The students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 15

Review of 4s Strategy

Lesson Objectives	• The student will build fluency with multiplication facts involving factors of 4 and 8. • The student will apply understanding of the term Associative Property by following the steps of the strategy.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Associative Property of Multiplication, Commutative Property of Multiplication, dividend, divisor, factor, number family, product, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 191-202) • Whiteboard with marker • Multiplication Strategy Poster (optional)	• Student Booklet (pp. 95-100) • Whiteboard with marker (1 per student) • Multiplication Strategy Bookmark (1 per student)

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Preview

Say: **Today we will practice using the doubling strategy.**

Engage Prior/Informal Knowledge — Time: 2 min

Review number families and finding the missing factor or product with students. Have students turn to the ***Engaged Practice Sheet.*** Students will identify the missing product or factor to complete the number family.

Ask question such as:

- What is missing: the product or a factor? *(factor, factor, factor, product, product, factor)*
- What is the missing factor/product? *(4, 2, 1, 6, 20, 2)*
- How can you check to see if the number you filled in is correct? *(multiply the factors to find the product)*

Modeled Practice — Time: 8 min

1. Students will work with number families and find missing factors for the 4s facts.

   Have students turn to ***Modeled Practice Sheet #1***. The teacher and students should complete the steps as the lesson progresses. Refer the students to the first problems done at the beginning of the lesson.

Say: **From the *Engaged Practice Sheet*, what 3 numbers make up the number family in the first triangle?** *(3, 4, and 12)* **What are the corresponding facts?** *(3 × 4 = 12; 4 × 3 = 12; 12 ÷ 3 = 4; 12 ÷ 4 = 3)*

What is another number family we reviewed today? *(answers may vary; students should refer to the previously discussed number families)*

The number families we are going to look at today include 4 or 8 as one of the factors.

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The Commutative Property of Multiplication says that in a multiplication problem, the order of the factors does not change the product. For division, can the order of the numbers in the problem be changed? *(no)*

Why? *(in division you must begin with the total to be able to divide it into equal parts)*

Look at the triangle on the left. What are the equations for this number family? *(24 ÷ 6 = 4; 24 ÷ 4 = 6; 6 × 4 = 24; 4 × 6 = 24)* **Write the number family equations.**

If I write 6 ÷ 24, does it equal 4? *(no)* **How do you know?** *(not enough parts to make 24 equal groups)*

If time allows, have students draw an explanation of why 6 cannot be divided by 24. Write "6 ÷ 24 ≠ 4"underneath the number family equations.

What is the product of 4 × 6 = 24? *(24)*

24 is the product for the multiplication equation, so it will be the number divided, or the dividend, in the division problem.

Continue using ***Modeled Practice Sheet #1*** for the next section on number families.

Say: **Look at the triangle on the right. For this number family triangle the factors and the product are missing. Look at the multiplication and division problems below.**

What numbers make up this number family? *(8, 8, and 64)*

Give students a chance to fill in the numbers of the number family in the triangle.

Say: **We only have to write 1 multiplication and 1 division equation for this number family. Why?** *(if you switch the factors, it is still 8 × 8)*

List another number family that only has 2 facts. *(students should list any doubles fact)*

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2. Use the doubling strategy to solve unknown multiplication facts.

   Have students turn to ***Modeled Practice Sheet #2***. Display the ***Multiplication Strategy Poster***. The teacher and students should complete the steps as the lesson progresses.

Say: **Let's review the doubling strategy for 4s. What are the 3 steps?** *(step 1: think of 4 as 2 × 2; step 2: double the other factor; step 3: double the product)* **Write the steps at the top of your sheet.**

The first step is to think of 4 as 2 × 2. Write the new multiplication problem for step 1. *(2 × 2 × 7)*

What is the second step? *(double the other factor)*

What is the other factor? *(7)*

What is 7 doubled? *(14)* **Write out the multiplication-doubling expression.**

What is the third step? *(double the product)*

Think aloud about how you would double 14 in your head. *(break 14 apart into 10 and 4, double 10 to make 20, double 4 to make 8, add 20 plus 8 to get 28.*

What is 4 × 7? *(28)* **Write the final multiplication-doubling expression.**

Complete the number family triangle for this fact. *(28 in the top space, 7 in the space on the bottom right)*

Write the 3 corresponding facts for this number family on the lines below. *(7 × 4 = 28; 28 ÷ 7 = 4; 28 ÷ 4 = 7)*

For which factors in a multiplication problem can we use the doubling strategy to help solve? *(4 or 8)*

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Practice Time: 13 min

Activity 1: Students will work with a math partner to complete problems involving number families.

Have students turn to the ***Practice Sheet*** on page 98.

Say: With your math partner, complete the practice problems on your activity sheet.

Allow students time to complete their work. Have students discuss their answers with each other. Provide assistance as students work, and then gradually fade assistance.

Activity 2: Students will practice the doubling strategy playing *Doubles Doom*. Distribute 2 6-sided number cubes to each pair of students. Have students turn to the ***Practice Sheet*** on page 99.

1 student will roll the number cubes, count up the dots on both cubes, and multiply that number by 4. The student will then record the multiplication problem on their practice sheet. The other student will complete the same steps.

If the fact is automatic, the student may just write down the answer. If it requires a strategy, the student should use the doubling strategy or another previously taught strategy to solve. The student must show their work so that their partner may check their answer.

If the student rolls doubles on the number cubes, their score is 0 for that round. Students should play 5 rounds (1 round being that each student rolls once). When 5 rounds are over the students will add up their answers to determine a winner. Calculators may be used for finding the total score.

Independent Practice Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: You will work independently for 5 minutes on number families and practicing the 4s and 8s strategies. Complete as much as

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you can. At the end of 5 minutes we will discuss our answers as a group.

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. The students should mark the total number correct at the top of the page.

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25
Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 16

Practice Solving Unknown Multiplication Facts

Lesson Objectives	• The student will choose an appropriate strategy to solve unknown multiplication facts. • The student will apply their knowledge of the Associative, Commutative, and Distributive Properties to solve unknown multiplication facts. • The student will use mathematically precise language to explain how to solve multiplication facts.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	Associative Property of Multiplication, Commutative Property of Multiplication, Distributive Property, number family	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 203-214) • Whiteboard with marker • Multiplication Strategy Poster • Solving an Unknown Fact Poster	• Student Booklet (pp. 101-106) • Whiteboard with marker (1 per student) • Multiplication Strategy Bookmark (1 per student)

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Preview

Say: **In this lesson you will practice solving unknown multiplication facts using skills we have learned in past lessons.**

Engage Prior/Informal Knowledge — Time: 3 min

Discuss with students various strategies they use to solve problems. The discussion does not have to only be about solving math problems.

For example:

What would be a strategy…

- for rescuing a kitten from a tree?
- for saving money for a new expensive game?
- For adding up 3 items in the store to see if you have enough money?

Once students have shared several problem-solving strategies for different situations, ask if there was only one way to solve each problem.

Modeled Practice — Time: 8 min

1. Discuss ways to solve a multiplication problem using 2 different strategies. Solve using the Doubling Strategy for 4s. Use think-aloud dialog to demonstrate mental calculations.

   Have students turn to ***Modeled Practice Sheet #1***. The teacher and students will complete the steps as the lesson progresses.

Say: **Read the problem together. Ready, read: "The taco truck sold 3 potato breakfast tacos and 4 bacon breakfast tacos each hour for 4 hours. How many breakfast tacos in all were sold in the 4 hours?"**

What is the question asking us to find? *(the number of breakfast tacos sold in 4 hours)*

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What is the important information in the story? *(3 potato tacos and 4 bacon tacos were sold each hour for 4 hours)* **Circle the important information.**

This problem has 2 steps. What do we do first? *(add 3 and 4 together first to find all the tacos sold in 1 hour)* **Why?** *(we have to know how many tacos are sold each hour)* **How many tacos are sold each hour?** *(7)*

Teacher Note
The problem could also be solved by finding the total of each type of taco, then adding the totals together. However, for this lesson follow the suggested solution in order to practice solving multiplication strategies.

Say: **What is the second step?** *(multiply 4 × 7)* **Why?** *(it asks about all the tacos sold, and there are 4 groups (hours) with 7 tacos in each group)*

What are the 2 factors in this problem? *(4 and 7)*

Write "4 × 7" on the sheet. Circle the 4 to solve this problem using the doubling strategy.

What is 1 strategy you could use to solve? *(the 4s; doubling)* **What is another way to solve this fact?** *(the 7s; break apart)* **Why are there 2 ways?** *(you can use either factor strategy)*

Let's first solve it using the Doubling Strategy for 4s.

Display the ***Multiplication Strategy Poster*** and refer students to the Doubling Strategy for 4s. Have students use their ***Multiplication Strategy Bookmarks*** as a reference tool.

Say: **What is the first step in the Doubling Strategy for 4s?** *(think of 4 as 2 × 2)*

Write "2 × 2 × 7." What is the second step*? (double the other factor)* **What is 7 doubled?** *(14)* **Write "14" underneath.**

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What is the third step in this strategy? *(double the product)*

Write "2 ×" in front of 14. How do we solve this in our heads? *(10 doubled equals 20, 4 doubled equals 8, 20 plus 8 equals 28)*

What is 4 × 7? *(28)* **28 what?** *(tacos)*

How many breakfast tacos were sold in 4 hours? *(28 tacos)*

2. Solve the same problem using the Break Apart Strategy with 7s to demonstrate that there are 2 ways to solve the problem. Refer to the ***Multiplication Strategy Poster*** and have students use their ***Multiplication Strategy Bookmarks***.

Say: **We can also solve this problem a different way and get the same answer.**

What is the other factor strategy I could use? *(Break Apart with 7s)*

Write "4 × 7" again on your sheet next to the using 4s Doubling Strategy. Circle the 7.

What is the first step in the Break Apart Strategy? *(break 7 apart into 2 and 5)* **Why?** *(2 and 5 can be solved quickly using skip counting)*

Write "4 × (2 + 5)" under 4 × 7.

What is the second step in the strategy? *(multiply both parts of the other factor)*

Write "(4 × 2) + (4 × 5)."

What is 4 × 2? *(8)* **What is 4 × 5?** *(20)*

What is the third step? *(add the 2 products together)*

Write "8 + 20."

What is 8 + 20? *(28)*

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4×7 equals 28. How many breakfast tacos were sold? *(28 tacos)*

Did we get the same answer as the first time we solved the problem? *(yes)* **Why?** *(it has the same factors, just used different strategies to find the answer)*

Does the answer 28 make sense? Could 28 be too big or too small when I multiplied 4×7? *(allow students to share their reasoning with the group)*

3. Review the steps to choose a strategy. Remind students that it is their choice on how to solve each problem as long as the students can explain their solution.

   Display the ***Solving an Unknown Fact Poster*** for students to refer to during practice time.

Say: **Look back at the 2 examples. What is something similar we did for both problems?** *(circled the factor being used in the strategy)*

Step 1 on the Solving an Unknown Fact poster is to look at both factors and then circle the factor you will be breaking apart.

Step 2 is to follow the strategy steps that you have already learned.

Step 3 is to check that your answer makes sense.

Review the steps with me. Ready, read. "Step #1: Look at both factors and circle the factor you will break apart. Step #2: Follow the strategy steps. Step #3: Check that the answer makes sense."

I can use either factor to solve because the Commutative Property of Multiplication states I can multiply numbers in any order and always get the same answer.

4. Students will work backward, determining the original multiplication problem based on the strategy that was used.

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Have students turn to ***Modeled Practice Sheet #2***. Student will become detectives in order to find the missing multiplication problem.

Say: **In this problem, the work has been done along with the answer, but it is missing the original multiplication problem.**

Using our knowledge of the strategies and our detective skills, let's determine the missing multiplication problem.

Look at step #1. What kind of strategy was used? *(a factor was broken apart into 2 factors that are known)*

Which factor was broken apart? *(6)* **How do you know?** *(broken into 5 and 1)*

What is the other factor in this problem? *(6)* **How do you know?** *(multiplied 6 to 5 and 1)*

Did they solve the problem correctly? *(yes)*

What do you think is the missing multiplication problem? *(6 × 6)*

Does the answer 36 sound reasonable for the problem 6 × 6? *(yes)* **Explain your thinking.** *(answers should include: 5 × 6 is 30, 6 × 6 is just a little more, 36 is just a little more than 30)*

Practice — Time: 8 min

Activity 1: Students will work on choosing a way to solve practice problems.

Have students turn to the ***Practice Sheet*** on page 103.

Say: **You will work 3 problems on your own, choosing your own way to solve the problem. After solving the 3 problems, you will discuss with your partner how you solved each problem and your reason for choosing the particular method.**

After students have completed their work and discussions, check over answers and ask if there are any questions. If time is available allow students to share one of their solutions.

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Activity 2: Students will work in pairs as detectives determining the original multiplication problem based on the strategy used.

Have students turn to the ***Practice Sheet*** on page 104.

Say: **The next set of problems is another math mystery. Work with your partner to determine the original multiplication problem.**

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes solving multiplication problems. You may choose your own strategy to solve but you must show your work. Complete as much as you can. At the end of 5 minutes we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 17

Solving for a Missing Factor

Lesson Objectives	• The student will solve division facts by using knowledge of number families and the corresponding multiplication problem with a missing factor. • The student will show understanding of mathematical vocabulary in order to solve problems with a missing factor.	
Vocabulary	***unknown*:** a letter, such as *n*, that represents a missing number or factor in a mathematical equation	
Reviewed Vocabulary	dividend, division, divisor, equal groups, equation, factor, multiplication, product, quotient	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 215-226) • Counters (15)	• Student Booklet (pp. 107-112) • Counters (16 for every 2 students) • Multiplication table (1 per student)

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Preview

Say: **Today we will solve division problems by thinking of it as a multiplication problem with a missing factor. We will use equal-groups models and multiplication strategies as tools to help us find the missing number.**

Engage Prior/Informal Knowledge Time: 3 min

Distribute a ***multiplication table*** to each student. Review with students 1 way to use a multiplication table.

Note to Teacher

If a student is unsure how to use the multiplication table, explicitly teach the student how to find products on a multiplication table. If students are familiar with the table, review finding the products for the multiplication problems listed.

Possible questions and instructions:

- What do the top rows and left columns represent? *(the factors)*
- Where are the products listed on a multiplication table? *(students should indicate all the numbers on the table except the top row and far left column)*
- How would you find the product of 6 × 7 using the multiplication table? *(accept reasonable answers, such as look across the top to find 6 and then down the side to find 7, or follow the 2 paths to the intersection of 6s and 7s to find the product)*
- Use the multiplication table to find the product of 8 × 9. *(72)*
- Find the product of 4 × 8. *(32)*
- Find the product of 9 × 5. *(45)*

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Modeled Practice — Time: 8 min

1. Use counters to show the relationship of multiplication to division. By creating equal groups, students will have a visual representation of solving for a missing factor.

   Spread 15 counters on the table. Show ***Modeled Practice Sheet #1***. The teacher and students should complete the steps as the lesson progresses.

Say: ***n* is a letter that stands for an *unknown* number or factor. If I said *n* + 1 = 4, what does *n* stand for?** *(3)* **Yes, 3 + 1 = 4. Sometimes different letters will be used, but it always stands for an *unknown*.**

***n* × 3 = 15 means there are 15 items in equal groups of 3. The *unknown* factor is the number of equal groups.**

Can we write the corresponding division equation for this number family to help us solve? *(yes)* **What is the division equation**? *(15 ÷ 3 = n)*

Write "15 ÷ 3 = *n*" under the multiplication equation on ***Modeled Practice Sheet #1***.

Say: **Multiplication is the joining of equal-sized groups. Division is the separation of the whole into equal-sized groups. To find the missing factor, make groups of 3.**

Have students help make groups of 3 using the 15 counters on the table.

Say: **Is every counter in a group of 3?** *(yes)*

How many groups of 3 can 15 make? *(5)*

What does *n* represent for *n* × 3 = 15? *(5)*

What is 5? *(the missing factor, n)*

Write "5 × 3 = 15" and "15 ÷ 3 = 5" on ***Modeled Practice Sheet #1***.

Say: **Read the complete multiplication equation.** *(5 × 3 = 15)*

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Read the complete division equation. *(15 ÷ 3 = 5)*

2. Use multiplication problems with a missing factor to solve division facts.

   Have students turn to ***Modeled Practice Sheet #2***. The teacher and students should complete the steps as the lesson progresses.

Say: **In your own words, what is multiplication? What is division? How are they related?** *(accept reasonable answers that include multiplication is the joining of equal groups, division is the separating into equal groups, both operations have to do with equal grouping)*

When given a division fact, we can turn it into a multiplication problem with a missing factor. Why? *(because of number families, multiplication and division are related)*

Look at the division problem on your sheet. The dividend is the number being separated, or divided, into equal groups. Which number is being divided? *(21)*

The divisor is the number doing the dividing. Which number is dividing 21? *(3)*

In a multiplication problem the divisor will be 1 of the factors.

From this division problem, what are the factors for the multiplication problem? *(3 and a)*

Under the division problem, write the multiplication problem. The multiplication problem will have a missing factor, or an *unknown*. What is the multiplication problem you wrote? *(3 × a = 21 or a × 3 = 21)*

When we see the division problem 21 ÷ 3, we think to ourselves, "3 × what number equals 21?"

I can't remember immediately what number times 3 equals 21, so I am going to do some estimation to get close to 21.

We will use what we know. We know 3 × 5 = 15. Write down "3 × 5 = 15" under your multiplication problem.

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Is the product, 21, greater than or less than 15? *(greater than)*

If the product is greater, then we must need a factor that is greater than 5.

We also know 3 × 10 = 30. Write down "3 × 10 = 30" on your sheet.

Is 30 greater than or less than 21? *(greater than)* **So we know the other factor must not be greater than 10.**

This means that the missing factor is between 5 and 10.

Let's start with 6. What is 3 × 6? We know 3 × 5 equals 15, and 3 × 6 is 1 more group of 3. If we add 1 more group of 3 to 15, what do we get? *(18)*

18 is still less than 21. Let's go to 7. What is 3 × 7? We know 3 x 6 equals 18, and 3 × 7 is 1 more group of 3. Adding 1 more group of 3 to 18, what do you get? *(21)*

Did we find the missing factor? *(yes, 7)* **Write it.**

21 is the product. What are 7 and 3 called? *(factors)*

What is the completed multiplication equation? *(3 × 7 = 21)*

Go back to the original division problem. Can we now solve 21 ÷ 3? *(yes, 7)*

Practice — Time: 9 min

Activity 1: Students will solve an application problem by writing a missing factor problem and using the multiplication table.

Have students turn to the ***Practice Sheets*** on pages 109 and 110. Have students write an equation with a missing factor and then solve for the missing factor.

Say: **Read the problem together. Ready, read: "Perla is buying hotdog buns, soda, and 5 bags of chips for a BBQ. She needs to**

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buy 48 buns. The buns come in packages of 8. How many packages of 8 buns will Perla need to buy to have a total of 48 buns?"

What is the question asking you to find? *(how many packages of 8 buns to buy)* **What is the important information that is needed to solve?** *(48 buns, packages of 8)* **Circle the important information.**

If we were going to solve this using division, what division problem would we write? *(48 ÷ 8)* **Write it.**

Let's think of the division problem as a multiplication problem with a missing factor. What factor do we know? *(8)*

What does 48 represent? *(the total number of buns)*

We were given 1 factor and the total. Let's choose a letter to represent the missing factor. I am going to use the letter *p* to stand for the number of packages Perla needs to buy.

Write a multiplication equation for this problem with *p* as the missing factor.

Write "$p \times 8 = 48$" under the story problem. Check that students have written the same. If a student writes "$8 \times p = 48$," talk about how this will still work because of the Commutative Property of Multiplication.

Say: **This equation says that *p* packages with 8 hotdog buns each equals 48 total hotdog buns.**

Use the facts we know to help us solve. What is 8 x 5? *(40)* **Is this greater than or less than 48?** *(less than)*

What do we now know about the missing factor? *(it is greater than 5)* **How do you know?** *(because 48 is greater than 40)*

We can add 1 more group of 8 to 40 to find 6 x 8. What is 6 x 8? *(48)*

What is the missing factor? *(6)*

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How many packages of 8 buns should Perla buy to equal 48 buns altogether? *(6 packages)*

What is 48 ÷ 8? *(6)*

Activity 2: Students will work with a math partner to solve division facts by using counters, drawing an equal-groups model, and writing multiplication problems with missing factors.

Have students work with a math partner on the other problems. Distribute a pencil and 16 counters to each student pair. Work along with students at first, then fade teacher assistance.

Say: **On the activity sheet the first problem is 16 ÷ 4. With your math partner, use your counters or draw an equal-groups model to solve for the unknown.**

Ask questions such as:

- How many total counters do you need? *(16)*
- How do you know? *(it is the dividend, the number being divided)*
- How many counters will be in each group? *(4)*
- How do you know? *(4 is the divisor, the number doing the dividing)*
- What is the multiplication problem you can use to help solve? *(4 × n = 16)*
- What numbers are in this number family? *(4, 4, and 16)*

Collect the counters after the first 2 problems have been answered. Have the students write multiplication problems with missing factors to help solve the division problems on the rest of the sheet.

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Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 18

Multiplicative Comparison

Lesson Objectives	• The student will use number lines and a strip diagram to multiply numbers in multiplicative comparison problems. • The student will show understanding of mathematical vocabulary in order to solve problems with a number line and a strip diagram.	
Vocabulary	**No new words are introduced**.	
Reviewed Vocabulary	division, equal groups, equation, multiplication, number line, strip diagram	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 227-240) • Number line 0-20 (laminated or in plastic sleeve) with marker	• Student Booklet (pp. 113-119)

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Preview

Say: **Today we will solve multiplication problems using the number line and a strip diagram.**

Engage Prior/Informal Knowledge — Time: 3 min

Have students turn to the ***Engaged Practice Sheet***. Discuss words that mean multiplicative comparisons.

After reading each sentence as a group (or having a student volunteer read each sentence), have the students identify the bold word and what the word means in the sentence.

Modeled Practice — Time: 8 min

1. Using the number line, demonstrate multiplicative comparison by jumping an equal amount a set number of times.

   Have students turn to the ***Modeled Practice Sheet***. The teacher and students should complete the steps as the lesson progresses.

Say: **Read the story on the sheet. Ready, read: "I ate 4 pancakes this morning for breakfast and drank 4 glasses of orange juice. My brother ate 5 times as many pancakes as I did, but only drank 2 glasses of orange juice. How many pancakes did he eat?"**

What is the question asking us to find? *(the number of pancakes the brother ate)*

Did my brother eat more or less pancakes than me? *(more)*

How many times more? *(5)*

Did he eat 5 more pancakes or 5 times more pancakes? *(5 times more)*

How does that affect what type of problem we will write? *(we will write a multiplication equation and not an addition equation)*

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Teacher Note
The word "times" should be emphasized so that students hear the difference between just adding 1 more set and repeatedly adding more sets.

Say: **Yes, if my brother ate 5 more pancakes than me, I would just add 5 to the 4 pancakes I ate.**

Because my brother ate 5 times more than me, I cannot just add 5. I will need to add the number of pancakes I ate, which was 4, 5 times.

I need to find out what 5 groups of 4 equals. Write "5 groups of 4 equals *p*" on your sheet under the number line. Why do you think I used the letter "*p*?" *(because that is our unknown, we are trying to find how many pancakes)*

Draw an arch on the number line from 0 to 4 with a dry erase marker. Write "× 1" above the arch.

Say: **On the number line, how much will be in 1 group?** *(4)* **Why?** *(because you ate 4 pancakes)* **Draw an arch and write "× 1" because I ate 1 set of 4 pancakes.**

How many pancakes did my brother eat? *(5 times as many pancakes)* **He ate 5 groups of 4 pancakes.**

On the number line, draw a second arch over the next set of 4 (from the number 4 to 8). Label the second arch "× 2." Run your finger over the arches 1 at a time to illustrate 2 sets of 4.

Say: **This shows 2 groups of 4: 1 group of 4, 2 groups of 4.**

What does 2 groups of 4 equal? *(8)*

Have we answered how many pancakes my brother ate? *(no)* **How do you know?** *(your brother ate 5 groups or 5 times as many, not 2 groups or 2 times as many)*

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My brother ate 5 times more than me. How many more groups of 4 do I need? *(3 more sets)*

Draw 3 more arches over 3 more sets of 4, ending at 20. Label the arches "× 3," "× 4," and "× 5." Run your finger over the arches 1 at a time to illustrate 5 groups of 4.

Say: **On the number line, we have shown 5 groups of 4: 1 group of 4, 2 groups of 4, 3 groups of 4, 4 groups of 4, and 5 groups of 4.**

What does 5 groups of 4 equal? *(20)*

Have your finger pointing to the 20 on the number line when asking this question.

Say: **Write "p = 20" after "5 groups of 4 equals," under the number line.**

How many equal groups of pancakes did my brother eat? *(5)*

How many pancakes are in each equal group? *(4)*

How many pancakes did my brother eat? *(20 pancakes)*

What is the multiplication equation for this problem? *(5 × 4 = 20)* **Write it.**

2. Demonstrate the same problem again using the strip diagram on the ***Modeled Practice Sheet***. The teacher and students should complete the steps as the lesson progresses.

Teacher Note
Remind students that there is always more than 1 way to solve a problem. Working this same problem twice will help all students with their flexible thinking and problem solving abilities.

Say: **We will work the same problem using a strip diagram.**

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Here is the mathematics story again: I ate 4 pancakes this morning for breakfast and drank 4 glasses of orange juice. My brother ate 5 times as many pancakes as I did, but only drank 2 glasses of orange juice. How many pancakes did he eat?

What is the question asking us to find? *(the number of pancakes the brother ate)*

Complete the strip diagram on your page. It has 2 strips, 1 below the other. The top strip will represent the whole, or total, while the bottom strip represents the parts, or equal groups.

What information did the story provide? *(I ate 4 pancakes, my brother ate 5 times more than me)*

Did the story give us the total? *(no)* **Write "*p*" in the top strip of your diagram.**

Which number is repeating? *(4)*

How many times is 4 repeating? *(5 times)*

Divide the bottom strip into 5 groups of 4. Write "4" in each group to show that 4 is being repeated 5 times.

What does the 4 represent? *(the number of pancakes I ate)*

Why is it written 5 times? *(because my brother ate 5 times as many as me)*

What is 5 × 4? *(20)* **Write "= 20" after the *p* in the top strip of your diagram.**

What is the multiplication equation for this problem? *(5 × 4 = 20)* **Write the multiplication equation under the strip diagram.**

How many pancakes did my brother eat? *(20 pancakes)* **Write it below the equation.**

What is a corresponding division equation for this problem? *(20 ÷ 4 = 5 or 20 ÷ 5 = 4)* **Write it.**

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3. Compare the 2 ways to solve the problem. Display the number line and the strip diagram. Allow students to explain their own thinking.

Say: **Think about the 2 ways we solved the same problem.**

Which way works best for you, the number line or the strip diagram? Why? *(accept reasonable answers)*

We will continue to work the same problem twice so we can become comfortable with both methods of solving the problem.

Practice Time: 8 min

Activity 1: Have students use the number line and strip diagram to answer the multiplicative comparison problems.

Have students turn to the ***Practice Sheet*** on page 115. Work along with the students at first, then fade teacher assistance. Check students' work throughout the lesson, providing corrective feedback when necessary.

Say: **Let's read the first problem together. Ready, read: "The tree in the front yard is 8 ft. tall. The house is twice as tall as the tree. The 4 kids who live in the house love climbing the tree! How tall is the house?"**

What is the question asking us to find? *(the height of the house)*

What information does the word problem provide about the tree and the house? *(the tree is 8 feet tall and the house is twice as tall as the tree)*

Teacher Note

Students may underline, circle, or highlight important information in the problem to follow any normal problem-solving procedures already established in the classroom.

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Say: **The house is twice as tall as the 8 ft. tree. What is the multiplication problem we are trying to solve?** *(2 x 8)*

1 way to solve this problem is using a number line. How big will the first jump be? *(8)*

Draw 1 set of 8 on your number line.

How many more sets of 8 will you need to make? *(1 more)*

Draw the second set of 8 on your number line. What does 2 sets of 8 equal? *(16)*

Watch For

Students may struggle making jumps with larger numbers on the number line, such as with 8. Have students count up 8 more spaces, place a dot on the number they landed on, and then draw an arch to that number. Remind students to count the spaces between the harsh marks, not the hash marks themselves.

Say: **What is the repeated addition equation for this problem?** *(8 + 8 = 16)*

The number line shows 2 sets of 8. What is the multiplication equation for this problem? *(2 × 8 = 16)* **Write the multiplication equation under the strip diagram.**

How tall is the house? *(16 feet tall)* **Write it.**

Students will use a strip diagram to solve the same problem.

Say: **Let's read the first problem together. Ready, read: "The tree in the front yard is 8 ft. tall. The house is twice as tall as the tree. The 4 kids who live in the house love climbing the tree! How tall is the house?"**

What is the question asking us to find? *(the height of the house)*

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Draw a strip diagram to represent this problem. How tall is the tree? *(8 feet)*

How tall is the house compared to the tree? *(twice as tall)*

How many parts will you divide the strip into to represent the house? *(2 parts)* **Why?** *(twice means 2 times)* **Divide the bottom strip into 2 parts and write "2" in each group.**

What is the repeated addition equation for this problem? *(8 + 8 = 16)* **Write "t = 16" in the top strip of your diagram.**

What is the multiplication equation for this problem? *(2 × 8 = 16)* **Write the multiplication equation under the strip diagram.**

How tall is the tree? *(8 feet)* **The house is twice as tall as the tree, which equals what?** *(2 times as tall, or 16 feet)*

How tall is the house? *(16 feet)* **Write it.**

Activity 2: Allow students to work with a math partner to solve the next problem using both a number line and a strip diagram.

Have students turn to the ***Practice Sheet*** on page 116.

Say: **You will work with your math partner to solve the next problem. Each pair will work together to solve each problem twice, once using the number line and a second time using a strip diagram.**

After students finish their work, check answers as a group. Use mathematical language from the lesson.

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Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as many problems as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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ESTAR INTERVENTION

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 19

Using Strip Diagrams to Solve

Lesson Objectives	• The student will solve multiplication and division problems with the use of a strip diagram. • The student will solve word problems by identifying the part and the whole of the problem.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	division, equation, multiplication, strip diagram, unknown	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 241-254)	• Student Booklet (pp. 120-126) • Multiplication Strategy Bookmark – optional (1 per student)

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Preview

Say: **Today we will solve multiplication and division problems using a strip diagram.**

Engage Prior/Informal Knowledge — Time: 3 min

Review strip diagrams for multiplication and division. Have students turn to the ***Engaged Practice Sheet.***

Say: **Look at the strip diagram at the top of your sheet. What is the total?** *(24)*

How much is in each group? *(8)*

How many groups of 8? *(3)*

Write the corresponding multiplication and division equations for this strip diagram. *(8 x 3 = 24; 3 x 8 = 24; 24 ÷ 8 = 3; 24 ÷ 3 = 8)*

Have students use the corresponding equations provided to complete the blank strip diagram. Discuss the different parts of the equations and the strip diagram.

Ask questions such as:

- What is the total that will go in the top strip of the diagram? *(54)*
- How many parts will you divide the bottom strip into? *(6 or 9)*

Modeled Practice — Time: 8 min

1. Use a strip diagram to solve a multiplication problem.

   Have students turn to the ***Modeled Practice Sheet***. The teacher and students will complete the steps as the lesson progresses.

Say: **Read the problem together and then we will use a strip diagram to help us solve. Ready, read: "The Millers are remodeling their house. They bought 4 large buckets of paint with 6 gallons of**

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paint in each bucket, 12 paintbrushes, and 4 rags. How many gallons of paint did the Millers buy altogether?"

What is the question asking you to find? *(the number of gallons of paint)* **Underline it.**

What is the important information we need in order to solve the question? *(4 buckets of paint, 6 gallons in each bucket)* **Circle the important information. What about the other numbers in the problem? Are these important?** *(no)* **Why not?** *(not needed to answer the question)*

To solve this problem using a strip diagram, we will need to identify what information is given and what is the unknown we are solving for.

How many buckets of paint did the Millers buy? *(4 buckets)*

How much paint is in each bucket? *(6 gallons)*

We know the Millers bought 4 buckets of 6 gallons. What is the unknown? What are we solving for? *(the total gallons of paint; the product)*

The problem has given us 2 parts, or factors, and is asking for the total. What mathematical operation should we use to find the total when we combine equal groups? *(multiplication)*

Write the multiplication problem we have to solve to answer the question below the strip diagram. *(4×6 or 6×4)*

Do we know the total gallons of paint? *(no)* **Write an "*n*" in the top portion of the strip diagram because this is the unknown.**

The bottom strip will be for the equal groups.

How many buckets of paint? *(4)* **Divide the bottom of your strip into 4 groups.**

How many gallons are in each bucket? *(6)* **Write "6" in each section to represent 6 gallons in each bucket.**

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Using the strip diagram, how can we find the total amount of paint? *(accept reasonable answers that include adding the 4 6s, or multiplying 4 × 6)*

What is 4 × 6? *(24)*

Teacher Note
Students may need time to answer this question. Allow students the chance to use a previously learned strategy to solve or use repeated addition for the strip diagram to solve.

Say: **24 what?** *(24 gallons of paint)*

After the *n*, write "= 24 gallons of paint." In this problem we were looking for the total amount and we had equal groups so we could use multiplication to solve.

Make a connection to the number family. What are the corresponding division equations? *(24 ÷ 6 = 4 and 24 ÷ 4 = 6)*

2. Use a strip diagram to solve a division problem.

Have students continue on the ***Modeled Practice Sheet*** to the next problem. The teacher and students will complete the steps as the lesson progresses.

Say: **Let's read the next problem together. Ready, read: "For the light fixture in the dining room, Mrs. Miller needs 16 light bulbs. Light bulbs are sold in packages of 4. Each package costs $2.25. How many packages will she need to buy?**

What is the question asking you to find? *(the number of packages to buy)* **Underline it. What important information do we need to solve the question?** *(16 light bulbs needed, sold in packages of 4)* **Circle the important information. Do we need to know how much each package of bulbs was?** *(no)* **Why?** *(it doesn't help us answer the question)*

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We will use the strip diagram again to solve. First, we must identify what information the story provided.

What is the total number of light bulbs the light fixture needs? *(16)* **On the strip diagram, where do we write the total?** *(in the top strip)* **Write it.**

What other information does the story tell us about the light bulbs? *(they come in packages of 4)*

This problem has provided the total and one of the parts. Are the parts equal groups? *(yes)* **Will we multiply the total times the equal parts, or divide the total by the equal parts to find the missing information?** *(divide)*

What is the division problem we need to use to solve? *(16 ÷ 4)* **Using *n* as our unknown, what is the corresponding multiplication equation?** *(4 × n = 16)*

How many light bulbs are in each package? *(4)* **Divide the bottom strip into groups of 4. Draw a line for the first group of 4 and label it "4."**

Draw a second group of 4. What does 2 equal groups of 4 equal? *(4 + 4 = 8)*

We need to create enough groups of 4 to equal 16. Draw another line to make a third group of 4.

What is the total now for 3 groups of 4? *(12)* **How many more groups of 4 do we need to equal 16?** *(1 more group)*

Write 4 in the last section of the bottom strip. How many groups of 4 equal 16? *(4)*

Mrs. Miller needs 16 light bulbs altogether and each package has 4 bulbs. How many packages will she buy? *(4)* **4 what?** *(4 packages of light bulbs)*

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What is the difference between this problem and the first problem? *(the first problem gave 2 parts and asked for the total, the second problem gave the total and asked for 1 part)*

We wrote the corresponding multiplication equation before we started. How could that have helped you solve this division problem? *(accept reasonable answers such as using our multiplication fact knowledge to answer the division problem)*

Practice Time: 8 min

Activity 1: Students will work with a math partner to solve application problems using the strip diagram.

Have students turn to the ***Practice Sheet*** on page 122. Work along with students at first, then fade teacher assistance.

Say: **With your math partner, use a strip diagram to solve the problems.**

Ask follow-up questions and give instructions such as:

- What important information do we need from the story to solve the equation? *(3 packages and 12 in each package; $8 an hour; $32 total)*
- What missing information are you looking for? *(how many screws are in the 3 packages; how many hours the handyman worked)* Write an "n" to show the unknown, the piece we are solving for.
- Write the equation. *($12 \times 3 = n$; $12 \times 3 = 36$; $32 \div 8 = n$; $n \times 8 = 32$; $32 \div 8 = 4$)*
- In the first problem, how many parts will you divide the bottom strip into? *(3 parts of 12)*
- In the second problem, how many equal parts of 8 are in 32? *(4)*

Activity 2: Have students continue to work on the ***Practice Sheet*** on page 123.

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Say: **The third problem you have to write. The strip diagram is completed for you and now you must fill in the story.**

After students have written a story problem for the strip diagram, have them share their application problems. The problem can either be a multiplication or division problem.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the *Independent Practice Sheets* and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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Total Time: 25 minutes
Instructional Time: 19 minutes
Independent Practice: 6 minutes

Module MDFS
Lesson 20

Solving With Strip Diagrams

Lesson Objectives	• The student will solve multiplication and division problems with the use of a strip diagram. • The student will determine if the problem can be solved with multiplication or division based on the language of the problem and the information that has been provided.	
Vocabulary	**No new words are introduced.**	
Reviewed Vocabulary	division, equation, multiplication, strip diagram, unknown	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters (pp. 255-266)	• Student Booklet (pp. 127-132)

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Preview

Say: **Today we will continue to practice solving multiplication and division problems using a strip diagram.**

Engage Prior/Informal Knowledge Time: 3 min

Review strip diagrams for multiplication and division. Have students turn to the ***Engaged Practice Sheet.***

Say: **Look at the strip diagram at the top of your sheet. What is the total?** *(42)*

How much is in each group? *(6)*

How many groups of 6? *(7)*

Write the corresponding multiplication and division equations for this strip diagram. *($6 \times 7 = 42$, $7 \times 6 = 42$, $42 \div 6 = 7$, $42 \div 7 = 6$)*

Have students use the corresponding equations provided to complete the blank strip diagram. Discuss the different parts of the equations and the strip diagram.

Ask questions such as:

- What is the total that will go into the top strip of the diagram? *(36)*
- How many parts will you divide the bottom strip into? *(4 or 9)*

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Modeled Practice — Time: 8 min

1. Use a strip diagram to solve a division problem.

   Have students turn to the ***Modeled Practice Sheet***. The teacher and students will complete the steps as the lesson progresses.

Say: **Let's read the problem together. Ready, read: "There are 56 sticks of gum. Each package has 8 sticks of gum. The gum is a mix of peppermint, spearmint, and sweet mint. How many packages of gum are there?"**

What is the question asking you to find? *(the number of packages of gum)* **Underline it. What is the important information in the story?** *(56 sticks of gum, 8 sticks per pack)* **Circle it.**

We will use the strip diagram to solve. What should we do first? *(identify what information the story provided)*

What is the total number of gum sticks? *(56)* **On the strip diagram, where do we write the total?** *(in the top strip)* **Write it.**

What other information does the story tell us about the gum? *(it comes in packages of 8)*

This problem has provided the total and 1 of the parts. Are the parts equal groups? *(yes)* **To find the missing information will we multiply the total times the equal parts, or divide the total by the equal parts?** *(divide)* **How do you know?** *(we know the total, but we are missing how much is in each part, or the number of packages of gum)*

Using the information we know, what is the unknown? *(how many parts)* **Write the equation.** *(students should write $56 \div 8 = n$)* **What is the corresponding multiplication equation for this number family?** *($8 \times n = 56$)* **Write it.**

Do we know how many are in each part? *(yes, 8)* **Draw a line for the first group of 8 and label it.**

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Draw a second group of 8. What does 2 equal groups of 8 equal? *(8 + 8 = 16)*

We need to create enough groups of 8 to equal 56. Draw another line to make a third group of 8.

What is the total for 3 groups of 8? *(24)* **We need more groups of 8 before we reach 56. Could we add 3 more groups of 8? How much is 3 groups of 8?** *(24)* **What is 24 + 24?** *(48)*

Prompt students with doubling language explanations if needed.

Say: **Draw 2 more lines to make 3 more groups and label each group "8." Now how many groups of 8 do we have?** *(6)* **And what is the total so far?** *(48)*

We have not reached 56 yet. Can we have 1 more group of 8? *(yes)*

Teacher Note
Students may need to write out calculations off to the side of their paper. Encourage students to create a "work space" on their paper in order to make some of the needed calculations.

Say: **Adding another group of 8, what is the total?** *(56)*

How many parts of 8 are in 56? *(7)*

There are 56 sticks of gum. Each package has 8 sticks of gum. How many packages of gum are there? *(7)* **7 what?** *(7 packages of gum)*

What is the completed division equation? *(56 ÷ 8 = 7)* **What is the other division equation for this number family?** *(56 ÷ 7 = 8)*

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2. Use a strip diagram to solve a multiplication problem.

   Have students continue on the ***Modeled Practice Sheet*** to the next problem. The teacher and students will complete the steps as the lesson progresses.

Say: **Let's read the problem together and then we will use a strip diagram to help us solve. Ready, read: "Kiya buys 7 packages of highlighters, 2 packages of pens, and 3 spiral notebooks for school. Each package contains 4 highlighters. How many highlighters did Kiya buy?"**

What is the question asking you to find? *(the number of highlighters she bought)* **Underline it. What is the important information we need in order to solve the equation?** *(7 packages of highlighters, each package has 4 highlighters)* **Circle the important information.**

What information do we need? *(how many highlighters in all)*

How many highlighters in all is the total, and we are given the 2 parts. What operation will we use to solve this problem? *(multiplication)*

Write an *n* on the top of the strip diagram to remind us that this is our unknown.

What is the multiplication problem to be solved? *(7 × 4 or 4 × 7)* **Write it.**

Where on the strip diagram do we need to show the equal parts? *(bottom strip)*

How many groups will we divide the bottom strip into? *(7)* **Divide your strip into 7 groups.**

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Teacher Note

Dividing the strip into 7 equal parts can be difficult. Encourage the students to try to make the parts as equal as possible, but it does not have to be exact.

Say: **How many highlighters are in each package?** *(4)* **So what number do we write in the 7 parts?** *(4)* **How do you know?** *(there are 4 highlighters in each package)*

What are we looking for? *(the total number of highlighters)*

Using the strip diagram, how can we find the total number? *(accept reasonable answers that include adding the 7 4s or multiplying 7 × 4)*

What is 7 × 4? *(28)* **How did you solve?** *(accept reasonable answers; the fact was memorized; break apart 7s strategy; double 4s strategy)* **28 what?** *(28 highlighters)*

What is the corresponding division equation? *(28 ÷ 7 = 4 or 28 ÷ 4 = 7)*

In this problem we were looking for the total amount. We had equal groups so we could use multiplication to solve.

What is different in this problem than in the first problem? *(the first problem gave the total and asked for 1 part, while the second problem gave 2 parts and asked for the total)*

Practice — Time: 8 min

Activity 1: Students will work with a math partner to solve application problems using the strip diagram.

Have students turn to the ***Practice Sheet*** on page 129. Work along with students at first, then fade teacher assistance.

Say: **With your math partner, use a strip diagram to solve the problems. Write an *n* to show what we are trying to find.**

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Ask follow-up questions such as:

- What information does the story provide? *(#1: $27 and $3 per candy bar; #2: 8 brushes and 5 tables)*
- What missing information are you looking for? *(#1: number of candy bars sold; #2: how many paintbrushes in all)*
- In the first problem, how many equal parts of 3 are in 27? *(9)*
- In the second problem, how many parts will you divide the bottom strip into? *(5 parts of 8)*

Activity 2: Have students continue to work on the ***Practice Sheet*** on page 130.

Say: **The third problem you have to write. The strip diagram is completed for you, and now you must fill in the story.**

After students have written a story problem for the strip diagram, have the students share their application problems. The problem can either be a multiplication or division problem.

Independent Practice — Time: 6 min

1. For 5 minutes: Have students turn to the ***Independent Practice Sheets*** and complete as many items as possible.

Say: **You will work independently for 5 minutes. Complete as much as you can. At the end of 5 minutes, we will discuss our answers as a group.**

2. For the remaining time: Have students share their answers with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

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