


# Introduction to the **Revised Mathematics TEKS**

COMPUTATIONAL FLUENCY AND  
MATHEMATICAL PROFICIENCY  
JOURNAL  
GRADES 3 - 5



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## Your Definitions

Computational Fluency

Mathematical Proficiency

Automaticity

Conceptual Understanding

Vertical Learning Progression Recording Sheet

Grades 3-5

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Whole Numbers	1(5)(B) Skip count by twos, fives, and tens 2(6)(A) Model, create, and describe contextual multiplication situations 2(6)(B) Model, create, and describe contextual division situations			

Vertical Learning Progression Recording Sheet

Grades 3-5

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Rational Numbers	6(3)(C) Represent integer operations		6(3)(D) Add/subtract integers fluently 7(3)(A) Add/subtract rational numbers fluently	7(3)(B) Solve problems using addition and subtraction of rational numbers

## Developing Mathematical Proficiency

Pairing a content standard with a process standard to solve problems allows students to become mathematically proficient with the content for each grade level.



How does pairing a process standard with a content standard allow students to become mathematically proficient?

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Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Adding Three Numbers Flexibly

The students in Mrs. Park's class were asked to add 34, 29, and 86.

How would you solve this problem?

Find the strategy that is similar to how you solved the problem.

	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center; padding-right: 20px;"> <math display="block">\begin{array}{r} 30 \\ 20 \\ +80 \\ \hline 130 \end{array}</math> </td> <td style="text-align: center;"> <math display="block">100 + 30</math> </td> <td style="text-align: center; padding-left: 20px;"> <math display="block">\begin{array}{r} 4 \\ 6 \\ +9 \\ \hline 19 \end{array}</math> </td> <td style="text-align: center; padding-left: 20px;"> <math display="block">10 + 9</math> </td> </tr> <tr> <td colspan="4" style="text-align: center; padding-top: 20px;"> <math display="block">130 + 19 = 149</math> </td> </tr> </table>	$\begin{array}{r} 30 \\ 20 \\ +80 \\ \hline 130 \end{array}$	$100 + 30$	$\begin{array}{r} 4 \\ 6 \\ +9 \\ \hline 19 \end{array}$	$10 + 9$	$130 + 19 = 149$			
$\begin{array}{r} 30 \\ 20 \\ +80 \\ \hline 130 \end{array}$	$100 + 30$	$\begin{array}{r} 4 \\ 6 \\ +9 \\ \hline 19 \end{array}$	$10 + 9$						
$130 + 19 = 149$									

Could all of the strategies be used to solve the problem? Why or Why not?

*Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.*



Teacher Directions:

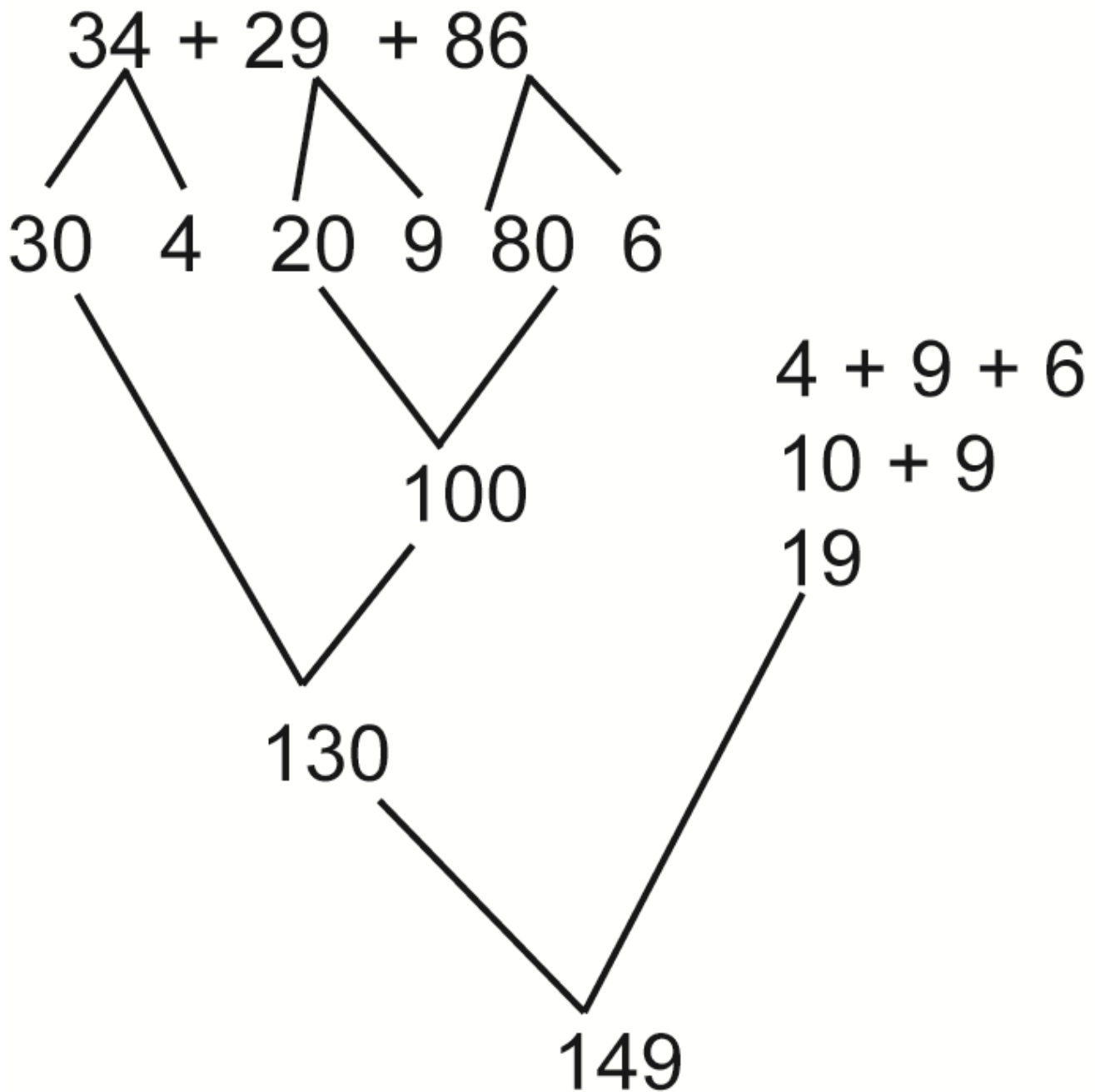
1. Display the problem.

The students in Mrs. Park's class were asked to add 34, 29, and 86.

2. Prompt the students to solve this problem.
3. Display the four strategy pages in four different corners of the room.
  - Display a blank piece of paper for students to form a group if their strategy did not match any of the four strategies represented.
4. Prompt the students to find the strategy that is similar to the way they solved the problem.
5. Once students have identified their strategy, prompt the students to justify why they belong in that group.
6. After students have had a chance to share their thinking, rearrange the strategy pages so that they can be seen by all of the students.
7. Prompt the students to respond to the question.

*Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.*



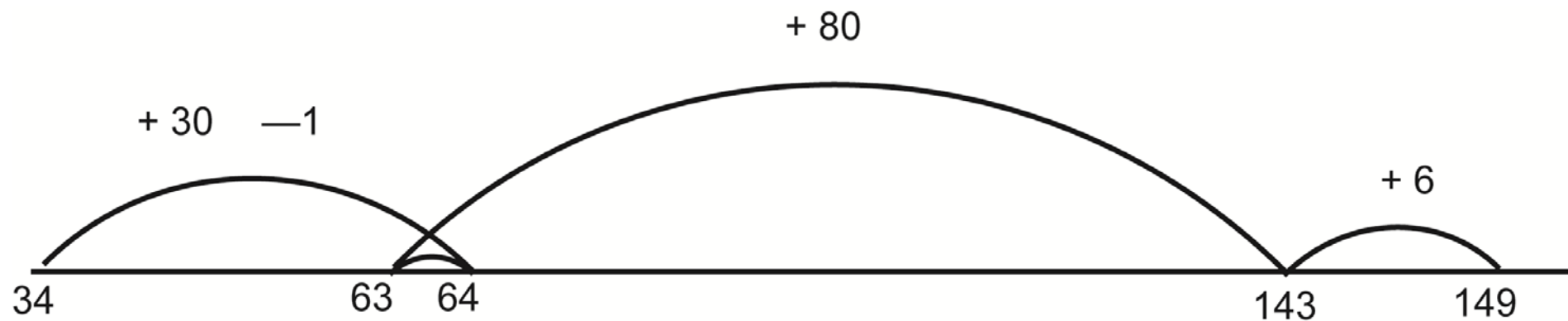


Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.

$$\begin{array}{r}
 30 \\
 20 \\
 +80 \\
 \hline
 130
 \end{array}
 \begin{array}{l}
 \\
 \rangle \\
 \rangle \\
 \rangle
 \end{array}
 100 + 30$$

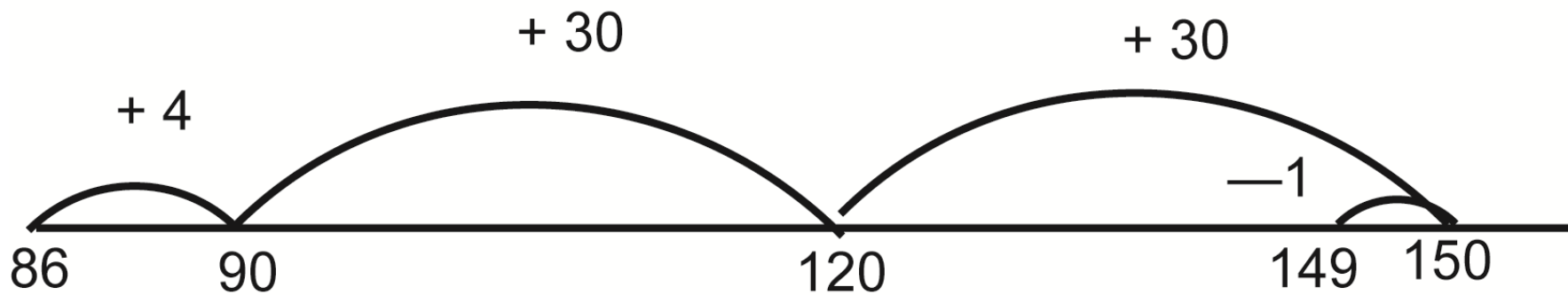
$$\begin{array}{r}
 4 \\
 6 \\
 +9 \\
 \hline
 19
 \end{array}
 \begin{array}{l}
 \\
 \rangle \\
 \rangle
 \end{array}
 10 + 9$$

$$130 + 19 = 149$$



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Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.



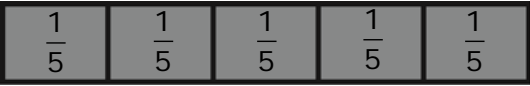



*Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.*

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Representing Fractions

Use the example below to complete the remaining items.

<p>Example:</p> <div style="text-align: center; margin-bottom: 10px;"> <math>\frac{3}{4}</math> </div>  <div style="text-align: center; margin-top: 10px;"> <math>\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}</math> </div>	<p>2</p> <div style="text-align: center; margin-bottom: 10px;"> <math>\frac{\square}{\square}</math> </div>  <div style="text-align: center; margin-top: 10px;"> <math>\frac{1}{3} + \frac{1}{3} = \frac{\square}{\square}</math> </div>
<p>1</p> <div style="text-align: center; margin-bottom: 10px;"> <math>\frac{\square}{\square}</math> </div>  <div style="text-align: center; margin-top: 10px;"> <math>\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{\square}{\square} = 1</math> </div>	<p>3</p> <div style="text-align: center; margin-bottom: 10px;"> <math>\frac{\square}{\square}</math> </div>  <div style="text-align: center; margin-top: 10px;"> <math>\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}</math> </div>

What patterns did you notice?

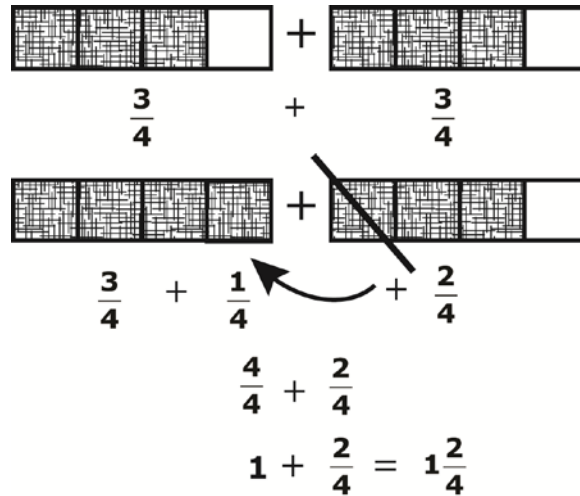
Grade 03 Fluency Activity

3(3)(D)- The student is expected to compose and decompose a fraction  $\frac{a}{b}$  with a numerator greater than zero and less than or equal to  $b$  as a sum of parts  $\frac{1}{b}$ .

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### James' Strategy

James used the strategy shown to add  $\frac{3}{4} + \frac{3}{4}$ .



Complete the problems below using James' strategy.

<p><b>1</b>            <math>\frac{3}{6} + \frac{4}{6}</math></p>	<p><b>3</b>            <math>\frac{3}{7} + \frac{4}{7}</math></p>
<p><b>2</b>            <math>\frac{2}{3} + \frac{2}{3}</math></p>	<p><b>4</b>            <math>\frac{7}{10} + \frac{6}{10}</math></p>

What patterns did you notice?

*Grade 4 Fluency Activity- 4(3)(B) The student is expected to decompose a fraction in more than one way into a sum of fractions with the same denominator using concrete and pictorial models and recording results with symbolic representations.*

Name: \_\_\_\_\_ Date: \_\_\_\_\_

### Francesca's Fractions

Below is Francesca's work from her class today.

$$\begin{aligned} & \frac{2}{3} + \frac{3}{4} \\ & \frac{8}{12} + \frac{9}{12} \\ & \left( \frac{8}{12} + \frac{4}{12} \right) + \frac{5}{12} \\ & \frac{12}{12} + \frac{5}{12} \\ & 1 \frac{5}{12} \end{aligned}$$

What was her strategy? Complete the four problems below using her strategy.

1 $\frac{2}{3} + \frac{5}{9}$	3 $\frac{5}{8} + \frac{2}{3}$
2 $\frac{1}{2} + \frac{4}{5}$	4 $\frac{7}{10} + \frac{3}{4}$

What patterns did you notice?

*Grade 5 Fluency Activity – 5(3)(K) The student is expected to add and subtract positive rational numbers fluently.*

## Drill or Practice?

**Drill** refers to repetitive, non-problem-based exercises designed to improve skills or procedures already acquired.

**Practice** refers to different problem-based tasks or experiences, spread over numerous class periods, each addressing the same basic ideas.

Van De Walle, 2004, pp.85-86



Van De Walle, J. (2004). *Elementary and middle school mathematics*. Boston: Pearson.



## Case Study Recording Sheet

Examine the case study documents provided for each student. What evidence do you see for each of the categories?

Student A			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

Student B			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

Case Study Student A

Work Sample

This summer Robert earned \$1,295 dollars mowing lawns. Robert placed all of the money he earned into his savings account, his checking account, his college fund account, and his wallet. Robert placed \$44 in his wallet and the remaining money was shared equally amongst the other accounts. How much money did Robert deposit into his savings account? Explain your thinking.

$$\begin{array}{r} \phantom{3} \\ 44 \overline{) 1295} \\ \underline{88} \\ 415 \\ \underline{- 396} \\ 19 \end{array}$$

The problem says he shared money equally so I divided.

Case Study Student B

Work Sample

This summer Robert earned \$1,295 dollars mowing lawns. Robert placed all of the money he earned into his savings account, his checking account, his college fund account, and his wallet. Robert placed \$44 in his wallet and the remaining money was shared equally amongst the other accounts. How much money did Robert deposit into his savings account? Explain your thinking.

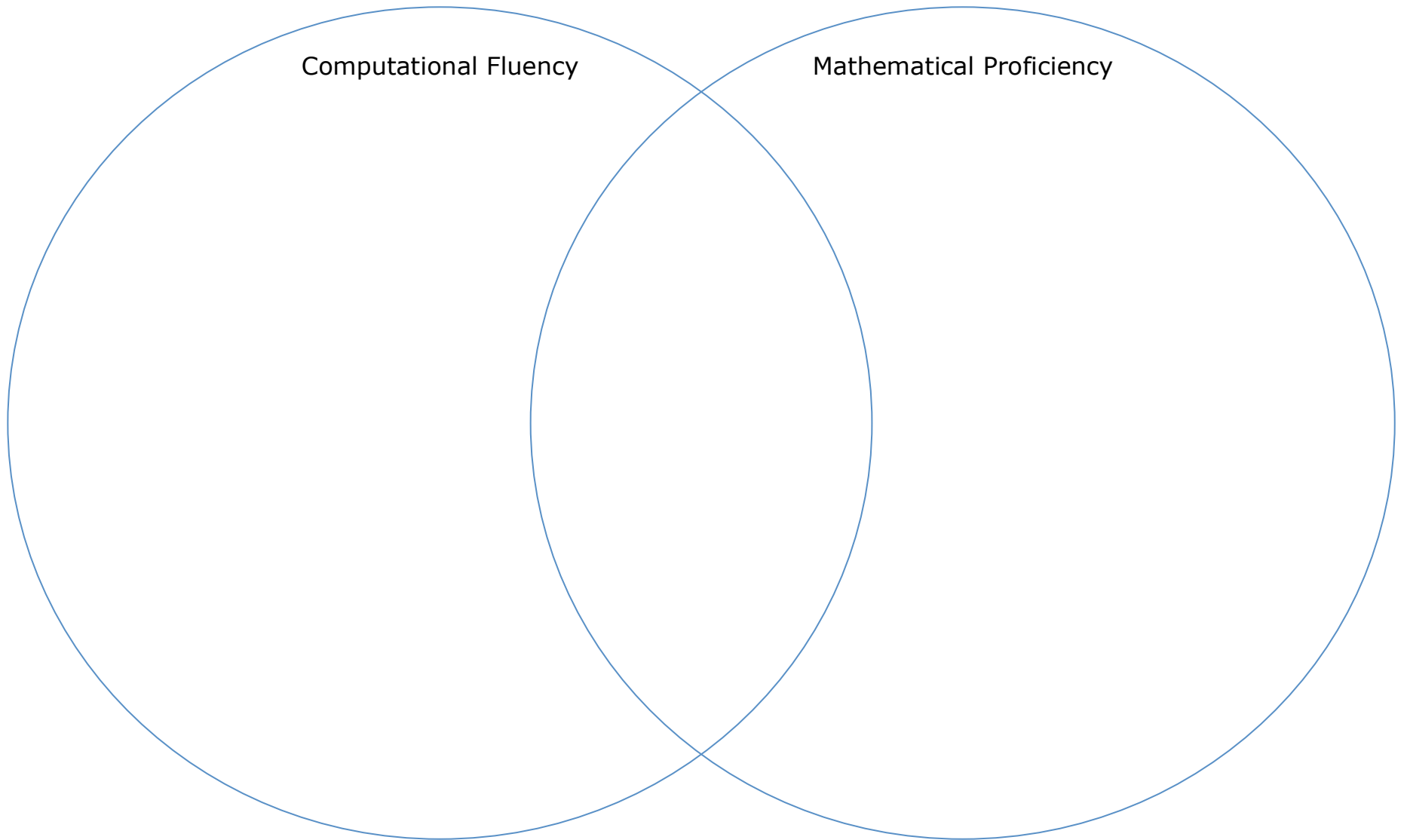
$$\begin{array}{r} 1295 \\ - 44 \\ \hline 1251 \end{array}$$

Savings \$433  
 Checking \_\_\_\_\_  
 College \_\_\_\_\_  
 Wallet \$44

- 3
- 6
- 9
- 12
- ~~14~~ 15
- ~~16~~ 18
- ~~18~~ 21
- ~~20~~ 28
- 30
- 33

$$\begin{array}{r} 4 \\ \cancel{9}33 \\ \hline 3 \overline{) 1251} \\ \underline{- 12} \phantom{0} \\ 11 \cancel{5} 1 \\ \phantom{11} \underline{\phantom{0} 9} \\ \phantom{11} \cancel{2} 2 \\ \phantom{11} \phantom{2} \underline{- 9} \\ \phantom{11} \phantom{2} \phantom{9} 3 \end{array}$$

Reflection



Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
<b>Adding and Subtracting with Whole Numbers</b>	<p><i>K(2)(A) Count forward and backward to 20 w/wo objects</i></p> <p><i>K(2)(F) Generate a number one more or one less</i></p> <p><i>K(2)(I) Compose and decompose number up to 10 with objects and pictures</i></p> <p><i>K(3)(A) Model the action of joining and separating</i></p> <p><i>1(3)(A) Use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number</i></p> <p><i>1(3)(B) Use objects and pictorial models to solve word problems</i></p> <p><i>1(3)(C) Compose 10 with two or more addends with and without concrete objects</i></p> <p><i>1(5)(D) Represent word problems involving addition and subtraction using concrete and pictorial models and number sentences</i></p> <p><i>3(5)(A) Represent one- and two-step addition and subtraction problems using pictorial models, number lines, and equations</i></p> <p><i>4(5)(A) Represent multi-step problems involving the four operations using strip diagrams and equations with a letter standing for the unknown quantity</i></p>	<p><i>2(4)(A) Recall basic facts with automaticity</i></p>	<p><i>1(3)(D) Apply basic fact strategies to add and subtract including making 10 and decomposing a number leading to a 10</i></p> <p><i>1(5)(F) Determine the unknown whole number in an addition or subtraction equation</i></p> <p><i>1(5)(G) Apply properties of operations to add and subtract two or three numbers</i></p> <p><i>2(4)(B) Add and subtract using mental strategies and algorithms</i></p> <p><i>3(4)(B) round or use compatible numbers to estimate solutions</i></p> <p><i>4(4)(A) Add and subtract using the standard algorithm</i></p>	<p><i>K(3)(C) Explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences</i></p> <p><i>K(3)(B) Solve word problems using objects and drawings</i></p> <p><i>1(3)(E) Explain the strategies used to solve problems using spoken words, concrete and pictorial models, and number sentences</i></p> <p><i>1(3)(F) Generate and solve problem situations when given a number sentence</i></p> <p><i>2(4)(C) Solve one-step and multi-step addition and subtraction problems using a variety of strategies</i></p> <p><i>2(7)(C) Represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem</i></p> <p><i>2(10)(C) Write and solve one-step addition and subtraction word problems using data represented with pictographs and bar graphs</i></p> <p><i>3(4)(A) Solve multi-step addition and subtraction problems using various strategies</i></p> <p><i>3(8)(B) Solve problems using data represented with a frequency table, dot plot, pictograph, or bar graph</i></p> <p><i>4(9)(B) Solve problems using data in a frequency table, dot plot, or stem-and-leaf plot</i></p> <p><i>5(3)(A) Estimate to determine solutions to mathematical and real-world problems</i></p> <p><i>5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity</i></p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
<b>Multiplying and Dividing with Whole Numbers</b>	<p>1(5)(B) Skip count by twos, fives, and tens</p> <p>2(6)(A) Model, create, and describe contextual multiplication situations</p> <p>2(6)(B) Model, create, and describe contextual division situations</p> <p>3(4)(D) Determine the total number of objects when arranged in arrays</p> <p>3(4)(E) Represent multiplication facts using a variety of approaches</p> <p>3(4)(H) Determine the number of objects in each group when a set of objects is partitioned into equal shares</p> <p>3(5)(B) Represent and solve one- and two-step multiplication and division problems using arrays, strip diagrams, and equations</p> <p>3(5)(C) Describe a multiplication expression as a comparison</p> <p>4(4)(C) Represent the product of 2 two-digit numbers using arrays, area models, or equations</p> <p>4(4)(E) Represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations</p> <p>4(5)(A) Represent multi-step problems involving the four operations using strip diagrams and equations with a letter standing for the unknown quantity</p> <p>4(5)(C) Use models to determine the formulas for the perimeter and area of a rectangle</p>	<p>3(4)(F) Recall multiplication facts with automaticity</p>	<p>3(4)(G) Use strategies and algorithms to multiply a two-digit number by a one-digit number</p> <p>3(4)(J) Determine a quotient using the relationship between multiplication and division</p> <p>3(5)(D) Determine the unknown whole number in a multiplication or division equation when the unknown is either a missing factor or product</p> <p>4(4)(B) Determine products of a number and 10 or 100</p> <p>4(4)(D) Use strategies and algorithms to multiply four-digit by one-digit or two-digit by two-digit numbers</p> <p>4(4)(F) Use strategies and algorithms to divide up to a four-digit dividend by a one-digit divisor</p> <p>4(4)(G) Round or use compatible numbers to estimate solutions involving whole numbers</p>	<p>3(4)(K) Solve one-step and two-step multiplication and division problems using various strategies</p> <p>3(8)(B) Solve one- and two-step problems using data from a frequency table, dot plot, pictograph or bar graph</p> <p>4(4)(H) Solve with fluency one- and two-step multiplication and division problems including interpreting remainders</p> <p>4(5)(D) Solve problems related to perimeter and area of rectangles</p> <p>4(8)(B) Convert measurements within the same measurement system</p> <p>4(8)(C) Solve problems that deal with measurement</p> <p>4(9)(B) Solve one- and two-step problems using data from a frequency table, dot plot, or stem-and-leaf plot</p> <p>5(3)(A) Estimate to determine solutions to mathematical and real-world problems</p> <p>5(3)(C) Solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor</p> <p>5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity</p> <p>5(7) Solve problems by calculating conversions within a measurement system</p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Rational Numbers	<p>3(3)(F) Represent equivalent fractions</p> <p>4(3)(B) Decompose fractions with models</p> <p>4(3)(E) Represent and solve addition and subtraction of fractions with equal denominators using models</p> <p>4(3)(F) Evaluate for reasonableness of sums and differences using benchmark fractions</p> <p>5(3)(H) Represent and solve for addition/subtraction of fractions using objects</p> <p>6(3)(C) Represent integer operations</p>		<p>4(4)(A) Add/subtract whole number and decimals using the standard algorithm</p> <p>5(3)(K) Add/subtract positive rational numbers fluently</p> <p>6(3)(D) Add/subtract integers fluently</p> <p>7(3)(A) Add/subtract rational numbers fluently</p>	<p>7(3)(B) Solve problems using addition and subtraction of rational numbers</p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Rational Numbers	<p>4(3)(A) <i>Compose and decompose fractions into unit fractions</i></p> <p>5(3)(D)(F) <i>Represent multiplication/division of decimals</i></p> <p>5(3)(I)(J) <i>Represent and solve problems involving multiplication/division of a whole number and a fraction</i></p> <p>6(3)(A) <i>Recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values</i></p> <p>6(3)(B) <i>Determine if a quantity is increased or decreased when multiplied by a fraction</i></p> <p>6(3)(C) <i>Represent integer operations</i></p>		<p>5(3)(E)(G) <i>Solve for products/quotients of decimals</i></p> <p>6(3)(D) <i>Multiply/divide integers fluently</i></p> <p>6(3)(E) <i>Multiply/divide positive rational numbers fluently</i></p> <p>7(3)(A) <i>Multiply/divide rational numbers fluently</i></p>	<p>7(3)(B) <i>Solve problems using multiplication and division of rational numbers</i></p>



## Reference Page

Beckmann, S. (2010). *Rtl for elementary and middle school mathematics* [PowerPoint slides]. Retrieved from [http://educationnorthwest.org/webfm\\_send/710/](http://educationnorthwest.org/webfm_send/710/)

National Research Council. (2002). *Helping children learn mathematics*. Mathematics Learning Study Committee, J. Kilpatrick, and J. Swafford Editors. Center for Education, Division of Behavioral and Social Sciences. Washington, DC: National Academies Press.

National Research Council. (2001). *Adding It up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences. Washington, DC: National Academies Press.

Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching children mathematics*, 7(3), 154-158. Retrieved from <http://lib-ezproxy.tamu.edu:2048/login?url=http://search.proquest.com/docview/214137345?accountid=7082>

Van De Walle, J. (2004). *Elementary and middle school mathematics*. Boston: Pearson.