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Mathematical Proficiency

Computational Fluency

Mathematical Proficiency

Automaticity

Conceptual Understanding

Vertical Learning Progression Recording Sheet

Grades 6-8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Rational Numbers	3(3)(F) Represent equivalent fractions 4(3)(B) Decompose fractions with models 4(3)(E) Represent and solve addition and subtraction with equal denominators using models 4(3)(F) Evaluate for reasonableness of sums and differences using benchmark fractions		4(4)(A) Add/subtract whole number and decimals using the standard algorithm	

Vertical Learning Progression Recording Sheet

Grades 6-8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Rational Numbers	4(3)(A) Compose and decompose fractions into unit fractions			

Name: _____ Date: _____

Franchesca's Fractions

Below is Franchesca's work from her class today.

$$\begin{aligned} & \frac{2}{3} + \frac{3}{4} \\ & \frac{8}{12} + \frac{9}{12} \\ & \left(\frac{8}{12} + \frac{4}{12} \right) + \frac{5}{12} \\ & \frac{12}{12} + \frac{5}{12} \\ & 1\frac{5}{12} \end{aligned}$$

What was her strategy? Complete the four problems below using her strategy.

1 $\frac{2}{3} + \frac{5}{9}$	3 $\frac{5}{8} + \frac{2}{3}$
2 $\frac{1}{2} + \frac{4}{5}$	4 $\frac{7}{10} + \frac{3}{4}$

What patterns did you notice?

Grade 5 Fluency Activity – 5(3)(K) The student is expected to add and subtract positive rational numbers fluently.

Name: _____ Date: _____

Inigo's Integers

Below is Inigo's work from his class today.

$$\begin{aligned} & -5 + 8 \\ & (-5 + 5) + 3 \\ & 0 + 3 \\ & 3 \end{aligned}$$

What was his strategy? Complete the four problems below using his strategy.

1 $-6 + 2$	3 $-3 + 15$
2 $7 + (-12)$	4 $8 + (-5)$

What patterns did you notice?

Grade 6 Fluency Activity – 6(3)(D) The student is expected to add, subtract, multiply, and divide integers fluently.

Name: _____ Date: _____

Ra'Neisha's Rationals

Below is Ra'Neisha's work from her class today.

$$\begin{aligned} & -1.2 + 3.4 \\ & (-1.2 + 1.2) + 2.2 \\ & 0 + 2.2 \\ & 2.2 \end{aligned}$$

What was her strategy? Complete the four problems below using her strategy.

1 $-6.4 + 2.3$	3 $-3.7 + 15.4$
2 $7.9 + (-12.4)$	4 $3.4 + (-1.2)$

What patterns did you notice?

Grade 7 Fluency Activity – 7(3)(A) The student is expected to add, subtract, multiply, and divide rational numbers fluently.

Drill or Practice?

Drill refers to repetitive, non-problem-based exercises designed to improve skills or procedures already acquired.

Practice refers to different problem-based tasks or experiences, spread over numerous class periods, each addressing the same basic ideas.

Van De Walle, 2004, pp.85-86



Van De Walle, J. (2004). *Elementary and Middle School Mathematics*. Boston: Pearson.

Case Study Recording Sheet

Examine the case study documents provided for each student. What evidence do you see for each of the categories?

Student A			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

Student B			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

Case Study Student A – Grade 7: Kris

Work Sample

Mr. Khan must collect a fuel tax of \$0.39 per gallon of gasoline sold at his gasoline station. Over the last seven days he collected \$6,683.04 in fuel taxes. The customers paid an average \$3.78 per gallon of gasoline during that time.

What was his revenue for gasoline sales after paying the fuel tax for the week? Explain your thinking.

$$\begin{array}{r} 6683.04 \\ \times 0.39 \\ \hline 6014736 \\ 20049120 \\ \hline 2606.3856 \\ \quad \underline{2121} \end{array}$$

$$\begin{array}{r} 2606.3856 \\ \times 3.78 \\ \hline 208510848 \\ 1824469920 \\ 7819156800 \\ \hline 9852.137568 \\ \quad \underline{4321} \quad \underline{21} \end{array}$$

$$\boxed{9852.137568}$$

I multiplied all of the numbers together.

Case Study Student B – Grade 7: Pat

Work Sample

Mr. Khan must collect a fuel tax of \$0.39 per gallon of gasoline sold at his gasoline station. Over the last seven days he collected \$6,683.04 in fuel taxes. The customers paid an average \$3.78 per gallon of gasoline during that time.

What was his revenue for gasoline sales after paying the fuel tax for the week? Explain your thinking.

$$\text{Revenue} = \text{total} - \text{fuel tax}$$

$$? = ? - 6,683.04$$

$$\text{Revenue} = \frac{\text{non-tax}}{\text{gallon}} \cdot \# \text{ gals}$$

$$\text{non tax} = \begin{array}{r} 3.78 \\ - 0.39 \\ \hline 3.39 \end{array}$$

$$\begin{array}{l} \# \text{ gallons} \\ \times ? \\ \hline \end{array}$$

$$\begin{array}{l} \$ \\ \div 0.39 \\ \hline \end{array} \begin{array}{l} \$0.39 \\ \times 1 \text{ gal} \\ \hline \end{array} = \begin{array}{l} \$6,683.04 \\ \div 0.39 \\ \hline \end{array} \begin{array}{l} \square \text{ gal} \\ \times ? \end{array}$$

$$\begin{array}{r} 39 \\ 39 \\ \hline 78 \\ +39 \\ \hline 156 \\ +39 \\ \hline 195 \\ +39 \\ \hline 234 \\ +39 \\ \hline 273 \end{array}$$

$$\begin{array}{l} 1 \times 39 = 39 \\ 2 \times 39 = 78 \\ 3 \times 39 = 117 \\ 4 \times 39 = 156 \\ 5 \times 39 = 195 \\ 6 \times 39 = 234 \\ 7 \times 39 = 273 \end{array}$$

$$\begin{array}{r} 171.36 \\ .39 \overline{) 6683.04} \\ \underline{-39} \\ 278 \\ \underline{-273} \\ 53 \\ \underline{-39} \\ 140 \\ \underline{117} \\ 234 \\ \underline{-234} \\ 0 \end{array}$$

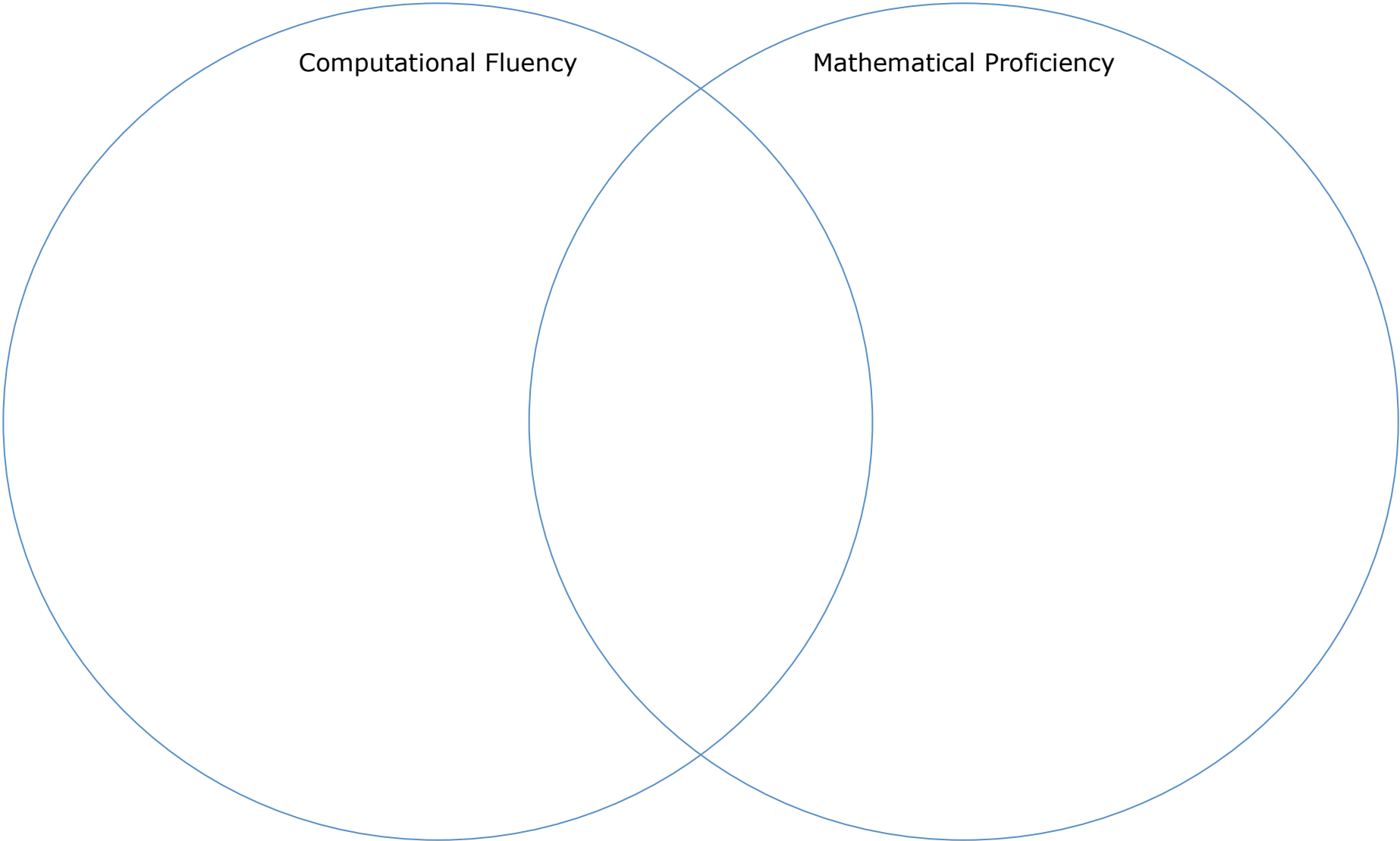
$$\begin{array}{r} 171.36 \\ \times 3.39 \\ \hline \end{array}$$

ran out of time! $\frac{4}{9}$



Reflection

Computational Fluency



Mathematical Proficiency

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Whole Numbers	<p><i>K(2)(A) Count forward and backward to 20 w/wo objects</i></p> <p><i>K(2)(F) Generate a number one more or one less</i></p> <p><i>K(2)(I) Compose and decompose number up to 10 with objects and pictures</i></p> <p><i>K(3)(A) Model the action of joining and separating</i></p> <p><i>1(3)(A) Use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number</i></p> <p><i>1(3)(B) Use objects and pictorial models to solve word problems</i></p> <p><i>1(3)(C) Compose 10 with two or more addends with and without concrete objects</i></p> <p><i>1(5)(D) Represent word problems involving addition and subtraction using concrete and pictorial models and number sentences</i></p> <p><i>3(5)(A) Represent one- and two-step addition and subtraction problems using pictorial models, number lines, and equations</i></p> <p><i>4(5)(A) Represent multi-step problems involving the four operations using strip diagrams and equations with a letter standing for the unknown quantity</i></p>	<p><i>2(4)(A) Recall basic facts with automaticity</i></p>	<p><i>1(3)(D) Apply basic fact strategies to add and subtract including making 10 and decomposing a number leading to a 10</i></p> <p><i>1(5)(F) Determine the unknown whole number in an addition or subtraction equation</i></p> <p><i>1(5)(G) Apply properties of operations to add and subtract two or three numbers</i></p> <p><i>2(4)(B) Add and subtract using mental strategies and algorithms</i></p> <p><i>3(4)(B) round or use compatible numbers to estimate solutions</i></p> <p><i>4(4)(A) Add and subtract using the standard algorithm</i></p>	<p><i>K(3)(C) Explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences</i></p> <p><i>K(3)(B) Solve word problems using objects and drawings</i></p> <p><i>1(3)(E) Explain the strategies used to solve problems using spoken words, concrete and pictorial models, and number sentences</i></p> <p><i>1(3)(F) Generate and solve problem situations when given a number sentence</i></p> <p><i>2(4)(C) Solve one-step and multi-step addition and subtraction problems using a variety of strategies</i></p> <p><i>2(7)(C) Represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem</i></p> <p><i>2(10)(C) Write and solve one-step addition and subtraction word problems using data represented with pictographs and bar graphs</i></p> <p><i>3(4)(A) Solve multi-step addition and subtraction problems using various strategies</i></p> <p><i>3(8)(B) Solve problems using data represented with a frequency table, dot plot, pictograph, or bar graph</i></p> <p><i>4(9)(B) Solve problems using data in a frequency table, dot plot, or stem-and-leaf plot</i></p> <p><i>5(3)(A) Estimate to determine solutions to mathematical and real-world problems</i></p> <p><i>5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity</i></p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Whole Numbers	<p>1(5)(B) Skip count by twos, fives, and tens</p> <p>2(6)(A) Model, create, and describe contextual multiplication situations</p> <p>2(6)(B) Model, create, and describe contextual division situations</p> <p>3(4)(D) Determine the total number of objects when arranged in arrays</p> <p>3(4)(E) Represent multiplication facts using a variety of approaches</p> <p>3(4)(H) Determine the number of objects in each group when a set of objects is partitioned into equal shares</p> <p>3(5)(B) Represent and solve one- and two-step multiplication and division problems using arrays, strip diagrams, and equations</p> <p>3(5)(C) Describe a multiplication expression as a comparison</p> <p>4(4)(C) Represent the product of 2 two-digit numbers using arrays, area models, or equations</p> <p>4(4)(E) Represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations</p> <p>4(5)(A) Represent multi-step problems involving the four operations using strip diagrams and equations with a letter standing for the unknown quantity</p> <p>4(5)(C) Use models to determine the formulas for the perimeter and area of a rectangle</p>	<p>3(4)(F) Recall multiplication facts with automaticity</p>	<p>3(4)(G) Use strategies and algorithms to multiply a two-digit number by a one-digit number</p> <p>3(4)(J) Determine a quotient using the relationship between multiplication and division</p> <p>3(5)(D) Determine the unknown whole number in a multiplication or division equation when the unknown is either a missing factor or product</p> <p>4(4)(B) Determine products of a number and 10 or 100</p> <p>4(4)(D) Use strategies and algorithms to multiply four-digit by one-digit or two-digit by two-digit numbers</p> <p>4(4)(F) Use strategies and algorithms to divide up to a four-digit dividend by a one-digit divisor</p> <p>4(4)(G) Round or use compatible numbers to estimate solutions involving whole numbers</p>	<p>3(4)(K) Solve one-step and two-step multiplication and division problems using various strategies</p> <p>3(8)(B) Solve one- and two-step problems using data from a frequency table, dot plot, pictograph or bar graph</p> <p>4(4)(H) Solve with fluency one- and two-step multiplication and division problems including interpreting remainders</p> <p>4(5)(D) Solve problems related to perimeter and area of rectangles</p> <p>4(8)(B) Convert measurements within the same measurement system</p> <p>4(8)(C) Solve problems that deal with measurement</p> <p>4(9)(B) Solve one- and two-step problems using data from a frequency table, dot plot, or stem-and-leaf plot</p> <p>5(3)(A) Estimate to determine solutions to mathematical and real-world problems</p> <p>5(3)(C) Solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor</p> <p>5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity</p> <p>5(7) Solve problems by calculating conversions within a measurement system</p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Rational Numbers	<p>3(3)(F) Represent equivalent fractions</p> <p>4(3)(B) Decompose fractions with models</p> <p>4(3)(E) Represent and solve addition and subtraction of fractions with equal denominators using models</p> <p>4(3)(F) Evaluate for reasonableness of sums and differences using benchmark fractions</p> <p>5(3)(H) Represent and solve for addition/subtraction of fractions using objects</p> <p>6(3)(C) Represent integer operations</p>		<p>4(4)(A) Add/subtract whole number and decimals using the standard algorithm</p> <p>5(3)(K) Add/subtract positive rational numbers fluently</p> <p>6(3)(D) Add/subtract integers fluently</p> <p>7(3)(A) Add/subtract rational numbers fluently</p>	<p>7(3)(B) Solve problems using addition and subtraction of rational numbers</p>

Vertical Learning Progression Recording Sheet Possible Progression

Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Rational Numbers	<p>4(3)(A) Compose and decompose fractions into unit fractions</p> <p>5(3)(D)(F) Represent multiplication/division of decimals</p> <p>5(3)(I)(J) Represent and solve problems involving multiplication/division of a whole number and a fraction</p> <p>6(3)(A) Recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values</p> <p>6(3)(B) Determine if a quantity is increased or decreased when multiplied by a fraction</p> <p>6(3)(C) Represent integer operations</p>		<p>5(3)(E)(G) Solve for products/quotients of decimals</p> <p>6(3)(D) Multiply/divide integers fluently</p> <p>6(3)(E) Multiply/divide positive rational numbers fluently</p> <p>7(3)(A) Multiply/divide rational numbers fluently</p>	<p>7(3)(B) Solve problems using multiplication and division of rational numbers</p>

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