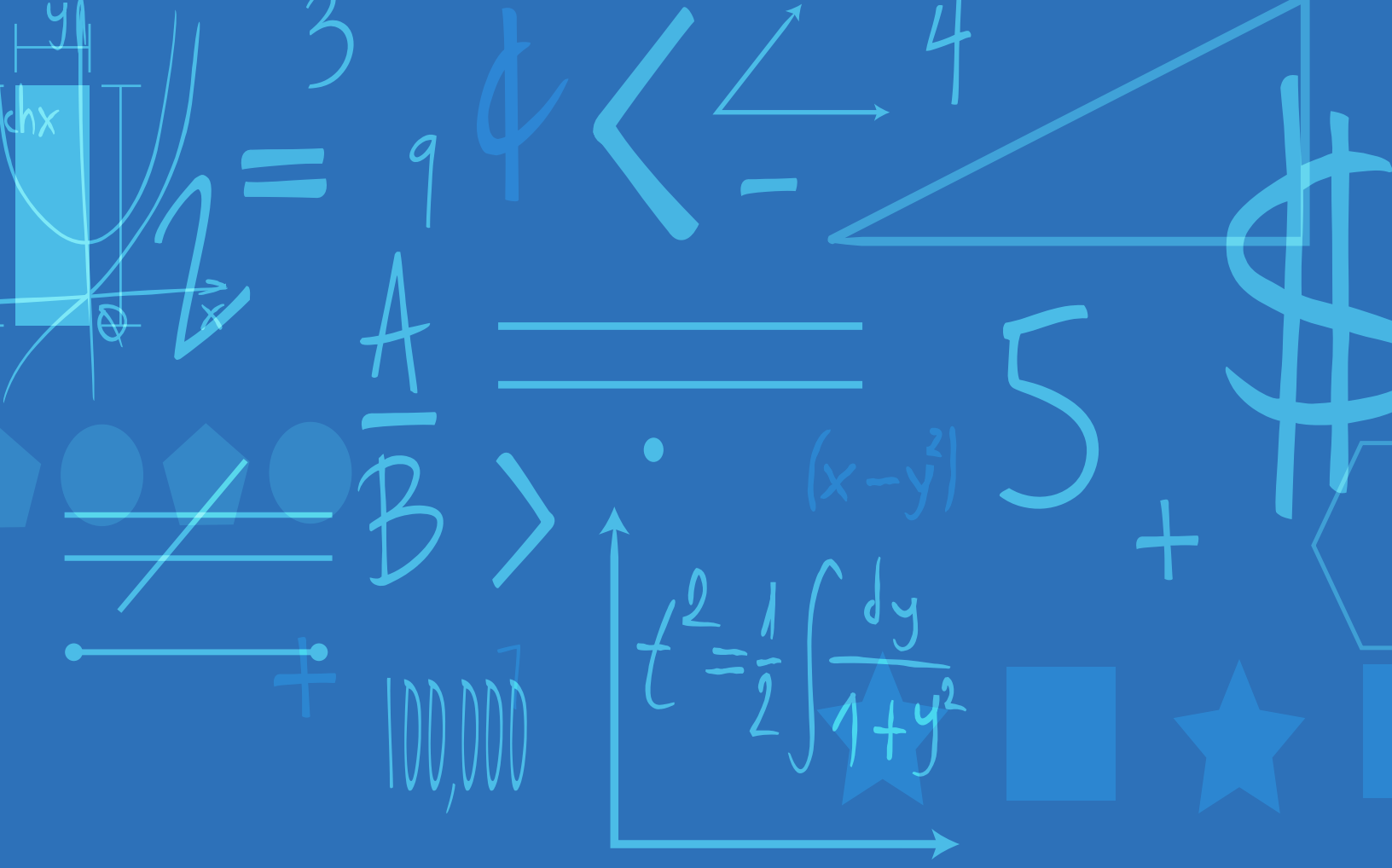

Introduction to the

Revised Mathematics TEKS

COMPUTATIONAL FLUENCY AND MATHEMATICAL PROFICIENCY JOURNAL KINDERGARTEN - GRADE 2

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Your Definitions

Computational Fluency

Mathematical Proficiency

Automaticity

Conceptual Understanding

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Vertical Learning Progression Recording Sheet Grades K-2

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Whole Numbers	4(5)(A) Represent multi-step problems using strip diagrams and equations with a letter standing for the unknown quantity		4(4)(A) Add and subtract using the standard algorithm	4(9)(B) Solve problems using data in a frequency table, dot plot, or stem-and-leaf plot 5(3)(A) Estimate to determine solutions to mathematical and real-world problems 5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity

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Vertical Learning Progression Recording Sheet Grades K-2

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Whole Numbers	4(4)(C) Represent the product of 2 two-digit numbers using arrays, area models, or equations 4(4)(E) Represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations 4(5)(A) Represent multi-step problems using strip diagrams and equations with a letter standing for the unknown quantity		4(4)(B) Determine products of a number and 10 or 100 4(4)(D) Use strategies and algorithms to multiply four-digit by one-digit or two-digit by two- digit numbers 4(4)(F) Use strategies and algorithms to divide up to a four-digit dividend by a one-digit divisor 4(4)(G) Round or use compatible numbers to estimate solutions involving whole numbers	4(4)(H) Solve one- and two-step multiplication and division problems including interpreting remainders 4(5)(C) Use models to determine the formulas for the perimeter and area of a rectangle 4(5)(D) Solve problems related to perimeter and area of rectangles 4(8)(B) Convert measurements within the same measurement system 4(8)(C) Solve problems that deal with measurement 4(9)(B) Solve one-and two-step problems using data from a frequency table, dot plot, or stem-and-leaf plot 5(3)(A) Estimate to determine solutions to mathematical and real-world problems 5(3)(C) Solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor 5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity 5(7) Solve problems by calculating conversions within a measurement system

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Developing Mathematical Proficiency

Pairing a content standard with a process standard to solve problems allows students to become mathematically proficient with the content for each grade level.

How does pairing a process standard with a content standard allow students to become mathematically proficient?

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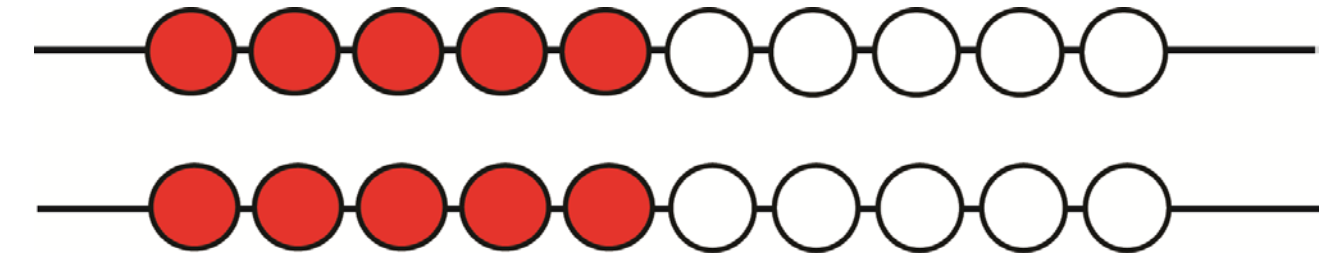
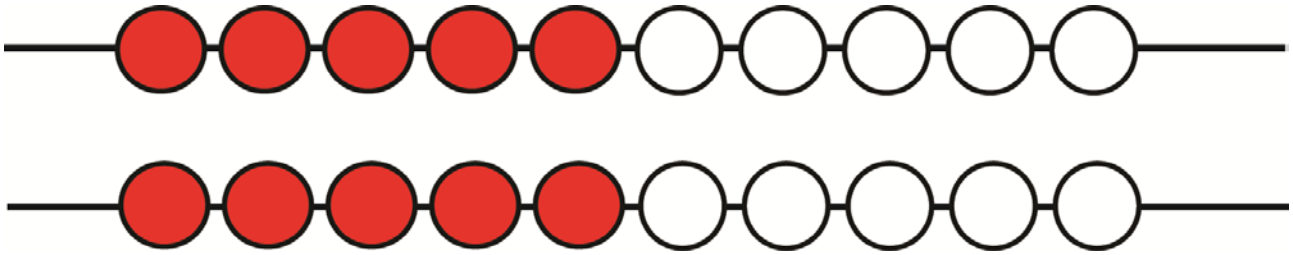
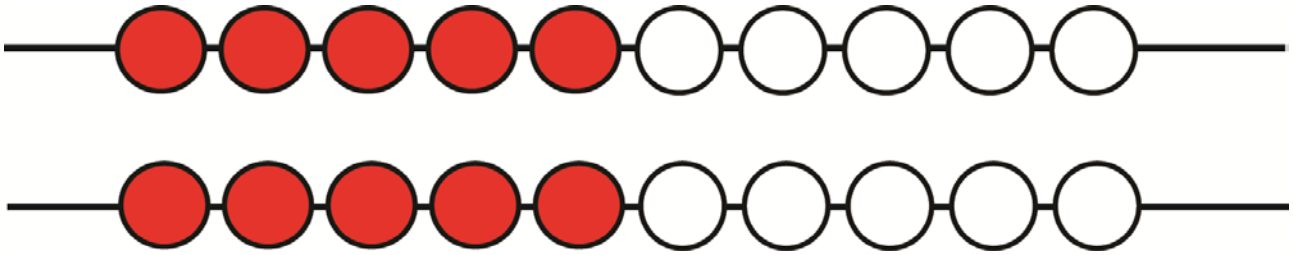
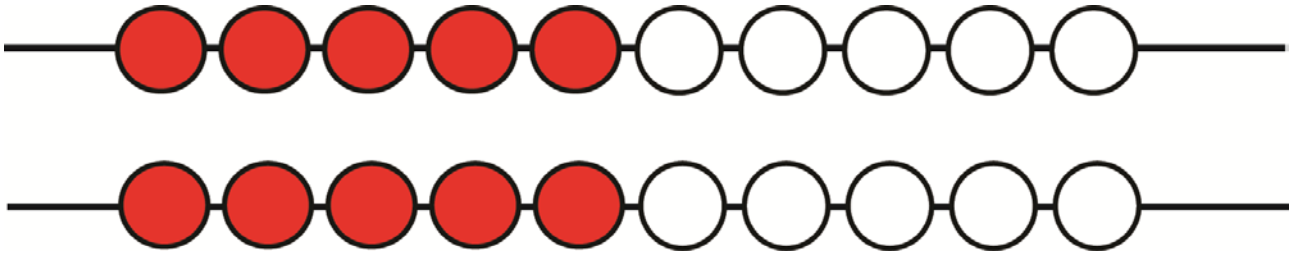
Make Ten









Kindergarten Fluency Activity - K(2)(I) The student is expected to compose and decompose numbers up to 10 with objects and pictures.

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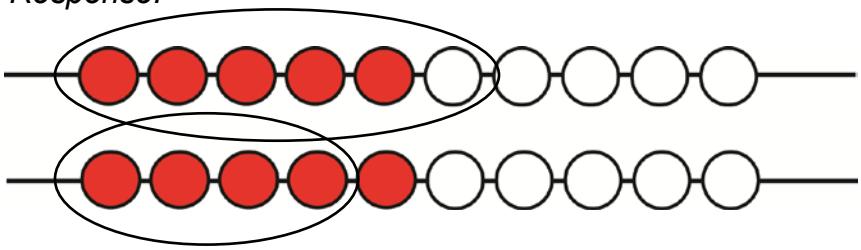
Teacher Directions:

1. Prompt the students to make a 10 on their rekenrek.
2. Prompt the students to circle the beads they used on the recording sheet.
   Possible Response:



3. Prompt the students to record the numbers of beads they used to make 10.
   Possible Response: 6 and 4 or 5 and 1 and 4
   Teacher note: The different colors of the rekenrek beads make it possible for students to have more than two addends when making 10 (i.e., 5 red, 1 white, and 4 red).
4. Repeat steps 1—3 to make the number 10 three different ways.
5. Prompt the students to share the different ways they represented the number 10.
6. Record and display the students' responses in the form of a number sentence.
   Possible Response: 6 + 4 = 10 or 5 + 1 + 4 = 10

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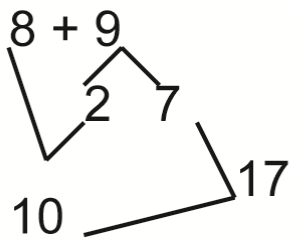
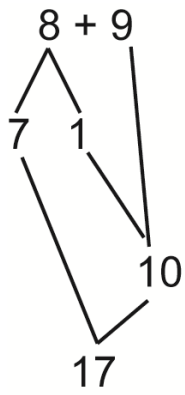
Name: _____ Date: _____

Flexible Facts

Sue has 8 red markers and 9 blue markers in her school bag. How many markers does Sue have in her school bag?

Find the strategy that is similar to how you solved the problem.

☐  8 + 2 = 10 10 + 7 = 17	☐  7 + 1 + 9 7 + 10 = 17
☐ 8 + 9 8 + 8 + 1 16 + 1 = 17	☐ 8 + 9 9 + 9 — 1 18 — 1 = 17

How is your strategy similar to your partner's?
How is your strategy different from your partner's?

Grade 1 Fluency Activity - 1(3)(D) The student is expected to apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a ten.

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Teacher Directions:

1. Display the problem.

   Sue has 8 red markers and 9 blue markers in her school bag. How many markers does Sue have in her school bag?

2. Prompt the students to solve this problem.
3. Display the four strategy pages in four different corners of the room.
   - Display a blank piece of paper for students to form a group if their strategy did not match any of the four strategies represented.
4. Prompt the students to find the strategy that is similar to the way they solved the problem.
5. Once students have identified their strategies, prompt them to justify why they belong in that group.
6. After the students have had an opportunity to share, prompt them to pair up with a student from another group.
7. Prompt the students to discuss how their strategies are similar and how they are different.

Grade 1 Fluency Activity - 1(3)(D) The student is expected to apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a ten.

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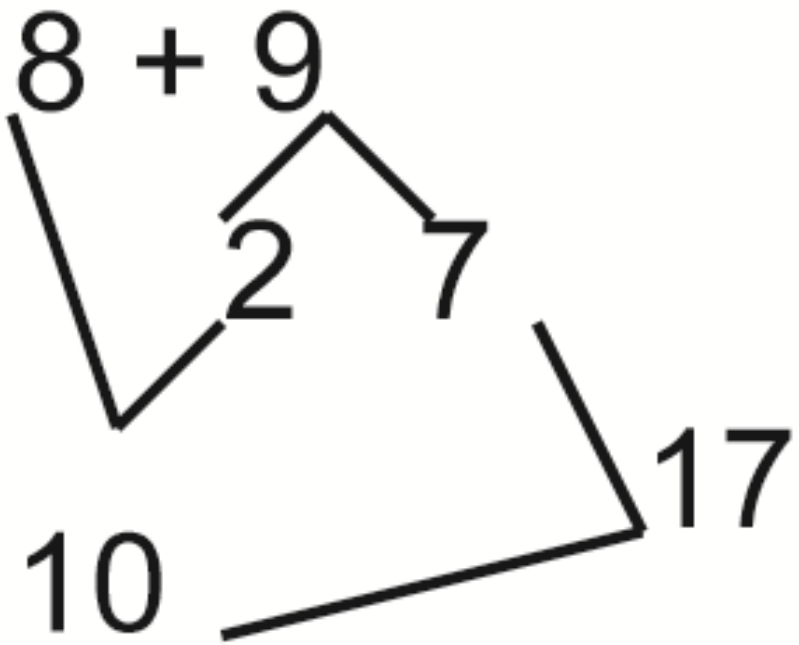

$8 + 2 = 10$

$10 + 7 = 17$

Grade 1 Fluency Activity - 1(3)(D) The student is expected to apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a ten.

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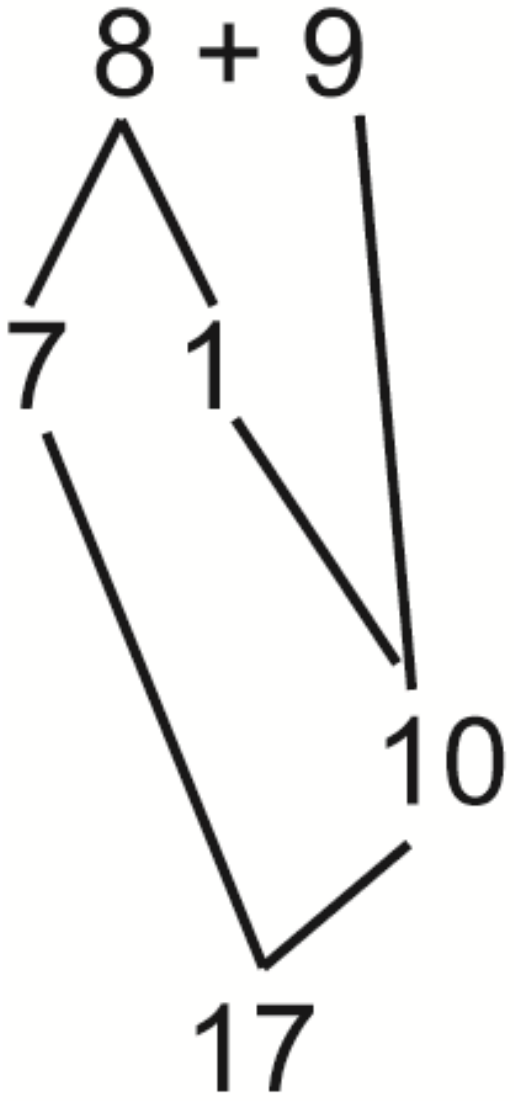

7 + 1 + 9

7 + 10 = 17

Grade 1 Fluency Activity - 1(3)(D) The student is expected to apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a ten.

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$$8 + 9$$

$$8 + 8 + 1$$

$$16 + 1 = 17$$

Grade 1 Fluency Activity - 1(3)(D) The student is expected to apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a ten.

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$$8 + 9$$

$$9 + 9 - 1$$

$$18 - 1 = 17$$

Grade 1 Fluency Activity - 1(3)(D) The student is expected to apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a ten.

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Name: _____ Date: _____

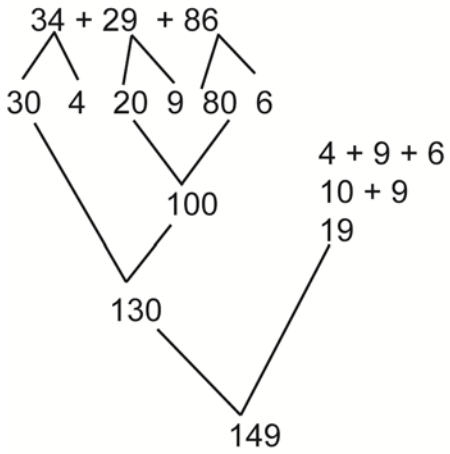
Adding Three Numbers Flexibly

The students in Mrs. Park's class were asked to add 34, 29, and 86.

How would you solve this problem?

Find the strategy that is similar to how you solved the problem.

☐



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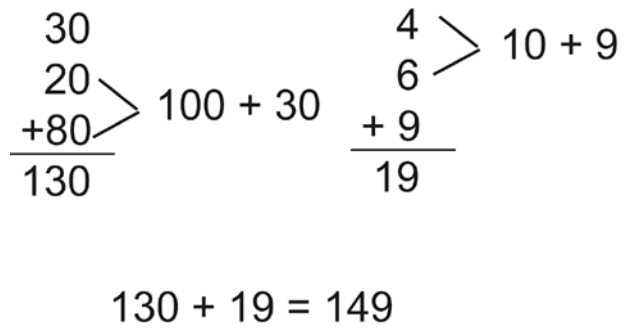

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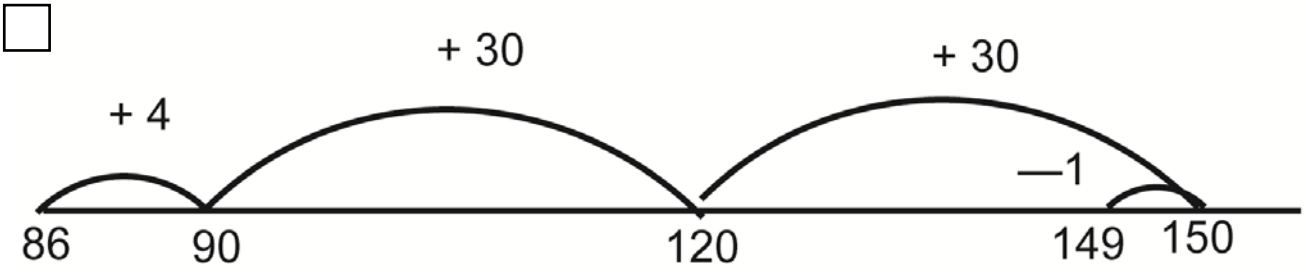

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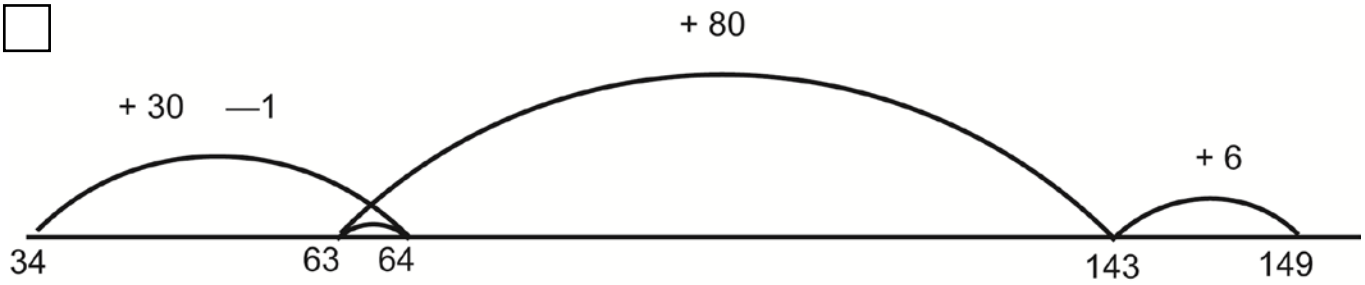

Could all of the strategies be used to solve the problem? Why or Why not?

Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.

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Teacher Directions:

1. Display the problem.

   The students in Mrs. Park's class were asked to add 34, 29, and 86.

2. Prompt the students to solve this problem.
3. Display the four strategy pages in four different corners of the room.
   - Display a blank piece of paper for students to form a group if their strategy did not match any of the four strategies represented.
4. Prompt the students to find the strategy that is similar to the way they solved the problem.
5. Once students have identified their strategy, prompt the students to justify why they belong in that group.
6. After students have had a chance to share their thinking, rearrange the strategy pages so that they can be seen by all of the students.
7. Prompt the students to respond to the question.

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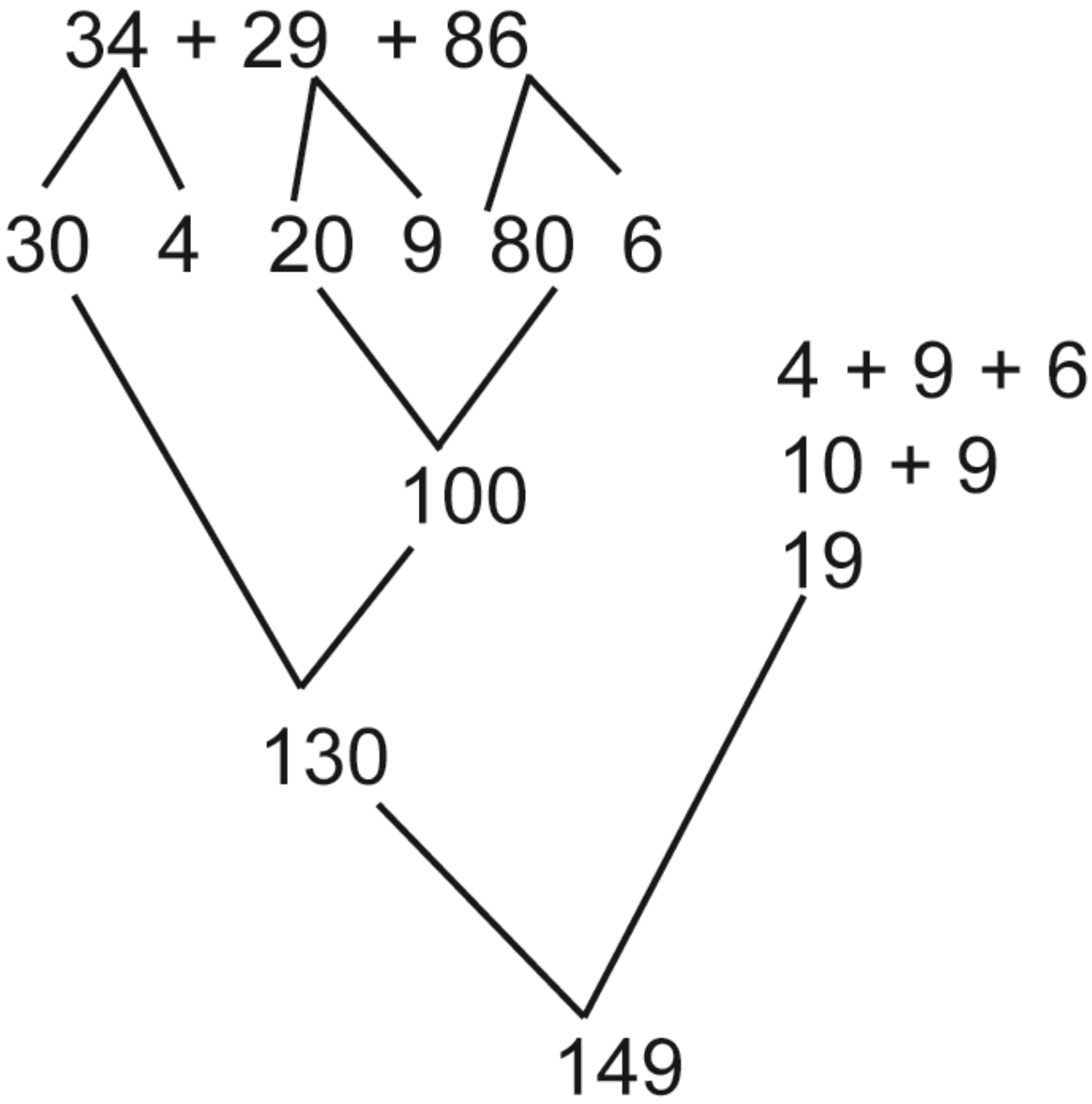

Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.

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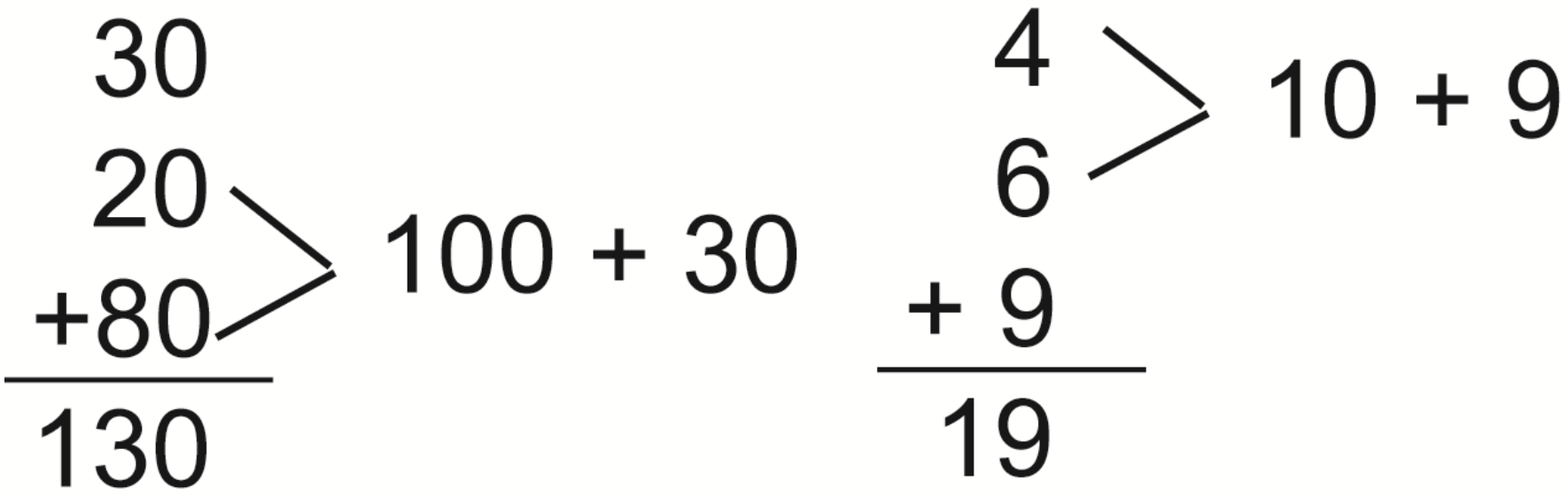

130 + 19 = 149

Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.

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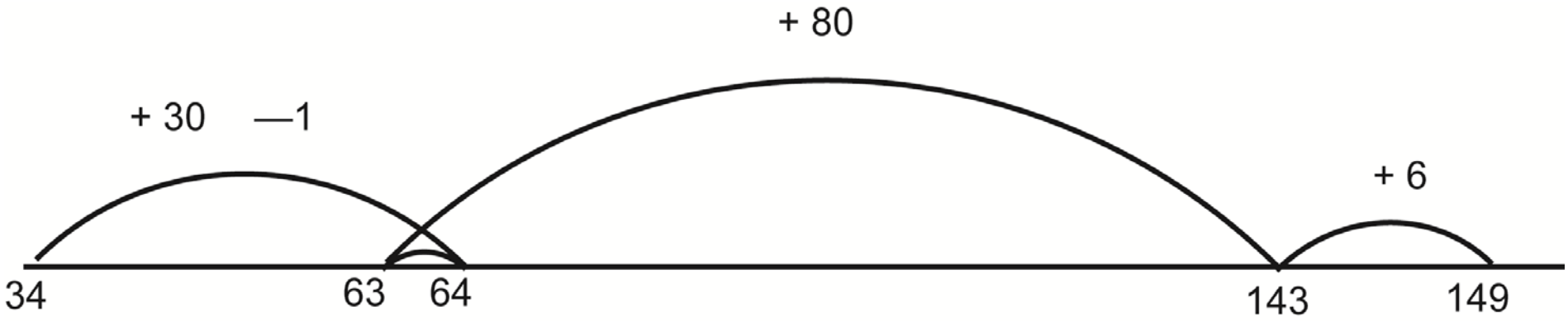

Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.

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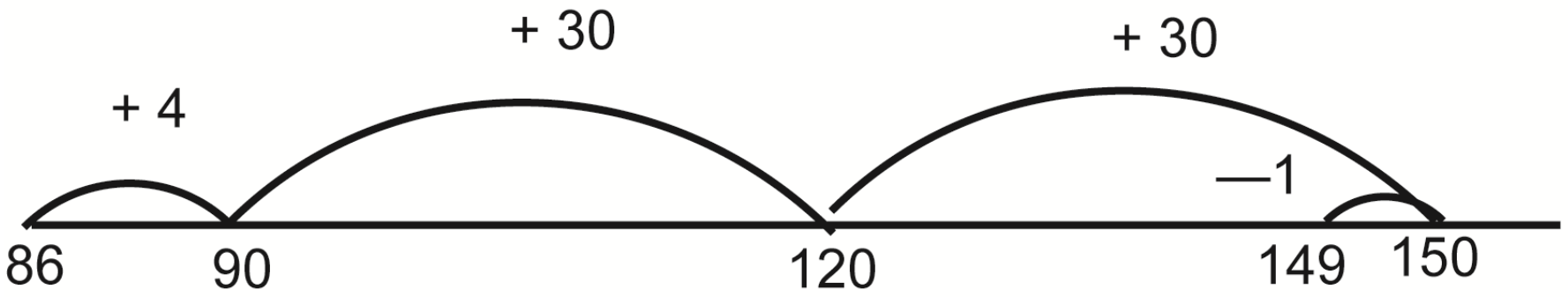

Grade 2 Fluency Activity - 2(4)(B) The student is expected to add up to four two-digit numbers and subtract two-digit numbers using mental strategies and algorithms based on knowledge of place value and properties of operations.

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Name: _____ Date: _____

Representing Fractions

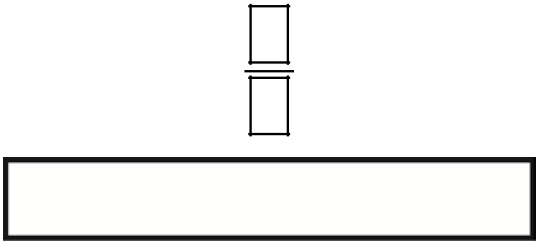
Use the example below to complete the remaining items.

Example:

$\frac{3}{4}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

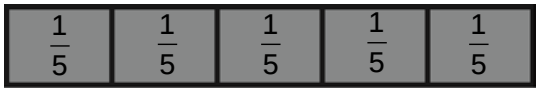
2



$\frac{1}{3} + \frac{1}{3} = \frac{\square}{\square}$

1

$\frac{\square}{\square}$



$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5} = \frac{\square}{\square} = 1$

3

$\frac{\square}{\square}$

$\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square} = \frac{\square}{\square}$

What patterns did you notice?

Grade 03 Fluency Activity

3(3)(D)- *The student is expected to compose and decompose a fraction* $\frac{a}{b}$ *with a numerator greater than zero and less than or equal to b as a sum of parts* $\frac{1}{b}$.

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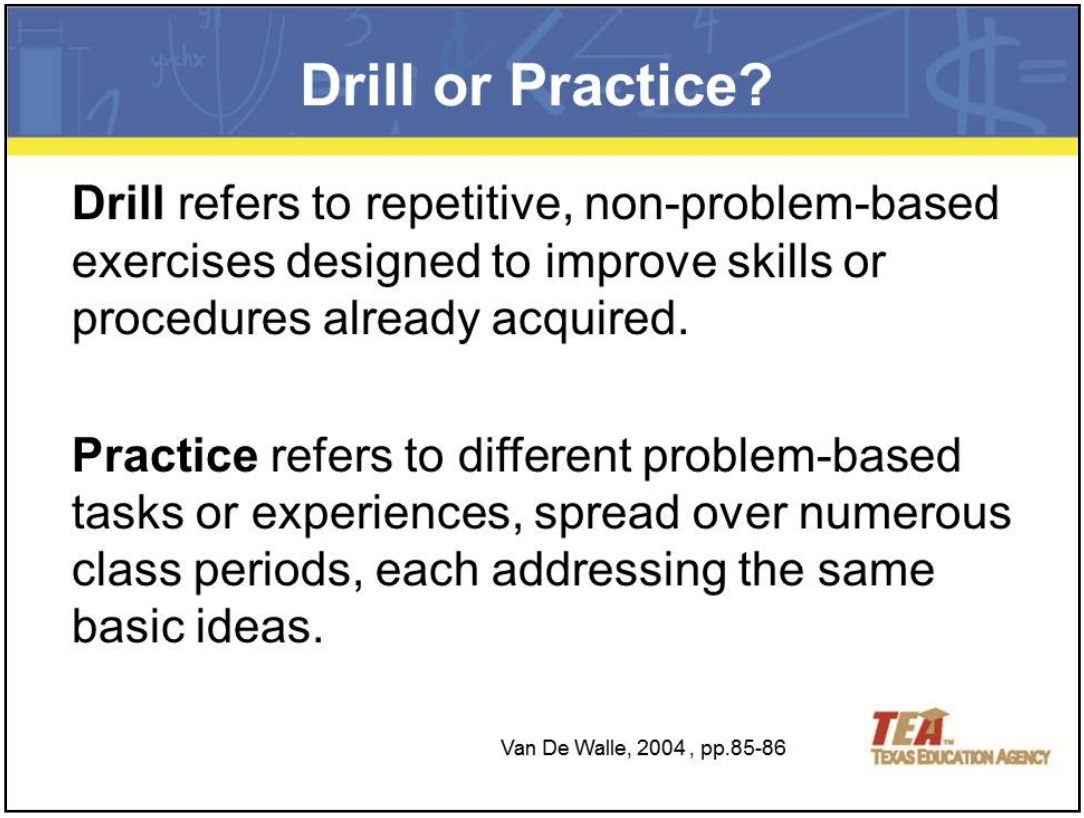

Notes:

Van De Walle, J. (2004). *Elementary and Middle School Mathematics*. Boston: Pearson.

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Case Study Recording Sheet

Examine the case study documents provided for each student. What evidence do you see for each of the categories?

Student A			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

Student B			
Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency

What additional evidence would you like to gather?

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Case Study Student A

Work Sample

Miss Jackson spent $163 at the grocery store on Monday. On Wednesday, Miss Jackson spent $15 less at the grocery store than she did on Monday. How much money did Miss Jackson spend at the grocery store on Monday and Wednesday? Explain your thinking.

$$\begin{array}{r} {\scriptstyle 5\ 13} \\ 1\cancel{6}\cancel{3} \\ -\ \ 15 \\ \hline 148 \end{array}$$

Miss Jackson had $163 then She had $15. $163 minus $15 is $148.

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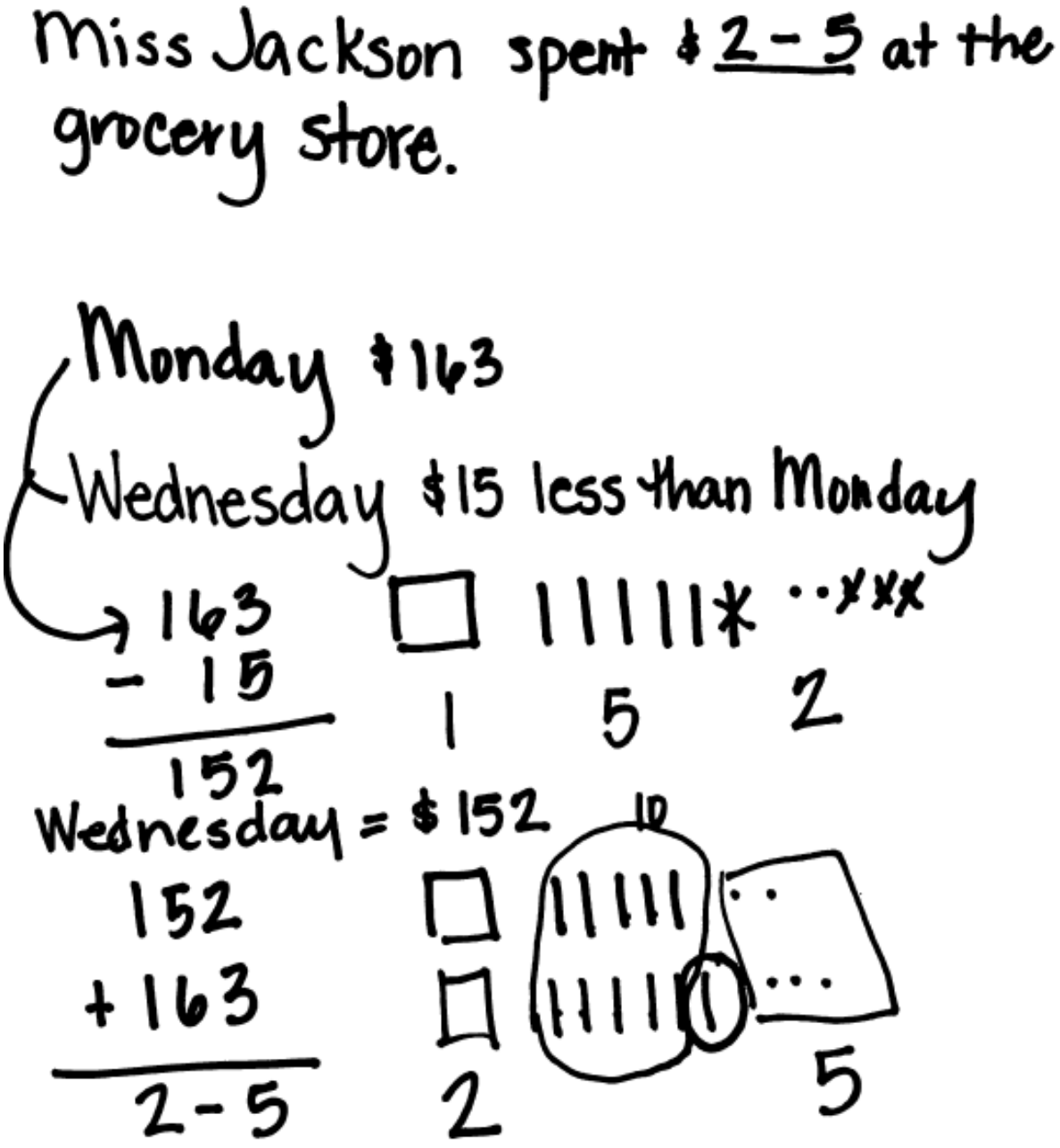
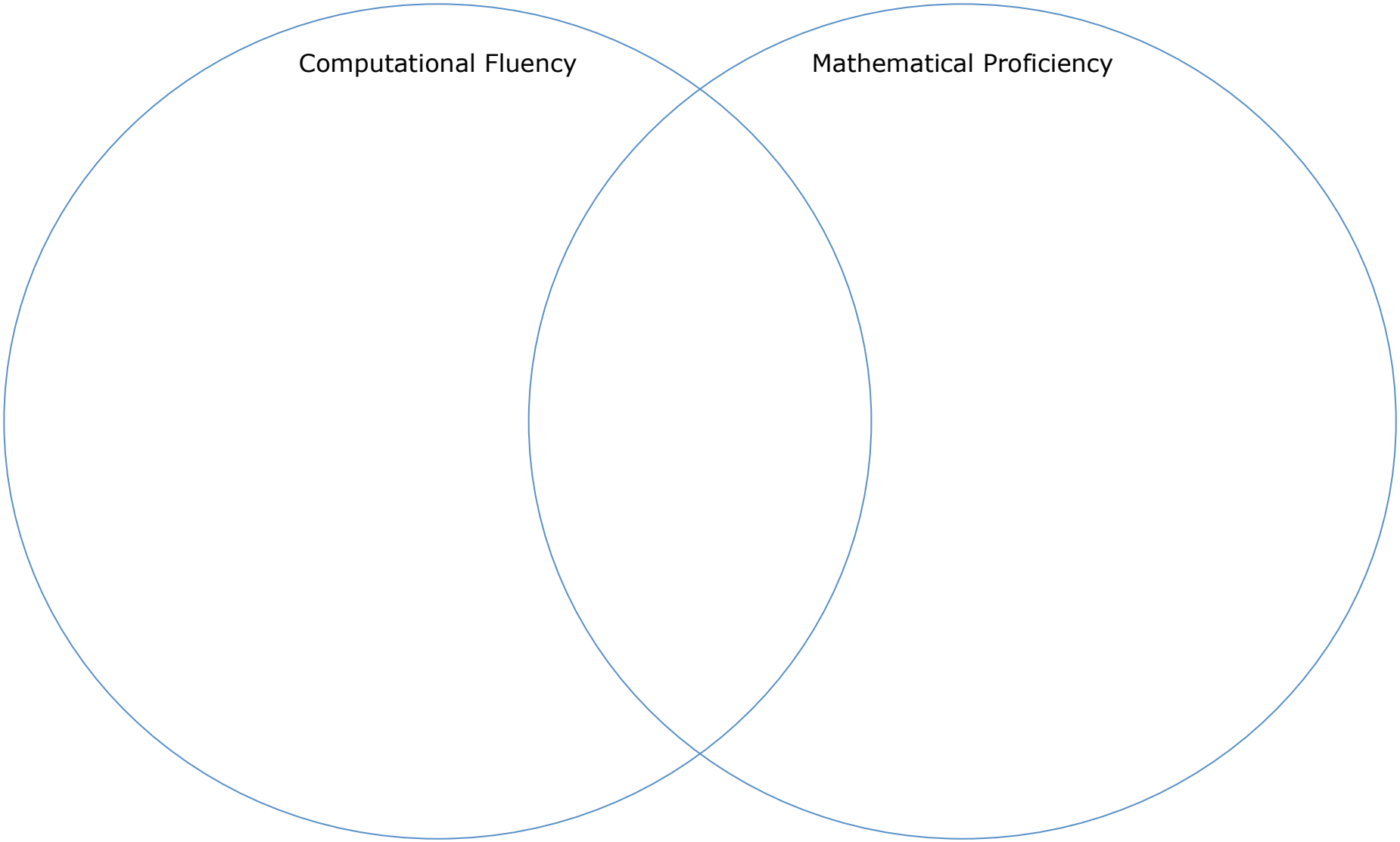
Case Study Student B

Work Sample

Miss Jackson spent $163 at the grocery store on Monday. On Wednesday, Miss Jackson spent $15 less at the grocery store than she did on Monday. How much money did Miss Jackson spend at the grocery store on Monday and Wednesday? Explain your thinking.



Reflection



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Vertical Learning Progression Recording Sheet Possible Progression Grades K - 8

	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Whole Numbers	*K(2)(A) Count forward and backward to 20 w/wo objects* *K(2)(F) Generate a number one more or one less* *K(2)(I) Compose and decompose number up to 10 with objects and pictures* *K(3)(A) Model the action of joining and separating* *1(3)(A) Use concrete and pictorial models to determine the sum of a multiple of 10 and a one-digit number* *1(3)(B) Use objects and pictorial models to solve word problems* *1(3)(C) Compose 10 with two or more addends with and without concrete objects* *1(5)(D) Represent word problems involving addition and subtraction using concrete and pictorial models and number sentences* *3(5)(A) Represent one- and two-step addition and subtraction problems using pictorial models, number lines, and equations* *4(5)(A) Represent multi-step problems involving the four operations using strip diagrams and equations with a letter standing for the unknown quantity*	*2(4)(A) Recall basic facts with automaticity*	*1(3)(D) Apply basic fact strategies to add and subtract including making 10 and decomposing a number leading to a 10* *1(5)(F) Determine the unknown whole number in an addition or subtraction equation* *1(5)(G) Apply properties of operations to add and subtract two or three numbers* *2(4)(B) Add and subtract using mental strategies and algorithms* *3(4)(B) round or use compatible numbers to estimate solutions* *4(4)(A) Add and subtract using the standard algorithm*	*K(3)(C) Explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences* *K(3)(B) Solve word problems using objects and drawings* *1(3)(E) Explain the strategies used to solve problems using spoken words, concrete and pictorial models, and number sentences* *1(3)(F) Generate and solve problem situations when given a number sentence* *2(4)(C) Solve one-step and multi-step addition and subtraction problems using a variety of strategies* *2(7)(C) Represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem* *2(10)(C) Write and solve one-step addition and subtraction word problems using data represented with pictographs and bar graphs* *3(4)(A) Solve multi-step addition and subtraction problems using various strategies* *3(8)(B) Solve problems using data represented with a frequency table, dot plot, pictograph, or bar graph* *4(9)(B) Solve problems using data in a frequency table, dot plot, or stem-and-leaf plot* *5(3)(A) Estimate to determine solutions to mathematical and real-world problems* *5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity*

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	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Whole Numbers	*1(5)(B) Skip count by twos, fives, and tens* *2(6)(A) Model, create, and describe contextual multiplication situations* *2(6)(B) Model, create, and describe contextual division situations* *3(4)(D) Determine the total number of objects when arranged in arrays* *3(4)(E) Represent multiplication facts using a variety of approaches* *3(4)(H) Determine the number of objects in each group when a set of objects is partitioned into equal shares* *3(5)(B) Represent and solve one- and two-step multiplication and division problems using arrays, strip diagrams, and equations* *3(5)(C) Describe a multiplication expression as a comparison* *4(4)(C) Represent the product of 2 two-digit numbers using arrays, area models, or equations* *4(4)(E) Represent the quotient of up to a four-digit whole number divided by a one-digit whole number using arrays, area models, or equations* *4(5)(A) Represent multi-step problems involving the four operations using strip diagrams and equations with a letter standing for the unknown quantity* *4(5)(C) Use models to determine the formulas for the perimeter and area of a rectangle*	*3(4)(F) Recall multiplication facts with automaticity*	*3(4)(G) Use strategies and algorithms to multiply a two-digit number by a one-digit number* *3(4)(J) Determine a quotient using the relationship between multiplication and division* *3(5)(D) Determine the unknown whole number in a multiplication or division equation when the unknown is either a missing factor or product* *4(4)(B) Determine products of a number and 10 or 100* *4(4)(D) Use strategies and algorithms to multiply four-digit by one-digit or two-digit by two-digit numbers* *4(4)(F) Use strategies and algorithms to divide up to a four-digit dividend by a one-digit divisor* *4(4)(G) Round or use compatible numbers to estimate solutions involving whole numbers*	*3(4)(K) Solve one-step and two-step multiplication and division problems using various strategies* *3(8)(B) Solve one- and two-step problems using data from a frequency table, dot plot, pictograph or bar graph* *4(4)(H) Solve with fluency one- and two-step multiplication and division problems including interpreting remainders* *4(5)(D) Solve problems related to perimeter and area of rectangles* *4(8)(B) Convert measurements within the same measurement system* *4(8)(C) Solve problems that deal with measurement* *4(9)(B) Solve one- and two-step problems using data from a frequency table, dot plot, or stem-and-leaf plot* *5(3)(A) Estimate to determine solutions to mathematical and real-world problems* *5(3)(C) Solve with proficiency for quotients of up to a four-digit dividend by a two-digit divisor* *5(4)(B) Represent and solve multi-step problems using equations with a letter standing for the unknown quantity* *5(7) Solve problems by calculating conversions within a measurement system*

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	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Adding and Subtracting with Rational Numbers	*3(3)(F) Represent equivalent fractions* *4(3)(B) Decompose fractions with models* *4(3)(E) Represent and solve addition and subtraction of fractions with equal denominators using models* *4(3)(F) Evaluate for reasonableness of sums and differences using benchmark fractions* *5(3)(H) Represent and solve for addition/subtraction of fractions using objects* *6(3)(C) Represent integer operations*		*4(4)(A) Add/subtract whole number and decimals using the standard algorithm* *5(3)(K) Add/subtract positive rational numbers fluently* *6(3)(D) Add/subtract integers fluently* *7(3)(A) Add/subtract rational numbers fluently*	*7(3)(B) Solve problems using addition and subtraction of rational numbers*

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	Conceptual Understanding	Automaticity	Computational Fluency	Mathematical Proficiency
Multiplying and Dividing with Rational Numbers	*4(3)(A) Compose and decompose fractions into unit fractions* *5(3)(D)(F) Represent multiplication/division of decimals* *5(3)(I)(J) Represent and solve problems involving multiplication/division of a whole number and a fraction* *6(3)(A) Recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values* *6(3)(B) Determine if a quantity is increased or decreased when multiplied by a fraction* *6(3)(C) Represent integer operations*		*5(3)(E)(G) Solve for products/quotients of decimals* *6(3)(D) Multiply/divide integers fluently* *6(3)(E) Multiply/divide positive rational numbers fluently* *7(3)(A) Multiply/divide rational numbers fluently*	*7(3)(B) Solve problems using multiplication and division of rational numbers*

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