## Mathematics TEKS SUPPORTING INFORMATION

# Advanced Quantitative Reasoning



The materials are copyrighted (c) and trademarked (tm) as the property of the Texas Education Agency (TEA) and may not be reproduced without the express written permission of TEA, except under the following conditions:

- Texas public school districts, charter schools, and Education Service Centers may reproduce and use copies of the Materials and Related Materials for the districts' and schools' educational use without obtaining permission from TEA.
- Residents of the state of Texas may reproduce and use copies of the Materials and Related Materials for individual personal use only without obtaining written permission of TEA.
- Any portion reproduced must be reproduced in its entirety and remain unedited, unaltered and unchanged in any way.
- No monetary charge can be made for the reproduced materials or any document containing them; however, a reasonable charge to cover only the cost of reproduction and distribution may be charged.

Private entities or persons located in Texas that are not Texas public school districts, Texas Education Service Centers, or Texas charter schools or any entity, whether public or private, educational or non-educational, located outside the state of Texas MUST obtain written approval from TEA and will be required to enter into a license agreement that may involve the payment of a licensing fee or a royalty.

For information contact: Office of Copyrights, Trademarks, License Agreements, and Royalties, Texas Education Agency, 1701 N. Congress Ave., Austin, TX 78701-1494; phone: 512-463-9041 email: copyrights@tea.texas.gov

©2016 Texas Education Agency. All Rights Reserved 2016

TEKS         Supporting Information           (a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.         The TEKS include descriptions of prerequisite coursework. Geometry and Algebra II are required prerequisites.           (b) Introduction.         (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.         A well-balanced mathematics future, and solid understanding allows for rich explorat in Advanced Quantitative Reasoning.           (b) Introduction.         (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards weave the other knowledge and skills together so that students may be successful problem solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving model that incorporates analyzing will solve the anorporiate noreal and the reasonalphanese of the solution.         The concept of generalization and abstraction in the text from AQR(1)(B) include introductory paragraphs from elementary TEKS	Readiness tion of the key ideas This highlights the demonstrate	
<ul> <li>(a) General requirements. Students shall be awarded one-half to one credit for successful completion of this course. Prerequisites: Geometry and Algebra II.</li> <li>(b) Introduction.</li> <li>(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.</li> <li>(b) Introduction.</li> <li>(c) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the razeonableness of the solution, and evaluating the problem-solving process and the razeonableness of the solution.</li> </ul>	Readiness tion of the key ideas This highlights the demonstrate	
Prerequisites: Geometry and Algebra II.       Geometry and Algebra II are required prerequisites.         (b) Introduction.       (1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.       A well-balanced mathematics curriculum includes the Texas College and Career Standards.         (b) Introduction.       (2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will age a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the vary free considered subsumed in inroductory paragraphs from elementary TEKS may be considered subsumed in inroductory paragraphs from elementary TEKS may be considered subsumed in inroductory paragraphs from elementary TEKS may be considered subsumed in inroductory paragraphs from elementary TEKS may be considered subsumed in inroductory paragraphs from elementary TEKS may be considered subsumed in inroductory paragraphs from elementary TEKS may be considered subsumed in introductory paragraphs from elementary TEKS may be considered subsumed in introductory paragraphs from elementary TEKS may be considered subsumed in introduc	Readiness tion of the key ideas Fhis highlights the demonstrate	
<ul> <li>(b) Introduction.</li> <li>(1) The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.</li> <li>(b) Introduction.</li> <li>(c) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills logether so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving model that incorporates of the solution, formulating the problem-solving model that incorporates of the solution, formulating the problem-solving model that incorporates onalyzing given information, formulating the problem-solving model that incorporates of the solution.</li> </ul>	Readiness tion of the key ideas Fhis highlights the demonstrate	
<ul> <li>(b) Introduction.</li> <li>(c) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use arising in every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process</li> <li>(b) Introduction.</li> <li>(c) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards are integrated at every grade level and course. When possible, students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process</li> <li>(c) The process standards describe ways in which students will select appropriate tools second be reasonable problem. Solving process</li> <li>(b) Introduction.</li> <li>(c) The process standards describe ways in which students are expected to engage in the content of the process standards weave the other model and course. When possible, students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process areal.</li> </ul>	tion of the key ideas	
<ul> <li>(b) Introduction.</li> <li>(2) The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process of the solution. Students will splet a appropriate tools such as real-</li> </ul>	Fhis highlights the demonstrate	
objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.	ed in the 1 this language. ogy. os to generate 1)(E).	
<ul> <li>(b) Introduction.</li> <li>(3) In Advanced Quantitative Reasoning, students will develop and apply skills necessary for college, careers, and life. Course content consists primarily of applications of high school mathematics concepts to prepare students to become well-educated and highly informed 21st century citizens. Students will develop and apply reasoning, planning, and communication to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.</li> </ul>	arized in this s standards. This e mathematical the key concepts ge and Career	
(b) Introduction. (b) Introduction.	ents within the TEKS	
(4) Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples. The phrases "including" and "such as" should not be considered as limiting facto expectations (SEs) in which they reside.	ors for the student	
Additional Resources are available online including		
Vertical Alignment Charts		
Texas Mathematics Resource Page		
Toyac College and Career Readiness Standards		

1

Advanced Quantitative Reasoning - Mathematics			
TEKS: Mathematical Process Standards.	Supporting Information		
AQR(1)(A) <b>Mathematical process standards.</b> The student uses mathematical processes to acquire and demonstrate mathematical understanding	This SE emphasizes application. The opportunities for application have been consolidated into three areas: everyday life, society, and the workplace.		
The student is expected to apply mathematics to problems arising in everyday life, society, and the workplace.	This SE, when paired with a content SE, allows for increased relevance through connections within and outside mathematics. <i>Example</i> : When paired with AQR(2)(D), students may be expected to solve geometric problems such as those encountered when building a bridge, constructing a skyscraper, or mapping planetary distances.		
AQR(1)(B) <b>Mathematical process standards.</b> The student uses mathematical processes to acquire and demonstrate mathematical understanding.	This process standard applies the same problem-solving model and is included in the TEKS for kindergarten through grade 12.		
The student is expected to use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.	This is the traditional problem-solving process used in mathematics and science. Students may be expected to use this process in a grade appropriate manner when solving problems that can be considered difficult relative to mathematical maturity.		
AQR(1)(C) <b>Mathematical process standards.</b> The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.	The phrase "as appropriate" indicates that students are assessing which tools and techniques to apply rather than trying only one or all of those listed. <i>Example:</i> When paired with AQR(3)(A), the student may be expected to estimate a prediction based upon the scatterplot or other model.		
AQR(1)(D) <b>Mathematical process standards.</b> The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.	<ul> <li>Students may be expected to address three areas: mathematical ideas, reasoning, and implications of these ideas and reasoning.</li> <li>Communication can be through the use of symbols, diagrams, graphs, or language. The phrase "as appropriate" implies that students may be expected to assess which communication tool to apply rather than trying only one or all of those listed.</li> <li>The use of multiple representations includes translating and making connections among the representations. <i>Example</i>: When paired with AQR(4)(R), students may justify the selected procentation format given the prode of a particular audiored.</li> </ul>		
AQR(1)(E) <b>Mathematical process standards.</b> The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to create and use representations to organize, record, and communicate mathematical ideas.	The expectation is that students use representations for three purposes: to organize, record, and communicate mathematical ideas. Representations include verbal, graphical, tabular, and algebraic representations. As students create and use representations, the students will evaluate the effectiveness of the representations to ensure that those representations are communicating mathematical ideas with clarity. <i>Example</i> : When paired with AQR(4)(M), students may be expected to collect and organize data from a population of interest.		
AQR(1)(F) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to analyze mathematical relationships to connect and communicate mathematical ideas.	Students may be expected to analyze relationships and form connections with mathematical ideas. Students may form conjectures about mathematical representations based on patterns or sets of examples and non-examples. Forming connections with mathematical ideas extends past conjecturing to include verification through a deductive process. <i>Example</i> : When paired with AQR(3)(G), students may be expected to look for and analyze various models for expenditures to determine a course of action.		
AQR(1)(G) <b>Mathematical process standards.</b> The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.	The expectation is that students speak and write with precise mathematical language to explain and justify the work. This includes justifying a solution. <i>Example</i> : When paired with AQR(4)(F), the student may be expected to explain in precise mathematical language why a given event can be considered to be mathematically fair.		

#### **TEKS: Numeric reasoning.** Supporting Information AQR(2)(A) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday The last student expectation involving the direct measuring of length is 2(9)(D) and (E), where life, society, and the workplace. The student extends existing knowledge and skills to analyze students are expected to measure to the nearest marked unit using rulers, yardsticks, meter real-world situations. sticks, or measuring tape. The student is expected to use precision and accuracy in real-life situations related to measurement and significant figures. AQR(2)(B) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday Students may be expected to compare and contrast attributes of data accuracy and mathematical life, society, and the workplace. The student extends existing knowledge and skills to analyze applications of data to analyze published ratings. real-world situations. Published ratings may include ratios, rates, and averages. The student is expected to apply and analyze published ratings, weighted averages, and indices to make informed decisions. AQR(2)(C) **Numeric reasoning**. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday Problems involving quantities that are not easily measured may include large or small quantities. life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations. Specificity includes the use of proportionality to solve problems. The student is expected to solve problems involving quantities that are not easily measured using proportionality. Authentic situations should involve indirect measurement. AOR(2)(D) **Numeric reasoning**. The student applies the process standards in mathematics to Specificity for paper techniques includes proportions with similar triangles, the Pythagorean generate new understandings by extending existing knowledge. The student generates new Theorem, Law of Sines, and Law of Cosines. mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze Specificity includes the mathematics that students may be expected to apply when solving real-world situations. geometric problems to include proportions with similar triangles, the Pythagorean Theorem, Law of Sines, and Law of Cosines, and the tool that students may be expected to use related to The student is expected to solve geometric problems involving indirect measurement, dvnamic geometry software. including similar triangles, the Pythagorean Theorem, Law of Sines, Law of Cosines, and the use of dynamic geometry software. When paired with AQR(1)(A), students may be expected to solve geometric problems such as those encountered when building a bridge, constructing a skyscraper, or mapping planetary distances. AQR(2)(E) Numeric reasoning. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze This SE builds on combinatorics introduced in Geometry in G(13)(A). real-world situations.

combinatorics.

The student is expected to solve problems involving large quantities using

Advanced Quantitative Reasoning - Mathematics

#### Advanced Quantitative Reasoning - Mathematics

#### **TEKS: Numeric reasoning.**

AQR(2)(F) **Numeric reasoning**. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations.

## The student is expected to use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems, including geometric transformations.

AQR(2)(G) **Numeric reasoning**. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations.

## The student is expected to analyze various voting and selection processes to compare results in given situations.

AQR(2)(H) **Numeric reasoning**. The student applies the process standards in mathematics to generate new understandings by extending existing knowledge. The student generates new mathematical understandings through problems involving numerical data that arise in everyday life, society, and the workplace. The student extends existing knowledge and skills to analyze real-world situations.

The student is expected to select and apply an algorithm of interest to solve real-life problems such as problems using recursion or iteration involving population growth or decline, fractals, and compound interest; the validity in recorded and transmitted data using checksums and hashing; sports rankings, weighted class rankings, and search engine rankings; and problems involving scheduling or routing situations using vertex-edge graphs, critical paths, Euler paths, and minimal spanning trees and communicate to peers the application of the algorithm in precise mathematical and nontechnical language.

#### Supporting Information

Specificity includes the use of matrices as a means to insure the efficient management of large collections of data.

Students may be expected to add, subtract, and multiply matrices when solving applied problems.

Specificity includes problems involving geometric transformations.

When paired with AQR(1)(A), given situations may include real-world problems.

Students may be expected to compare different processes in an effort to analyze the given situation. The emphasis is on the comparison of results.

Students may be expected to select and apply an algorithm of interest.

This complements AQR(1)(D) and (G) with the inclusion of nontechnical language.

The intended audience for this communication is identified as peers who may or may not be classmates.

Advanced Quantitative Reasoning - Mathematics	
TEKS: Algebraic reasoning (expressions, equations, and generalized relationships).	Supporting Information
AQR(3)(A) <b>Algebraic reasoning (expressions, equations, and generalized relationships)</b> . The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve	Students are first introduced to scatterplots in grade 5 [5(9)(B)]. Students fit lines, parabolas, and exponential curves to data in Algebra I [A(4)(C), A(8)(B), and A(9)(E)].
The student is expected to collect numerical bivariate data to create a scatterplot, select a function to model the data, justify the model selection, and use the model to interpret results and make predictions.	Students are expected to select the model of best fit in Algebra II [2A(8)(A), (B), and (C)].
AQR(3)(B) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to describe the degree to which uncorrelated variables may or may not be related and analyze situations where correlated variables do or do not indicate a cause-and-effect relationship.	This SE builds on the relationship between association and causation in Algebra I [A(4)(B)].
AQR(3)(C) <b>Algebraic reasoning (expressions, equations, and generalized relationships)</b> . The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems.	Students may be expected to determine an appropriate growth or decay model or to analyze a given model for growth or decay for a problem situation. This is the introduction of the logistic function.
for problem situations, including linear, exponential, and logistic functions.	
AQR(3)(D) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems.	Students may be expected to determine an appropriate cyclical model or to analyze a given cyclical model for a problem situation. Periodic functions may include cyclical models that are not trigonometric functions.
The student is expected to determine or analyze an appropriate cyclical model for problem situations that can be modeled with periodic functions.	
AQR(3)(E) <b>Algebraic reasoning (expressions, equations, and generalized relationships)</b> . The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems.	Students may be expected to determine an appropriate piecewise model or to analyze a given piecewise model for a problem situation.
The student is expected to determine or analyze an appropriate piece-wise model for problem situations.	

Advanced Quantitative Reasoning - Mathematics	
TEKS: Algebraic reasoning (expressions, equations, and generalized relationships).	Supporting Information
AQR(3)(F) <b>Algebraic reasoning (expressions, equations, and generalized relationships)</b> . The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems.	Students may be expected to create or generate, rather than determine or identify, mathematical models for income calculations. Specificity includes the purpose of representing and analyzing models. The purpose is to determine the best option for a given situation.
The student is expected to create, represent, and analyze mathematical models for various types of income calculations to determine the best option for a given situation.	
AQR(3)(G) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to create, represent, and analyze mathematical models for expenditures, including those involving credit, to determine the best option for a given situation.	Students may be expected to create or generate, rather than determine or identify, mathematical models for expenditures. Specificity includes the purpose of representing and analyzing models. The purpose is to determine the best option for a given situation.
AQR(3)(H) Algebraic reasoning (expressions, equations, and generalized relationships). The student applies the process standards in mathematics to create and analyze mathematical models of everyday situations to make informed decisions related to earning, investing, spending, and borrowing money by appropriate, proficient, and efficient use of tools, including technology. The student uses mathematical relationships to make connections and predictions. The student judges the validity of a prediction and uses mathematical models to represent, analyze, and solve dynamic real-world problems. The student is expected to create, represent, and analyze mathematical models and appropriate representations, including formulas and amortization tables, for various types of loans and investments to determine the best option for a given situation	Students may be expected to create or generate, rather than determine or identify, mathematical models for loans and investments. Mathematical models may include formulas and amortization tables. Specificity includes the purpose of representing and analyzing models. The purpose is to determine the best option for a given situation.

### mathematical and real-world problems.

the critical analysis of published statistical studies.

AQR(4)(C) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to calculate conditional probabilities and probabilities of compound events using tree diagrams, Venn diagrams, area models, and formulas.

AQR(4)(D) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to interpret-conditional probabilities and probabilities of compound events by analyzing representations to make decisions in problem situations.

AQR(4)(E) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

The student is expected to use probabilities to make and justify decisions about risks in everyday life.

#### Supporting Information

Mathematics TEKS: Supporting Information

Students may be expected to analyze the relationship between two events using a two-way frequency table as part of determining probabilities of compound events that may reflect conditional probabilities. A two-way frequency table, such as the one shown below, may be used to compare probabilities where voters were polled as to how they might vote on a bill related to fines for speeding tickets and whether or not they have received a speeding ticket.

	Vote: Yes	Vote No:	Undecided	Total
Has received a				
Has not received				
a ticket Unwilling to				
disclose				
Total				

The addition rule is sometimes stated as  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  where A and B are subsets of potential outcomes.

The use of formulas to calculate conditional probabilities and probabilities of compound events is included.

Conditional probabilities are introduced in Geometry [G(13)(D)].

This SE focuses on the day-to-day use of probability and may lead to actuarial decisions.

#### Advanced Quantitative Reasoning - Mathematics

#### **TEKS:** Probability and statistical reasoning.

AQR(4)(A) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

The student is expected to use a two-way frequency table as a sample space to identify

AQR(4)(B) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple

representations to communicate effectively the results of student-generated statistical studies and

The student is expected use the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), in

whether two events are independent and to interpret the results.

#### Advanced Quantitative Reasoning - Mathematics

#### TEKS: Probability and statistical reasoning.

AQR(4)(F) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to calculate expected value to analyze mathematical fairness, payoff, and risk.

AQR(4)(G) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to determine the validity of logical arguments that include compound conditional statements by constructing truth tables.

AQR(4)(H) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to identify limitations and lack of relevant information in studies reporting statistical information, especially when studies are reported in condensed form.

AQR(4)(I) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to interpret and compare statistical results using appropriate technology given a margin of error.

AQR(4)(J) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

The student is expected to identify potential misuses of statistics to justify particular conclusions, including assertions of a cause-and-effect relationship rather than an association, and missteps or fallacies in logical reasoning.

#### Supporting Information

This is the students' introduction to expected value, mathematical fairness, payoff, and risk.

Mathematical fairness is an expected value of zero.

This SE builds on the Geometry skills [G(4)(B) and (C)].

This is the first appearance of truth tables in the TEKS.

When paired with AQR(1)(A), students may be expected to identify limitations and the lack of relevant information in a published study.

When paired with AQR(1)(C), students may be expected to use technology to interpret and compare statistical results.

Students may be expected to identify the misuse of association, which is sometimes confused with the cause-and-effect relationship. Students may also be expected to identify the misuse of logical reasoning by applying missteps or producing fallacies.

Advanced Quantitative Reasoning - Mathematics	
TEKS: Probability and statistical reasoning.	Supporting Information
AQR(4)(K) <b>Probabilistic and statistical reasoning</b> . The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media.	Results that are published in the media are included.
AQR(4)(L) <b>Probabilistic and statistical reasoning</b> . The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions.	Students may be expected to determine whether a statistical investigation is needed based on the posed question or set of questions. Students may be expected to determine the purpose, or outcome, of a statistical investigation in order to determine the appropriate statistical analysis.
AQR(4)(M) <b>Probabilistic and statistical reasoning</b> . The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to identify the population of interest for a statistical investigation, select an appropriate sampling technique, and collect data.	Specificity has been provided as to the type of investigation for which students may be expected to identify the population of interest.
AQR(4)(N) <b>Probabilistic and statistical reasoning</b> . The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.  The student is expected to identify the variables to be used in a study.  AOR(4)(O) Probabilistic and statistical reasoning. The student uses the process standards in	When paired with AQR(1)(G), students may be expected to justify their selection of these variables.
mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies. The student is expected to determine possible sources of statistical bias in a study and how bias may affect the validity of the results.	In determining validity, students may be expected to consider whether the study produces generalizable results.

9

#### Advanced Quantitative Reasoning - Mathematics

#### TEKS: Probability and statistical reasoning.

AQR(4)(P) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to create data displays for given data sets to investigate, compare, and estimate center, shape, spread, and unusual features of the data.

AQR(4)(Q) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to analyze possible sources of data variability, including those that can be controlled and those that cannot be controlled.

AQR(4)(R) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied.

AQR(4)(S) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

## The student is expected to justify the design and the conclusion(s) of statistical studies, including the methods used.

AQR(4)(T) **Probabilistic and statistical reasoning**. The student uses the process standards in mathematics to generate new understandings of probability and statistics. The student analyzes statistical information and evaluates risk and return to connect mathematical ideas and make informed decisions. The student applies a problem-solving model and statistical methods to design and conduct a study that addresses one or more particular question(s). The student uses multiple representations to communicate effectively the results of student-generated statistical studies and the critical analysis of published statistical studies.

The student is expected to communicate statistical results in oral and written formats using appropriate statistical and nontechnical language.

#### Supporting Information

Unusual features of data may include the existence of outliers and skewed data distributions.

This SE builds on 6(13)(B) with the addition of the concept of control.

When paired with AQR(1)(G), students may be expected to justify the selected presentation format given the particular audience.

When paired with AQR(1)(D) and (E), students may be expected to use various representations to support the reasoning for design and conclusions choices.

Specificity has been added to include both statistical and nontechnical language when communicating results.