Grade 6- Additive verses Multiplicative Relationships

6(4)(A) **Proportionality.** The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to compare two rules verbally, numerically, graphically, and symbolically in the form of $y = ax$ or $y = x + a$ in order to differentiate between additive and multiplicative relationships.

6(5)(A) **Proportionality.** The student applies mathematical process standards to solve problems involving proportional relationships. The student is expected to represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions.

Materials:

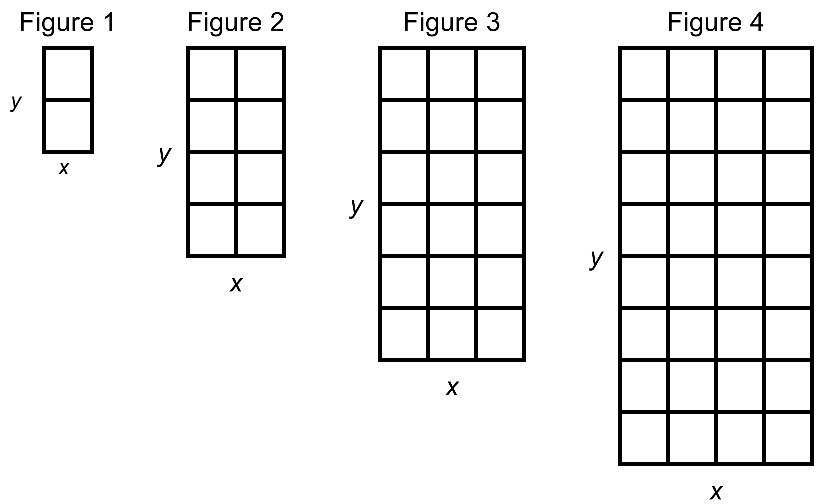
- **Additive verses Multiplicative Relationships** – one per group of two
- Colored Tiles – approximately 25 tiles per group of two

Additive Verses Multiplicative Relationships

- With a partner, determine who will be partner A and who will be partner B.
- Partner A completes **Activity A**. Partner B completes **Activity B**.
- Color tiles are provided to build the figures.

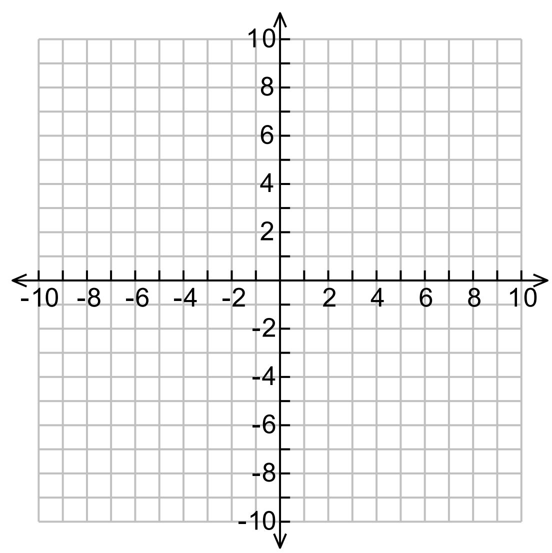
Activity A

1. What is the relationship between the length and width of this “family” of rectangles?



2. Use the relationship between the width and the length to complete the table and the graph.

Width x	Sketch	Length y
1		
2		
3		
4		
5		
x		



3. Based on your graph, if you were to continue the pattern, what ordered pair represents the length of a rectangle with a width of zero?

4. Does the ordered pair make sense in this situation? Why or why not?

5. Write an equation that could be used to determine y, the length of the rectangle, given x, the width of a rectangle, in this “family.”

6. With your partner, compare and contrast the two “families of rectangles.” How are the graphs, tables, and equations similar and different?

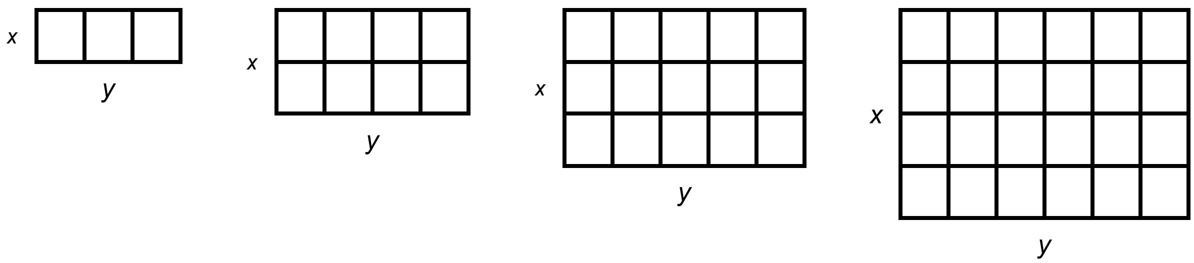
Additive Verses Multiplicative Relationships

- With a partner, determine who will be partner A and who will be partner B.
- Partner A completes **Activity A**. Partner B completes **Activity B**.
- Color tiles are provided to build the figures.

Activity B

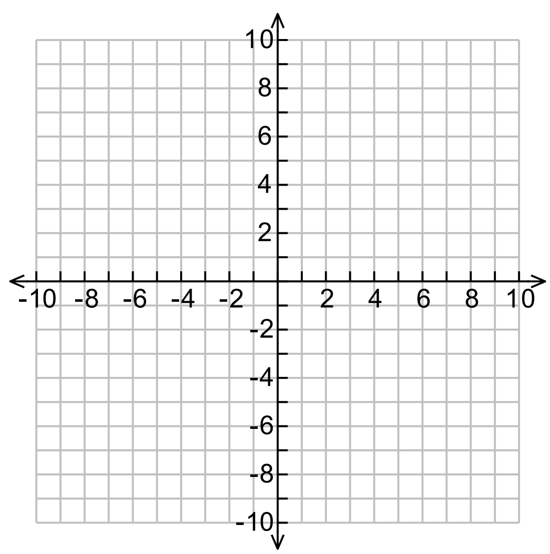
1. What is the relationship between the length and the width of this “family” or rectangles?

Figure 1 Figure 2 Figure 3 Figure 4



2. Use the relationship between the width and the length to complete the table and the graph.

Width *x*	Sketch	Length *y*
1		
2		
3		
4		
5		
x		



3. Based on your graph, if you were to continue the pattern, what ordered pair represents the length of a rectangle with a width of zero?

4. Does the ordered pair make sense in this situation? Why or why not?

5. Write an equation that could be used to determine *y*, the length of the rectangle, given *x*, the width of a rectangle, in this “family.”

6. With your partner compare and contrast the two “families of rectangles.” How are the graphs, tables, and equations similar and different?

Answer key:

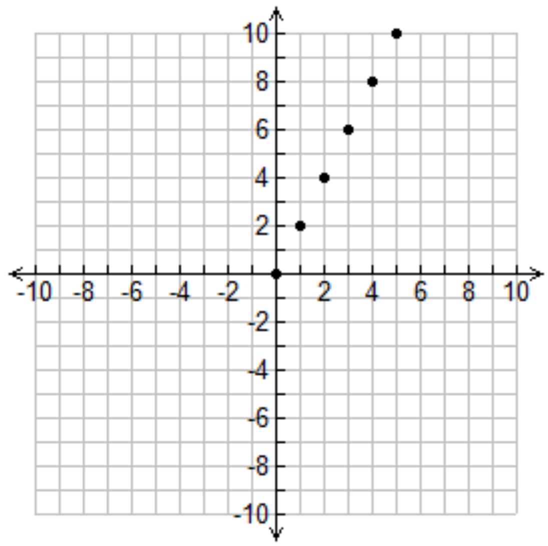
Additive Verses Multiplicative Relationships

Activity A

1. What is the relationship between the length and width of this “family” of rectangles?
   The length of each rectangle is two times the width.

2. Use the relationship between the width and the length to complete the table and the graph.

Width *x*	Length *y*
1	**2**
2	**4**
3	**6**
4	**8**
5	**10**



3. Based on your graph, if you were to continue the pattern, what ordered pair represents the length of a rectangle with a width of zero? **(0,0)**

4. Does the ordered pair make sense in this situation? Why or why not? **Yes, because if a rectangle does not have a width, then it cannot have a length.**

5. Write an equation that could be used to determine *y*, the length of the rectangle, given *x*, the width of a rectangle, in this “family.” $\boldsymbol{y = 2x}$.

6. With your partner, compare and contrast the two “families of rectangles.” How are the graphs, tables, and equations similar and different?
   In both rules, the data points on the graph form a straight line.

   Rule A**:**
   2*x* The origin is represented in the table and in the graph.

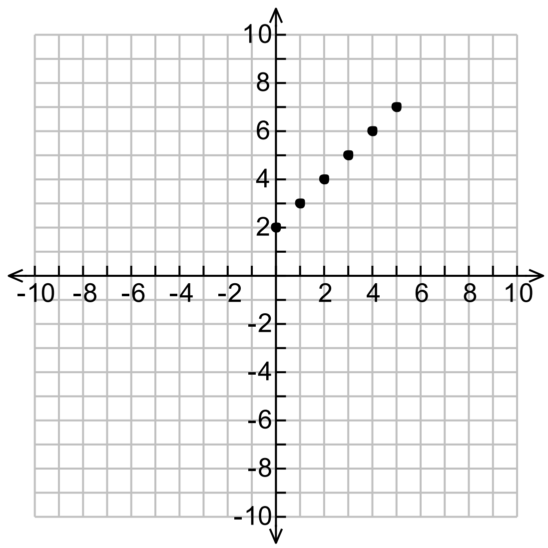
   Rule B**:**
   ***x* + 2 The origin is not represented in the table and the graph.**

Activity B

1. What is the relationship between the length and width of this "family" of rectangles?
   The length of each rectangle is two more than the width of each rectangle.

2. Use the relationship between the width and the length to complete the table and the graph.

Width x	Length y
1	**3**
2	**4**
3	**5**
4	**6**
5	**7**



3. Based on your graph, if you were to continue the pattern, what ordered pair represents the length of a rectangle with a width of zero? **(0, 2) Note: In context, (0, 2) does not make sense; however, having this conversation with students prepares them to think about what numbers may work in context.**

4. Does the ordered pair make sense in this situation? Why or why not? **No, because a rectangle that has a width of zero cannot have a length of two.**

5. Write an equation that could be used to determine *y*, the length of the rectangle, given *x*, the width of a rectangle, in this "family."
   ***y* = *x* + 2**

6. With your partner, compare and contrast the two "families of rectangles." How are the graphs, tables, and equations similar and different?
   In both rules, the data points on the graph form a straight line.

   Rule A:
   2x The origin is represented in the table and in the graph.

   Rule B:
   x + 2 The origin is not represented in the table and the graph.

Grade 7- Critical Attributes of Similarity

7(5)(A) **Proportionality.** The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to generalize the critical attributes of similarity, including ratios within and between similar shapes.

Materials:

- **Testing Triangle Ratios** (print with "No Scaling")
- Metric Ruler – 1 per student
- Protractor or Patty Paper (1-2 sheets per student)

Testing Triangle Ratios

Part A

Use the **Shape Templates** shown below and a metric ruler to complete the following tables. Measure to the nearest tenth of a centimeter.

Comparing Triangle *B* to Triangle *A*

1.

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *A*			
Triangle *B*			

How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?

Comparing Triangle *C* to Triangle *A*

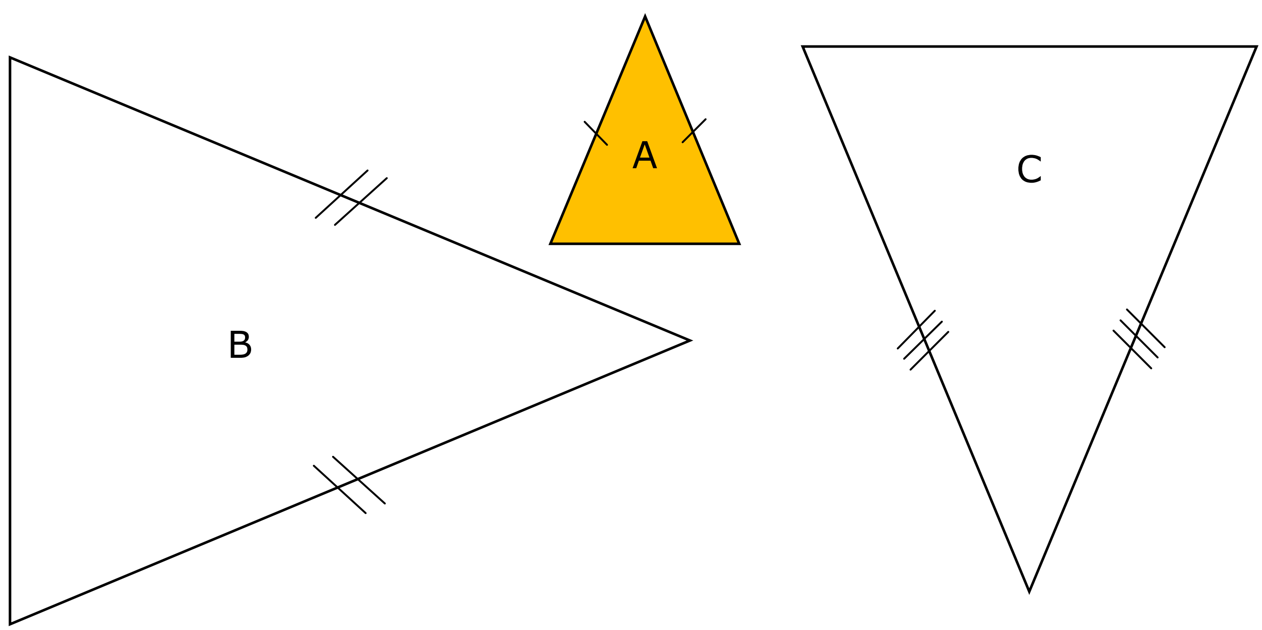
2.

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *A*			
Triangle *C*			

How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?

Do you think these three triangles are similar to each other? Justify your conjecture.

Shape Templates (Part A)



Testing Triangle Ratios (Continued)

Part B

Use the **Shape Templates** shown below and a metric ruler to complete the following. Measure to the nearest tenth of a centimeter.

Comparing Triangle *E* to Triangle *D*

3.

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *D*			
Triangle *E*			
How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?			

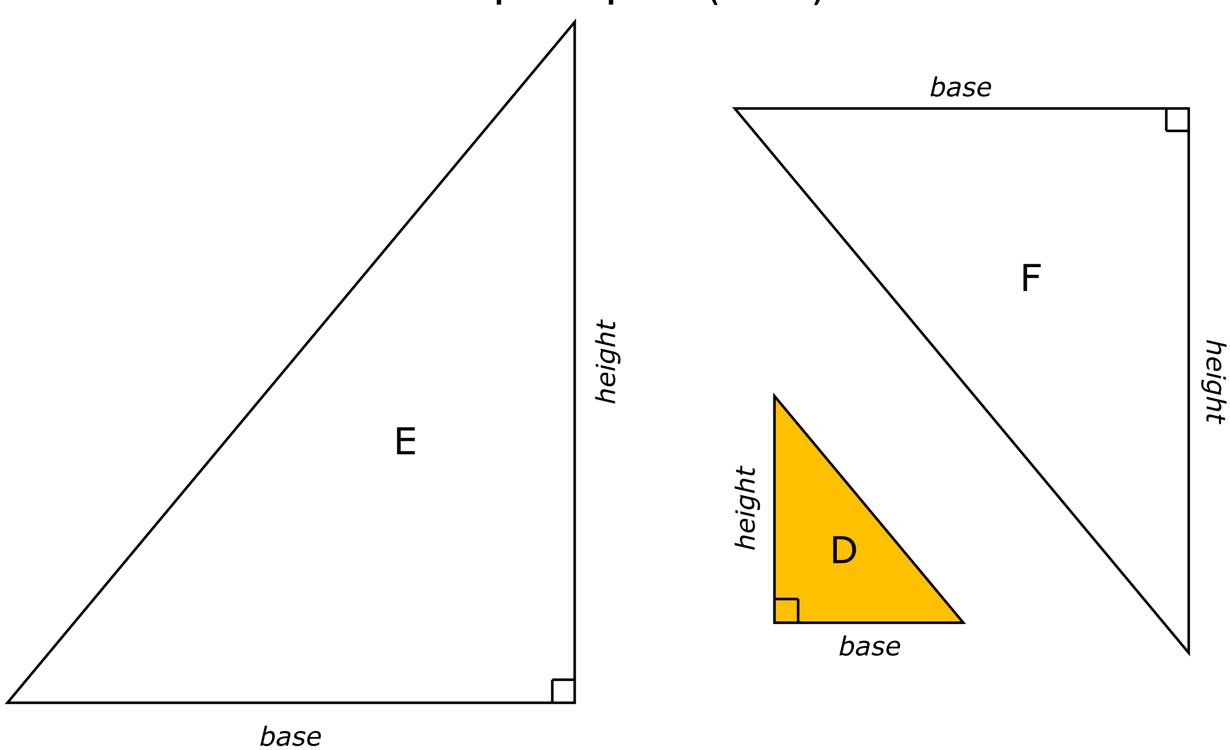
Comparing Triangle *F* to Triangle *D*

4.

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *D*			
Triangle *F*			
How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?			

Do you think these three triangles are similar to each other? Justify your conjecture.

Shape Templates (Part B)



Testing Triangle Ratios (Continued)

Summary

Work with a partner to answer the following questions:

5. What do you notice about the height-to-base ratios of all six of the triangles, *A, B, C, D, E,* and *F*? Do you think the six triangles are all similar to each other? Why or why not?

6. Based on the height-to-base ratios and the angle relationships, what inferences can you make about triangles *A, B,* and *C*?

7. Based on the height-to-base ratios and the angle relationships, what inferences can you make about triangles *D, E,* and *F*?

8. Based on the height-to-base ratios and the angle relationships, what inferences can you make about triangles *A, B, C, D, E,* and *F*?

9. If you are given a new set of triangles, describe a process you could use to determine if they are similar to each other.

Answer Key:

Part A

Comparing Triangle *B* to Triangle *A*

1

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *A*	2.5	3	$\mathbf{\frac{3}{2.5} = \frac{6}{5}}$
Triangle *B*	7.5	9	$\mathbf{\frac{9}{7.5} = \frac{90}{75} = \frac{6}{5}}$

How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?

They are the same.

Comparing Triangle *C* to Triangle *A*

2

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *A*	2.5	3	$\mathbf{\frac{3}{2.5} = \frac{6}{5}}$
Triangle *C*	6	7.2	$\mathbf{\frac{7.2}{6} = \frac{72}{60} = \frac{6}{5}}$

How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?

They are the same.

Do you think these three triangles are similar to each other? Justify your conjecture.

Possible responses include the following: Yes, I think these triangles are all similar to each other. The equivalent base-to-height ratios within each of the triangles lets me know they all have the same shape.

No, I don't think these triangles are necessarily similar because I don't know if the corresponding angles are congruent.

Part B

Comparing Triangle *E* to Triangle *D*

3

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *D*	2.5	3	$\mathbf{\frac{3}{2.5} = \frac{6}{5}}$
Triangle *E*	7.5	9	$\mathbf{\frac{9}{7.5} = \frac{90}{75} = \frac{6}{5}}$

How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?

They are the same.

Comparing Triangle *F* to Triangle *D*

4

	Base Length (cm)	Height (cm)	$\frac{h}{b}$
Triangle *D*	2.5	3	$\mathbf{\frac{3}{2.5} = \frac{6}{5}}$
Triangle *F*	6	7.2	$\mathbf{\frac{7.2}{6} = \frac{72}{60} = \frac{6}{5}}$

How do the ratios of $\frac{h}{b}$ within each of these two triangles compare?

They are the same.

Do you think these three triangles are similar to each other? Justify your conjecture.

Possible responses include the following: Yes, I think these triangles are all similar to each other. The equivalent base to height ratios within each of the triangles lets me know they all have the same shape.

No, I don't think these triangles are necessarily similar because I don't know if the corresponding angles are congruent.

Summary

Possible responses include:

5. What do you notice about the height-to-base ratios of all six of the triangles, *A, B, C, D, E,* and *F*? Do you think the six triangles are all similar to each other? Why or why not?
   I noticed that the base-to-height ratios of the six triangles are all congruent. Originally, I thought this meant the triangles must be similar to each other, but I can see from looking at the triangles that they are not all similar. I think they are not similar because the corresponding angles are not congruent.

6. Based on the height-to-base ratios and the angle relationships, what inferences can you make about triangles *A, B,* and *C*?
   I used patty paper to compare the measures of corresponding angles and verified that for these three triangles, the corresponding angles are congruent. I already knew the base-to-height ratios within each of these three triangles are also congruent. Because both of these conditions are true, I know that triangles *A, B*, and *C* are all similar to each other.

7. Based on the height-to-base ratios and the angle relationships, what inferences can you make about triangles *D, E,* and *F*?
   I used patty paper to compare the measures of corresponding angles and verified that for these three triangles, the corresponding angles are congruent. I already knew the base-to-height ratios within each of these three triangles are also congruent. Because both of these conditions are true, I know that triangles *D, E*, and *F* are all similar to each other.

8. Based on the height to base ratios and the angle relationships, what inferences can you make about triangles *A, B, C, D, E,* and *F*?
   Triangles *D, E*, and *F* are right triangles. Triangles *A, B*, and *C* are acute triangles. Therefore I know that they do not have congruent corresponding angles. Even though the base-to-height ratios among all six triangles are congruent, they cannot be all similar to each other because both conditions are not true.

9. If you are given a new set of triangles, describe a process you could use to determine if they are similar to each other.
   I would have to show that both the corresponding angles are congruent and the corresponding side lengths are proportional.

Grade 8- Slope Triangles

8(4)(A) **Proportionality.** The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $(y_2 – y_1) / (x_2 – x_1)$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line.

8(8)(D) **Expressions, equations, and relationships.** The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Materials:

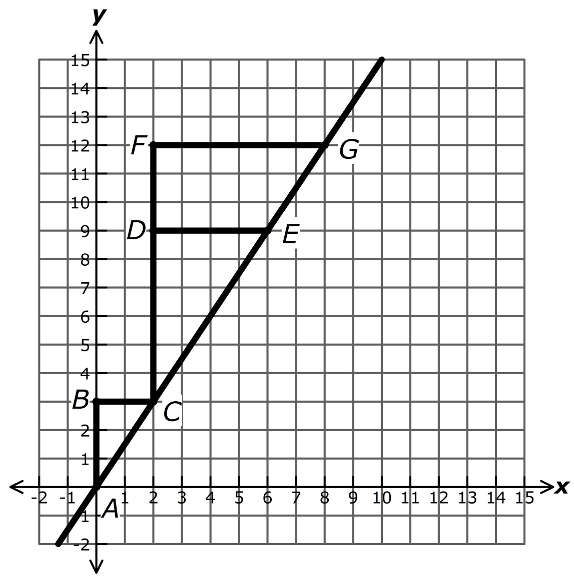
- **Slope Triangles**

Prior knowledge: It is expected that students will have explored special angle pairs formed when parallel lines are cut by a transversal prior to completing this activity.

Slope Triangles

Two triangles are **similar** if corresponding angles are ____________ and corresponding side lengths are ___________.

Right triangles *ABC*, *CDE*, and *CFG* are all represented on the following coordinate plane.



1. If $AB \parallel CD$, what is true about $\angle BAC$ and $\angle DCE$? Justify your answer.

2. If $BC \parallel DE$ and $DE \parallel FG$, what is true about $\angle BCA$, $\angle DEC$, and $\angle FGE$? Justify your answer.

3. Complete the table.

	Triangle *ABC*	Triangle *CDE*	Triangle *CFG*
$\dfrac{\text{length of vertical leg}}{\text{length of horizontal leg}}$			

4. Use the angle relationships and the ratios of the lengths of the legs to verify that triangles *ABC*, *CDE*, and *CFG* are all similar to each other.

5. If a new right triangle with a hypotenuse on the same line is added to the graph, do you think it would be similar to triangles *ABC*, *CDE*, and *CFG*? Why or why not?

6. Complete the table.

Hypotenuse	Endpoints	Use the coordinates of the endpoints to determine the lengths of the legs.	
$\overline{AC}$	A (____, ____) C (____, ____)	Vertical leg:	Horizontal leg:

$\overline{CE}$	C (____, ____) E (____, ____)	Vertical leg:	Horizontal leg:

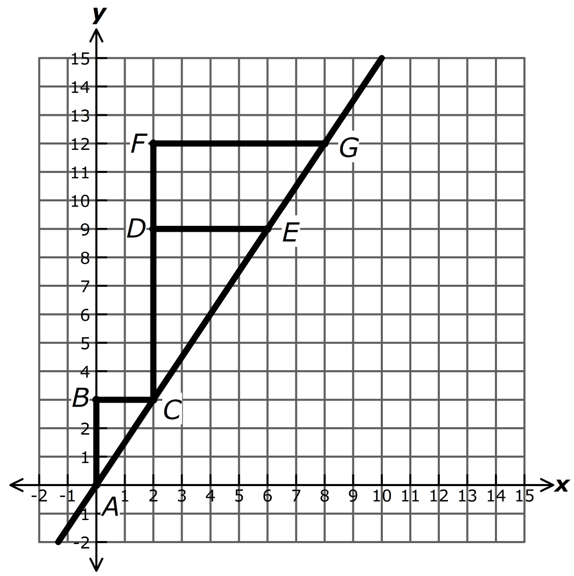
$\overline{CG}$	C (____, ____) G (____, ____)	Vertical leg:	Horizontal leg:

7. A new right triangle with a hypotenuse that lies on the same line is added to this graph. The coordinates of the endpoints of the hypotenuse are (x_1, y_1) and (x_2, y_2). How could you determine the ratio of the length of the vertical leg to the length of the horizontal leg of this triangle?

Answer Key:

Two triangles are **similar** if corresponding angles are **congruent** and corresponding side lengths are **proportional**.

Right triangles *ABC*, *CDE*, and *CFG* are all represented on the following coordinate plane.



1. If $AB \parallel CD$, what is true about $\angle BAC$ and $\angle DCE$? Justify your answer.
Since $AB \parallel CD$, $\angle BAC$ and $\angle DCE$ are corresponding angles formed when parallel lines are cut by a transversal. Therefore, $\angle BAC \cong \angle DCE$.

2. If $BC \parallel DE$ and $DE \parallel FG$, what is true about $\angle BCA$, $\angle DEC$, and $\angle FGE$? Justify your answer.
Since, $BC \parallel DE$ and $DE \parallel FG$, $\angle BCA$, $\angle DEC$, and $\angle FGE$ are all corresponding angles formed when parallel lines are cut by a transversal. Therefore, $\angle BCA \cong \angle DEC \cong \angle FGE$.

3. Complete the table.

	Triangle *ABC*	Triangle *CDE*	Triangle *CFG*
$\frac{\text{length of vertical leg}}{\text{length of horizontal leg}}$	$\frac{AB}{BC} = \frac{3}{2}$	$\frac{CD}{DE} = \frac{6}{4} = \frac{3}{2}$	$\frac{CF}{FG} = \frac{9}{6} = \frac{3}{2}$

4. Use the angle relationships and the ratios of the lengths of the legs to verify that triangles *ABC*, *CDE*, and *CFG* are all similar to each other.
Corresponding side lengths are proportional because the ratios of the lengths of the vertical legs to the lengths of the horizontal legs are congruent.
Corresponding angles are congruent because $AB \parallel CD$, so $\angle BAC$ and $\angle DCE$ form corresponding angles when parallel lines are cut by a transversal, and are congruent. Also, $BC \parallel DE$ and $DE \parallel FG$, $\angle BCA$, $\angle DEC$, and $\angle FGE$ are also corresponding when parallel lines are cut by a transversal, and are also congruent.

5. If a new right triangle with a hypotenuse on the same line is added to the graph, do you think it would be similar to triangles *ABC*, *CDE*, and *CFG*? Why or why not?
Possible response includes the following: Yes, because the ratio of the length of the vertical

leg to the length of the horizontal leg would be the same as these three triangles. The horizontal leg would be parallel to these horizontal legs and the vertical leg would be parallel to these vertical legs. I could form corresponding angles when parallel lines are cut by a transversal to show the angles are congruent.

6. Complete the table.

Hypotenuse	Endpoints	Use the coordinates of the endpoints to determine the lengths of the legs.	
$\overline{AC}$	*A* (**0**, **0**) *C* (**2**, **3**)	Vertical leg: **3 – 0 = 3**	Horizontal leg: **2 – 0 = 2**

$\overline{CE}$	*C* (**2**, **3**) *E* (**6**, **9**)	Vertical leg: **9 – 3 = 6**	Horizontal leg: **6 – 2 = 4**

$\overline{CG}$	*C* (**2**, **3**) *G* (**8**, **12**)	Vertical leg: **12 – 3 = 9**	Horizontal leg: **8 – 2 = 6**

7. A new right triangle with a hypotenuse that lies on the same line is added to this graph. The coordinates of the endpoints of the hypotenuse are (x_1, y_1) and (x_2, y_2). How could you determine the ratio of the length of the vertical leg to the length of the horizontal leg of this triangle?

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Note that this ratio represents a rate of the vertical change to horizontal change between two points.